

## SSNV177 - Test of Willam with model ENDO\_ORTH\_BETON

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### Summarized:

One presents here the test of Willam applied to the anisotropic constitutive law unilateral dedicated to the behavior of concrete ENDO\_ORTH\_BETON (cf [R7.01.09]) developed in [Feeding-bottle 1]. It is about a test of tension-shears which make it possible in the case of to observe the rotation of the clean reference of the damage a loading nonproportional.

## 1 Presentation of the test of Willam

the purpose of the test suggested by Willam and al. (cf [Feeding-bottle 2]) is to observe the response of the model when the clean reference of the loading turns. This test is purely theoretical and it is not very probable that experimental results are one day available for this test, taking into account as of difficulties dependant under investigation the fracture the concrete in tension. Its major interest is to compare the influence of the description of the anisotropy of the damage on the response of the material compared to the isotropic models.

This test is the succession of two phases of loading:

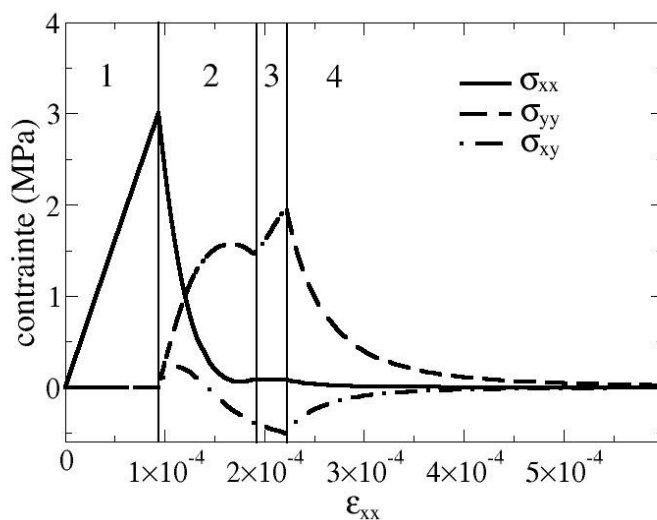
- 1) First phase: simple tension. One imposes  $\Delta \varepsilon_{xx}$  until nonthe linearity, i.e. the beginning of the damage of tension.
- 2) Second phase: on the basis of the strain state at the end of the first phase, one imposes two loadings of tension and a loading of shears in the loading plan of tension with the following proportionality factors:

$$(\Delta \varepsilon_{xx}, \Delta \varepsilon_{yy}, \Delta \varepsilon_{xy}) = (0.5, 0.75, 0.5) .$$

Appear 1-a shows us the evolution of the various components of the stress according to the strain  $\varepsilon_{xx}$ .

One can distinguish 4 phases in the behavior:

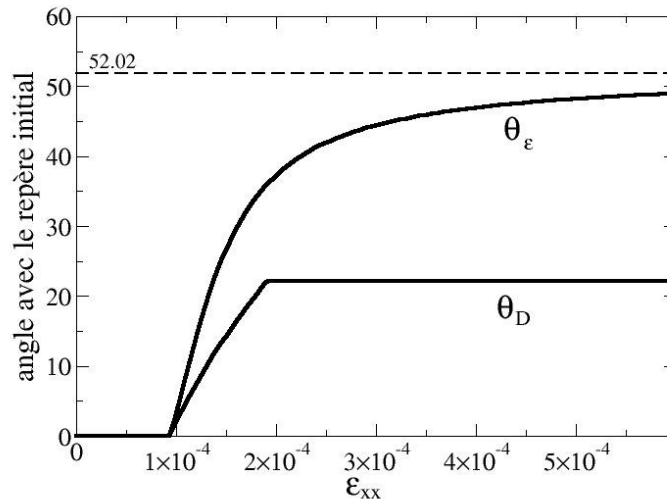
- 1) Phase 1 is the elastic phase under uniaxial loading.
- 2) Phase 2 corresponds to the growth of the damage in a particular direction. The stress  $\sigma_{xx}$  decrease insofar as the damage is initiated first of all according to the axis  $x$ . The stress  $\sigma_{yy}$  does not grow linearly, which indicates that the clean reference of the damage turns.
- 3) Phase 3 begins when the damage is total in a direction. The clean reference of the damage is then blocked as one sees it on Appear 1-b which shows the evolution of the angle between the clean reference of the damage and the initial reference (as well as the angle between the clean reference of the strains and the initial reference). One observes during this phase 3 an elastic behavior.
- 4) Phase 4 begins then when the damage in the orthogonal direction with the blocked direction is initiated. One observes a softening of the stresses then generated by the growth of the damage.



Appear 1-a Evolution of the stresses for the test of Willam

One notices moreover on Appear 1-b which the clean reference of the damage does not turn in the same way as the clean reference of the strains. This comes owing to the fact that the law of evolution

of the damage is not written according to the strains but according to the thermodynamic forces, which depend at the same time on the strain state and the damage. Insofar as the test of Willam is a theoretical test, it is difficult for us to make a precise assessment on this prediction of model ENDO\_ORTH\_BETON.



### Appear 1-b Evolution of the angles of the clean references of the strains and damage compared to the initial reference

the behavior obtained with our model for the test of Willam seems characteristic of until one can wait of the anisotropic models (cf [Feeding-bottle 3]). The principal difference, compared to the results of Carol [1999], lies in the blocking of the clean reference which one the model observes with ENDO\_ORTH\_BETON when the damage is complete in a direction, which does not appear in [Feeding-bottle 3].

## 2 Problem of reference

### 2.1 Geometry and boundary conditions

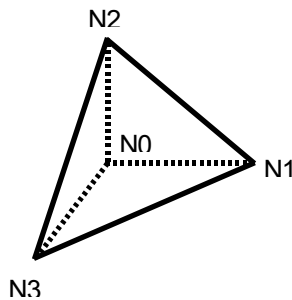
the element used is a tetrahedron at a point of gauss. There is thus no problem of homogeneity of the fields in the element.

The conditions of linear blockings and the relations between the nodes which should be applied are summarized on [Figure 3]. The edges  $N0N1$ ,  $N0N2$  and  $N0N3$  are length 1.

Taking into account the geometry of the element, conditions of blockings and relations linear, the strain is directly connected to displacements of the nodes:

$$\begin{aligned}\varepsilon_{xx} &= DX(N1) \\ \varepsilon_{yy} &= DY(N2) \\ \varepsilon_{zz} &= DZ(N3) \\ \varepsilon_{xy} &= DX(N2) = DY(N1) \\ \varepsilon_{xz} &= DX(N3) = DZ(N1) \\ \varepsilon_{yz} &= DY(N3) = DZ(N2)\end{aligned}$$

A imposed strain, it is thus enough to impose displacements on the adequate nodes.



Blockings :

$$N0 : \\ DX = DY = DZ = 0$$

Linear relations :

$$\begin{aligned}DY(N1) &= DX(N2) \\ DZ(N1) &= DX(N3) \\ DZ(N2) &= DY(N3)\end{aligned}$$

Loadings :

Phase 1: Tension in imposed displacement  
 $DX = F^{trac}$  imposed on  $N1$

Phase 2: Tension/Shears in imposed displacement

$$\begin{aligned}DX = DY &= 0.5 * F^{cisa} \text{ imposed on } N1 \\ DY &= 0.75 * F^{cisa} \text{ imposed on } N2\end{aligned}$$

Where  $F^{trac}$  and  $F^{cisa}$  is functions closely connected increasing of time

Appears 2.1-a Geometry, boundary conditions and loadings of the test of Willam

### 2.2 Material properties

the characteristic materials are identical for the 5 tests which are presented.

The elastic characteristics of the materials are the following ones:

$$E = 32000 \text{ MPa} ; \nu = 0.2$$

One uses the set of parameters following for the constitutive law:

ALPHA	K0 ( Mpa )	ECROB ( MJ/m <sup>3</sup> )	ECROD ( MJ/m <sup>3</sup> )	K <sub>1</sub> ( Mpa )	K <sub>2</sub>
0.87	2.634e-4	0	0.06	10.5	6.e-4

## 3 Reference solution

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This test is a test of non regression.

### 3.1 Bibliographical references

- [1] V. GODARD: Modelization of the anisotropic damage of the concrete with taking into account of the unilateral effect: Application to the computational simulation of the containment systems. Thesis of the University Paris VI, 2005.
- [2] K. WILLAM, E. PRAMONO, S. STURE: Fundamental exits of smeared ace models. Proc. Of the SEM-RILEM Int. Conf. One Fractures of Concrete and Rock'n'roll, Shah S.P., Schwartz S.E. (eds), Society of Engineering Mechanics, p. 193-207, 1987.
- [3] I. CAROL: Anisotropic ramming evolution using has pseudo-logarithmic tensor missed. Mechanics of the heterogeneous materials, Grenoble, 1999.

## 4 Modelization A

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### 4.1 Characteristic of the modelization

Modelization 3D

Element MECA\_TETRA4.

### 4.2 Characteristics of the mesh

Many nodes: 4

Number of meshes and types: 1 TETRA4

### 4.3 Way of loading

the loading breaks up into two phases:

- Phase 1: Tension in imposed displacement  
 $DX = F^{trac}$  imposed on  $N1$
- Phase 2: Tension/Shears in imposed displacement  
 $DX = DY = 0.5 * F^{cisa}$  imposed on  $N1$   
 $DY = 0.75 * F^{cisa}$  imposed on  $N2$

where  $F^{trac}$  and  $F^{cisa}$  is functions closely connected increasing of time

### 4.4 Quantities tested and results

the test of non regression is carried out on the value of the swing angles of the clean references of the strain and the tensor of damage.

For that, one extracts the strain fields (EPSI\_ELGA) and from damage (VARI\_ELGA) at time 2, and one creates the matrixes (in python) corresponding to the strain tensors and of damage. Then, one to compute: uses the LinearAlgebra library of python the eigenvectors of the matrixes associated with the strain tensors and of damage. Lastly, one calculates the swing angle of these eigenvectors compared to the initial reference.

Urgent	Name of the field	Component	Place	Aster
2	EPSI_ELGA	Angle swing of the clean reference	VOLUME , point 1	45.7160
2	VARI_ELGA	Angle swing of the clean reference	VOLUME , point 1	23.0825

## 5 Summary of the results

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the test of Willam is a simple test making it possible to compare the influence of the description of the anisotropy of the damage of model ENDO\_ORTH\_BETON on the response of the material compared to the isotropic models. It makes it possible moreover to observe the response of the model when the clean reference of the loading turns. One thus notices on the values tested, that the damage does not turn in the same way as the strain tensor. Moreover, the swing angle of the tensor of damage reaches a plate when the damage is complete in a direction, which one can associate with the creation of a macro-crack.