SSNV168 – Triaxial compression test drained with a softening `behavior` DRUCK_PRAGER

Summarized:

This case test makes it possible to during simulate a triaxial compression test drained on four different modelizations a nonlinear computation. That makes it possible to propose the effect of type of the negative, parabolic or linear hardening, in the case of model `AXIS` or `3D`.

Modelization a:
- models of type “ `DRUCK_PRAGER` ” with linear negative hardening for a containment of $2\,MPa$.
- model `AXIS` with meshes `QUAD4` .

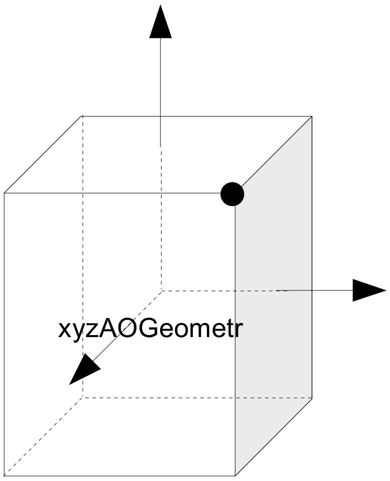
Modelization b:
- models of type “ `DRUCK_PRAGER` ” with parabolic negative hardening for a containment of $2\,MPa$.
- modelization `AXIS` with meshes `QUAD4` .

Modelization C:
- model of type “ `DRUCK_PRAGER` ” with linear negative hardening for a containment of $2\,MPa$.
- modelization `3D` with meshes `HEXA20` .

Modelization D:
- model of type “ `DRUCK_PRAGER` ” with parabolic negative hardening for a containment of $2\,MPa$.
- modelization `3D` with meshes `HEXA20` .

1 Problem of reference

1.1



- Dimension of the cube: $1\text{m}\times 1\text{m}\times 1\text{m}$.
- Center cube: $O:(0.,0.,0.)$

1.2 Properties of the Elastic

- material

$E=5800.0\,E6\,Pa$	Young Modulus
$\nu=0.3$	Poisson's ratio

- `DRUCK_PRAGER` with negative hardening linear

$\alpha=0.33$	Coefficient of dependence in pressure
$p_{ultm}=0.01$	cumulated Plastic strain ultimate
$\sigma^{Y}=2.57\,E6\,Pa$	plastic Stress
$h=-2.00\,E8\,Pa$	Hardening modulus

- `DRUCK_PRAGER` with negative hardening parabolic

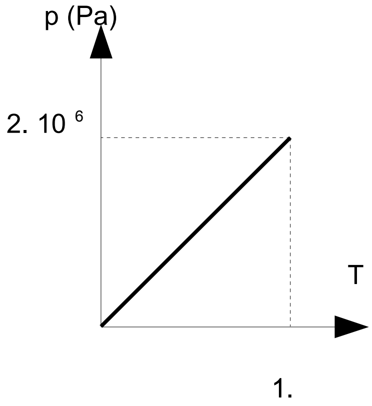
$\alpha=0.33$	Coefficient of dependence in pressure
$p_{ultm}=0.01$	cumulated Plastic strain ultimate
$\sigma^{Y}=2.57\,E6\,Pa$	ultimate Forced
$\sigma_{ultm}^{Y}=0.57\,E6\,Pa$	plastic Stress

1.3 Boundary conditions and loadings

the boundary conditions and the loadings applied are the following:

- Stage A : $t\in[0,1.]$

One gradually applies a compression $p=2.10^{6}\,Pa$ to 3 sides of the cube (high, in front, right) according to the function presented on the figure below, and of the conditions of symmetry on the 3 other sides (low, behind, left).

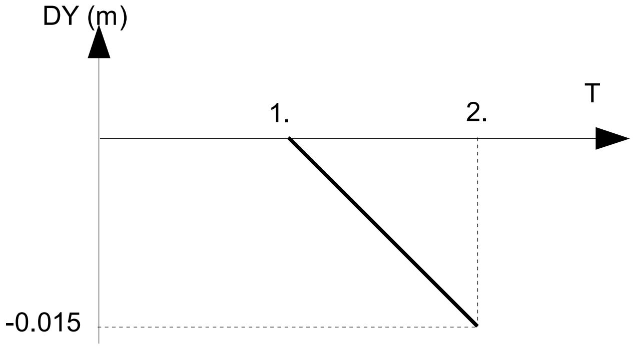


- Stage b: $t \in]1{,}2.]$

From the stress state at time $t = 1.s$, one applies to the sides of the cube the following conditions:

Imposed displacements:

- displacement varies gradually on the face of straight lines according to the function presented on the figure below:



- Conditions of symmetry on the 3 sides (low, behind, left).

Imposed loadings:

One applies a pressure of $p = 2.10^6 Pa$ to the 2 other sides (in front of and high).

2 Reference solution

2.1 Method of calculating used for the reference solution

2.1.1 Displacement DY

displacement DY of reference to the point A , corresponds to imposed displacement.

$$DY=-0.015(t-1)$$

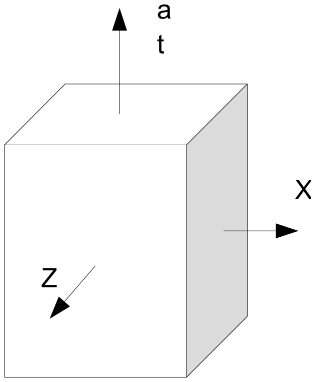
2.1.2 Stress $SIXX$

the stress $SIXX$ corresponds to the loading applied.

2.1.3 Stress $SIYY$ and cumulated plastic strain $V1$

triaxial Computation in conditions drained with the model of `DRUCK_PRAGER` .

Comparison with an analytical solution



$$\sigma_{eq}+\alpha I_1-R(p)=0$$

$$\sigma_{eq}+\alpha I_1-\sigma^Y=0 \text{ the top One} \qquad (1)$$

imposes there $\sigma_{xx}=\sigma_{zz}=-2\,MPa=\sigma^0$

$$\sigma_{eq}=\sqrt{\frac{3}{2}}S_{II}$$

$$S=\begin{pmatrix}\sigma_{xx}-\frac{1}{3}tr\,\sigma\\ \sigma_{yy}-\frac{1}{3}tr\,\sigma\\ \sigma_{zz}-\frac{1}{3}tr\,\sigma\end{pmatrix} \text{ with } tr\,\sigma=\sigma_{xx}+\sigma_{yy}+\sigma_{zz}=\sigma_{yy}+2\sigma^0$$

$$S=\begin{pmatrix} \sigma^0-\frac{1}{3}\sigma_{yy}-\frac{2}{3}\sigma^0 \\ \sigma_{yy}-\frac{1}{3}\sigma_{yy}-\frac{2}{3}\sigma^0 \\ \sigma^0-\frac{1}{3}\sigma_{yy}-\frac{2}{3}\sigma^0 \end{pmatrix}=\frac{1}{3}\begin{pmatrix} -\sigma_{yy}+\sigma^0 \\ 2\sigma_{yy}-2\sigma^0 \\ -\sigma_{yy}+\sigma^0 \end{pmatrix}$$

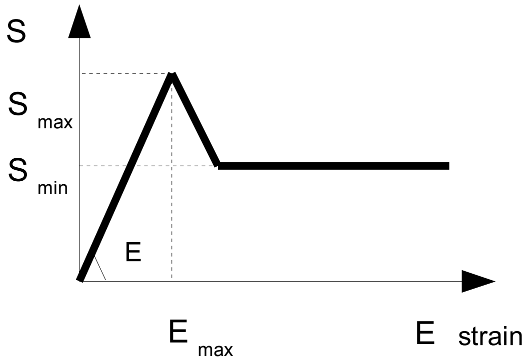
$$S_{II}=S.S=\frac{1}{3}\sqrt{2(\sigma_{yy}-\sigma^0)^2+(2\sigma_{yy}-2\sigma^0)^2}$$

$$\sigma_{eq}=\sqrt{\left(\frac{3}{2}\right)}S_{II}$$

what gives us $\boxed{\sigma_{eq}=|(\sigma_{yy}-\sigma^0)|}$ (2)

While introducing (2) into (1) one obtains

$$|(\sigma_{yy}-\sigma^0)|+\alpha(\sigma_{yy}+2\sigma^0)-\sigma^Y=0$$



$$\epsilon_{max}\Rightarrow\ \epsilon_{max}=\frac{\sigma_{max}}{E}=\frac{\sigma_{xx}-\sigma_{yy}}{E}>0 \quad \Rightarrow\sigma_{xx}>\sigma_{yy}$$
$$\Rightarrow\sigma_{yy}<\sigma^0$$

from where (2) => $(-\sigma_{yy}+\sigma^0)+\alpha(\sigma_{yy}+2.\sigma^0)-\sigma^Y=0$

$$\sigma_{max}\Rightarrow\boxed{\sigma_{yy}^{max}=\frac{\sigma^Y-\sigma^0(2\alpha+1)}{\alpha-1}}$$

from where $\epsilon_{yy}^{max}=\frac{\sigma^0-\sigma_{yy}^{max}}{E}$, one replaces σ_{yy}^{max} by his value and one obtains

$$\boxed{\epsilon_{max}^{yy}=\frac{3\alpha\sigma^0-\sigma^Y}{E(\alpha-1)}}$$

$$\sigma_{min} \Rightarrow \ \sigma_{eq}+\alpha I_1-R(p_{ultm})=0$$

$$\sigma_{yy}(\alpha-1)+\sigma^0(2\alpha+1)-R(p_{ultm})=0$$

$$\boxed{\sigma_{yy}^{min}=\frac{R(p_{ultm})-\sigma^0(2\alpha+1)}{\alpha-1}} \qquad (3)$$

$$\begin{cases}\epsilon-\epsilon^P=\dfrac{1+\nu}{E}\sigma-\dfrac{\nu}{E}(\mathrm{tr}\,\sigma).I\\ \sigma_{eq}+\alpha I_1-R(p)=0\end{cases}$$

$$\dot{\epsilon}^P=\lambda\left(\frac{3}{2}\frac{S}{\sigma_{eq}}+\alpha I\right) \text{ with } \lambda=\dot{p}$$

gold $S=\frac{1}{3}\begin{pmatrix}\sigma^0-\sigma_{yy}\\ 2\sigma_{yy}-2\sigma^0\\ \sigma^0-\sigma_{yy}\end{pmatrix} \quad \sigma_{eq}=|\sigma_{yy}-\sigma^0|=\sigma^0-\sigma_{yy}$

from where $\frac{S}{\sigma_{eq}}=\frac{1}{3}\begin{pmatrix}1\\-2\\1\end{pmatrix}$

$$\dot{\epsilon}_x^p=\dot{\epsilon}_z^p=\dot{p}\left(\frac{3}{2}\frac{1}{3}+\alpha\right)=\dot{p}\left(\alpha+\frac{1}{2}\right)$$

$$\dot{\epsilon}_y^p=\dot{p}\left(\frac{3}{2}\left(\frac{-2}{3}\right)+\alpha\right)=\dot{p}(\alpha-1)$$

$$\Rightarrow \epsilon_y^p=p(\alpha-1)+cste$$

$$\epsilon_y-\epsilon_y^p=\frac{1+\nu}{E}\sigma^Y-\frac{\nu}{E}(\sigma^Y+2\sigma^0)$$

$$\epsilon_y^{max}-cste=\frac{1+\nu}{E}\sigma_{yy}^{max}-\frac{\nu}{E}(\sigma_{yy}^{max}+2\sigma^0)=\frac{\sigma_{yy}^{max}}{E}-\frac{2\nu}{E}\sigma^0$$

$$\frac{3\alpha\sigma^0-\sigma^Y}{E(\alpha-1)}-cste=\frac{1}{E}\left(\frac{\sigma^Y-\sigma^0(2\alpha+1)}{\alpha-1}\right)-\frac{2\nu}{E}\sigma^0$$

$$\boxed{cste=\frac{\sigma^0\left[(5-2\nu)\alpha+(1+2\nu)\right]-2\sigma^Y}{E(\alpha-1)}} \qquad (4)$$

and

$$\boxed{\epsilon_y^p=p(\alpha-1)+cste}$$

from where

$$\epsilon_y-\epsilon_y^p=\frac{1+\nu}{E}\sigma_{yy}-\frac{\nu}{E}(\sigma_{yy}+2\sigma^0)$$

$$\boxed{\epsilon_y-p(\alpha-1)-cste=\frac{1}{E}\sigma_{yy}-\frac{2\nu}{E}\sigma^0} \qquad (5)$$

the statement below is a direct application of the statement (5)

$$\epsilon_y^{min}=p_{ultm}(\alpha-1)+cste+\frac{\sigma_{yy}^{min}}{E}-2\frac{\nu}{E}\sigma^0 \qquad (6)$$

By introducing the statements (3) and (4) in (6) one obtains:

$$\boxed{\epsilon_y^{min}=p_{ultm}\left[\frac{E(\alpha-1)^2+h}{E(\alpha-1)}\right]+\frac{\sigma^Y}{E(1-\alpha)}+\sigma^0\left[\frac{3\alpha+4\nu(1-\alpha)}{E(\alpha-1)}\right]}$$

The statement below is a direct application of the statement (5)

$$\boxed{p=\frac{-\sigma_{yy}}{E(\alpha-1)}+\frac{2\nu\sigma^0}{E(\alpha-1)}+\frac{\epsilon_y}{(\alpha-1)}-\frac{cste}{(\alpha-1)}} \qquad (7)$$

For reasons of simplification one poses

$$\boxed{p=A\sigma_{yy}+B}$$

with

$$\begin{cases} A=\dfrac{1}{E(1-\alpha)} \\ B=\dfrac{2\nu\sigma^0}{E(\alpha-1)}+\dfrac{\epsilon_y}{\alpha-1}-\dfrac{cste}{\alpha-1} \end{cases}$$

on the basis of the equation $\sigma_{eq}+\alpha I_1-R(p)=0$

with in the case:

- linear $R(p)=hp+\sigma^Y$
- parabolic $R(p)=\frac{6c\, f(p)\cos(\phi)}{3-\sin(\phi)}$

with $$f(p)=\begin{cases}\left(1-(1-\sqrt{\frac{\sigma^Y_{ultm}}{\sigma^Y}})\frac{p}{p_{ultm}}\right)^2 & \text{if } 0<p<p_{ultm}\\ \frac{\sigma^Y_{ultm}}{\sigma^Y} & p_{ultm}<p\end{cases}$$

linear Case

$$\sigma_{eq}+\alpha I_1-R(p)=0 \quad \text{case where} \quad \underline{0<p<p_{ultm}}$$

$$\sigma_{yy}(\alpha-1)+\sigma^0(2\alpha+1)-hp-\sigma^Y=0 \qquad (8)$$

By introducing the statement of p (7) into (8) one obtains the following statement

$$\boxed{\sigma_{yy}=\epsilon_y\left(\frac{E\,h}{h+E(\alpha-1)^2}\right)+\sigma^Y\left(\frac{E(\alpha-1)}{h+E(\alpha-1)^2}\right)+\sigma^0\left(\frac{2\nu h-(2\alpha+1)E(\alpha-1)}{h+E(\alpha-1)^2}\right)+\frac{hE.cste}{h+E(\alpha-1)^2}}$$

parabolic Case where $\underline{o<p<p_{ultm}}$

$$\sigma_{yy}(\alpha-1)+\sigma^0(2\alpha+1)-R(p)=0$$

$$R(p)=\left(1-(1-\alpha_2)\frac{p}{p_{ultm}}\right)^2\frac{6c\cos(\phi)}{3-\sin(\phi)}$$

$$R(p)=\sigma^Y\left(1-\left(1-\sqrt{\frac{\sigma^Y_{ultm}}{\sigma^Y}}\right)\frac{p}{p_{ultm}}\right)^2$$

By replacing this new statement of p (7) in the preceding equation

$$R(p)=\sigma^Y\left[1-\left(1-\sqrt{\frac{\sigma^Y_{ultm}}{\sigma^Y}}\right)\frac{A\sigma_{yy}+B}{p_{ultm}}\right]^2$$

while developing one obtains the following statement:

$$R(p)=a\sigma_{yy}^2+b\sigma_{yy}+c$$

$$avec \begin{cases} a=\frac{A^2}{p_{ultm}^2}\left(1-\sqrt{\frac{\sigma_{ultm}^Y}{\sigma^Y}}\right)^2 \sigma^Y \\ b=-2\sigma^Y\left[1-\left(1-\sqrt{\frac{\sigma_{ultm}^Y}{\sigma^Y}}\right)\frac{B}{p_{ultm}}\right]\left[\frac{A}{p_{ultm}}\left(1-\sqrt{\frac{\sigma_{ultm}^Y}{\sigma^Y}}\right)\right] \\ c=\left[1-\left(1-\sqrt{\frac{\sigma_{ultm}^Y}{\sigma^Y}}\right)\frac{B}{p_{ultm}}\right]^2 \end{cases}$$

One finds after simplification:

$$\sigma_{yy}(\alpha-1)+\sigma^0(2\alpha+1)-a\,\sigma_{yy}^2-b\,\sigma_{yy}-c=0$$

that is to say

$$a\,\sigma_{yy}^2+(1+b-\alpha)\sigma_{yy}+(c-\sigma^0(2\alpha+1))=0$$

Resolution of the polynomial of order 2:

$$\Delta=(1+b-\alpha)^2-4a(c-\sigma^0(2\alpha+1))$$

$$\begin{cases} \sigma_1=\frac{-(1+b-\alpha)-\sqrt{\Delta}}{2\mathrm{a}} \\ \sigma_2=\frac{-(1+b-\alpha)+\sqrt{\Delta}}{2\mathrm{a}} \end{cases}$$

2.2 Variables reference

- •Forced $SIXX$ to the node A
- •Forced $SIYY$ with the node A
- •Plastic strain cumulated $V1$ with the node A
- •Displacement DY with the node A

2.3 Result of reference

Quantity	Not	$Inst$	Référence*	linear Reference
$SIXX\ (N/m^2)$	**	2.0	$-2.0\,E6$	$-2.0\,E6$
$SIYY\,(N/m^2)$	A	1.07	$-8.09\,E6$	$-8.09\,E6$
		1.16	$-8.20\,E6$	$-8.01\,E6$
		1.34	$-6.89\,E6$	$-6.63\,E6$
		1.53	$-5.80\,E6$	$-5.81\,E6$
$V1$	A	1.07	0	0
		1.16	$1.99\,E-3$	$2.04\,E-3$
		1.34	$6.35\,E-3$	$6.42\,E-3$
		1.53	$1.09\,E-2$	$1.09\,E-2$
$DY\ (m)$	A	1.07	$-1.05\,E-3$	$-1.05\,E-3$
		1.16	$-2.40\,E-3$	$-2.40\,E-3$
		1.34	$-5.10\,E-3$	$-5.10\,E-3$
		1.53	$-7.95\,E-3$	$-7.95\,E-3$

A * hardening ** parabolic hardening

2.4 Uncertainty on the solution

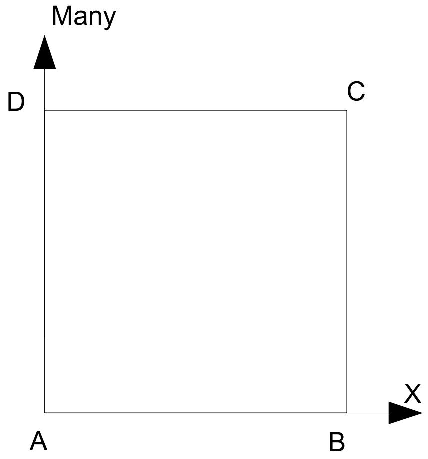
analytical Solution

3 Modelization A

3.1 Characteristic of the modelization A

Modelization `AXIS`.

Model `DRUCK_PRAGER` with linear negative hardening.



nodes Number of meshes	4			
Is	5	there :		
			`SEG2`	4
			`QUAD4`	1

the square is in space $[0.,1.]\mathrm{x}[0.,1.]$.

Coordinates of the points (m) :

$$A:(0.,0.)$$
$$B:(1.,0.)$$
$$C:(1.,1.)$$
$$D:(0.,1.)$$

Meshes :

$M1$: surface $ABDC$
$M2$: segment AB
$M3$: segment BC
$M4$: segment CD
$M5$: segment DA

Nodes groups:

A, B

3.2 Quantities tested and Quantity

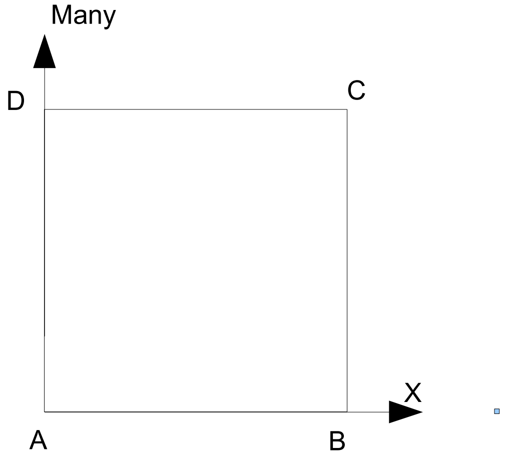
results	Not	$Inst$	Reference	Tolerance ($\%$)
$SIXX\ (Pa)$	C	2.0	$-2.0\,E6$	0.1
$SIYY\,(Pa)$	C	1.07	$-8.09\,E6$	0.1.0.1.0.1
		1.16	$-8.20\,E6$	
		1.34	$-6.89\,E6$	
		1.53	$-5.80\,E6$	0.1
$V1$	C	1.07	0	0.1.0.1.0.1
		1.16	$1.99\,E-3$	
		1.34	$6.35\,E-3$	
		1.53	$1.09\,E-2$	0.1
$DY\ (m)$	C	1.07	$-1.05\,E-3$	0.1.0.1.0.1
		1.16	$-2.40\,E-3$	
		1.34	$-5.10\,E-3$	
		1.53	$-7.95\,E-3$	0.1

4 Modelization B

4.1 Characteristic of the modelization B

Modelization `AXIS`.

Model `DRUCK_PRAGER` with parabolic negative hardening.



nodes Number of meshes	4		
Is	5	there :	
		`SEG2`	4
		`QUAD4`	1

the square is in space $[0.,1.]\mathrm{x}[0.,1.]$.

Coordinates of the points (m) :

$A:(0.,0.)$
$B:(1.,0.)$
$C:(1.,1.)$
$D:(0.,1.)$

Meshes :

$M1$: surface $ABDC$
$M2$: segment AB
$M3$: segment BC
$M4$: segment CD
$M5$: segment DA

Nodes groups:
A, B

4.2 Quantities tested and Quantity

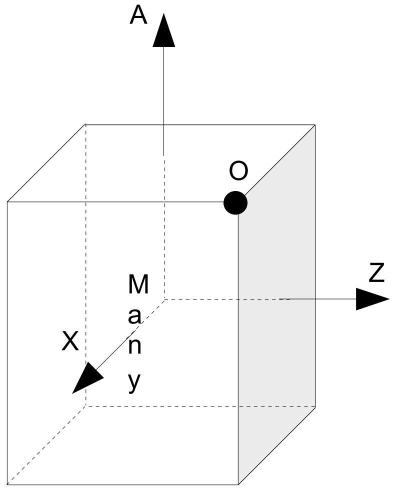
results	Not	$Inst$	Reference	Tolerance (%)
$SIXX\ (Pa)$	C	2.0	$-2.0\,E6$	0.1
$SIYY\,(Pa)$	C	1.07	$-8.09\,E6$	0.1.0.1.0.1
		1.16	$-8.01\,E6$	
		1.34	$-6.63\,E6$	
		1.53	$-5.81\,E6$	0.1
$V1$	C	1.07	0	0.1.0.1.0.1
		1.16	$2.04\,E-3$	
		1.34	$6.42\,E-3$	
		1.53	$1.09\,E-2$	0.1
$DY\ (m)$	C	1.07	$-1.05\,E-3$	0.1.0.1.0.1
		1.16	$-2.40\,E-3$	
		1.34	$-5.10\,E-3$	
		1.53	$-7.95\,E-3$	0.1

5 Modelization C

5.1 Characteristic of the modelization C

Modelization `3D`.

Model `DRUCK_PRAGER` with linear negative hardening.



nodes Number of meshes	20		
Is	7	there :	
		`QUAD8`	6
		`HEXA20`	1

Geometry of the cube (m) :

Center $O(0.,0.,0.)$
Side $C=1\,m$

Mesh groups:

BAS : surface cube belonging to plane $Z=-0.5$
$HAUT$: surface cube belonging to plane $Z=+0.5$
$DROITE$: surface cube belonging to plane $Y=+0.5$
$GAUCHE$: surface cube belonging to plane $Y=-0.5$
$DERRIERE$: surface cube belonging to plane $X=-0.5$
$DEVANT$: surface cube belonging to plane $X=+0.5$

5.2 Quantities tested and Quantity

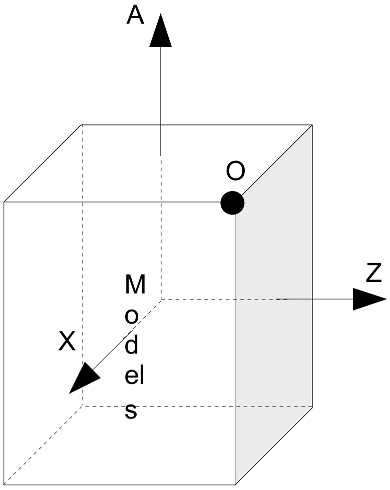
results	Not	$Inst$	Reference	Tolerance (%)
$SIXX\ (Pa)$	A	2.0	$-2.0\,E6$	0.1
$SIZZ\,(Pa)$	A	1.07	$-8.09\,E6$	0.1.0.1.0.1
		1.16	$-8.20\,E6$	
		1.34	$-6.89\,E6$	
		1.53	$-5.80\,E6$	0.1
$V1$	A	1.07	0	0.1.0.1.0.1
		1.16	$1.99\,E-3$	
		1.34	$6.35\,E-3$	
		1.53	$1.09\,E-2$	0.1
$DZ\ (m)$	A	1.07	$-1.05\,E-3$	0.1.0.1.0.1
		1.16	$-2.40\,E-3$	
		1.34	$-5.10\,E-3$	
		1.53	$-7.95\,E-3$	0.1

6 Modelization D

6.1 Characteristic of the modelization D

Modelization `3D.`

DRUCK_PRAGER `with` negative hardening there parabolic



Number of nodes	20		
Number of meshes	7	Is:	
		`QUAD8`	6
		`HEXA20`	1

Geometry of the cube (m) :

Center $O(0.,0.,0.)$
Side $C=1m$

Mesh groups:

BAS :	surface cube belonging to plane	$Z=-0.5$
$HAUT$:	surface cube belonging to plane	$Z=+0.5$
$DROITE$:	surface cube belonging to plane	$Y=+0.5$
$GAUCHE$:	surface cube belonging to plane	$Y=-0.5$
$DERRIERE$:	surface cube belonging to plane	$X=-0.5$
$DEVANT$:	surface cube belonging to plane	$X=+0.5$

6.2 Quantities tested and Quantity

results	Not	$Inst$	Reference	Tolerance ($\%$)
$SIXX\ (Pa)$	A	2.0	$-2.0E6$	0.1
$SIZZ(Pa)$	A	1.07	$-8.09E6$	0.1.0.1.0.1
		1.16	$-8.01E6$	
		1.34	$-6.63E6$	
		1.53	$-5.81E6$	0.1
$V1$	A	1.07	0	0.1.0.1.0.1
		1.16	$2.04E-3$	
		1.34	$6.42E-3$	
		1.53	$1.09E-2$	0.1
$DZ\ (m)$	A	1.07	$-1.05E-3$	0.1.0.1.0.1
		1.16	$-2.40E-3$	
		1.34	$-5.10E-3$	
		1.53	$-7.95E-3$	0.1

7 Summary of the results

the constitutive law of the type `DRUCK_PRAGER` with a linear negative hardening and a parabolic negative hardening gives satisfactory results with modelizations `AXIS` and `3D`.