SSNV166 – Roll fissured under multiple loadings

Abstract

This test is computation of the stress intensity factors along the crack tip for a cylinder comprising an axisymmetric crack.

The influence of the degree of the elements and the type of the method is studied through various modelizations.

1) The modelization A tests $K1$ and $K3$ with a linear mesh $3\mathrm{D}$ and a classical method with the finite elements (FEM).
2) The modelization B tests $K1$ and $K3$ with a quadratic mesh $3\mathrm{D}$ (elements of Barsoum) around the crack tip and one FEM .
3) The modelization C tests $K1$ and $K3$ with a linear mesh $3\mathrm{D}$ with a classical resolution but an extraction of the factors of intensity based on an energy computation.

Moreover, for each modelization, various cases of loadings are studied:
- tension (request in mode I);
- torsion (request in mode III);
- bending (opening of dimensioned, closing of the other) with and without taking into account of the contact.

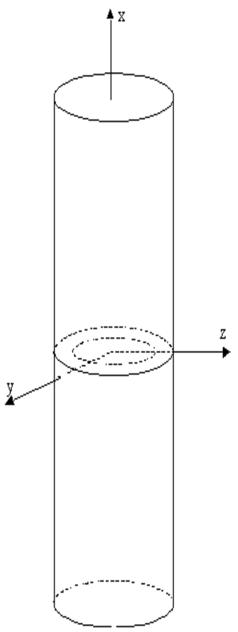
The cases of tension and torsion do not put concerned the contact.

Although symmetries exist in certain cases (axisymetry for case 1, plane symmetry for 2nd) the representation is made in $3\mathrm{D}$ to make the test generalizable under multiple loading.

1 Problem of reference

1.1 Geometry

the crack is a circular ring in an orthogonal plane with the axis of the cylinder [Figure 1.1-a]. The parameters a and b determine the radius of the cylinder and the depth of crack. [Figure 1.1-b] is a cut of the cylinder in the plane of crack (plane Oyz). So that the medium is regarded as infinite, the height of the cylinder is $h=10b$.



Appear 1.1-a : Geometry of the cylinder fissured

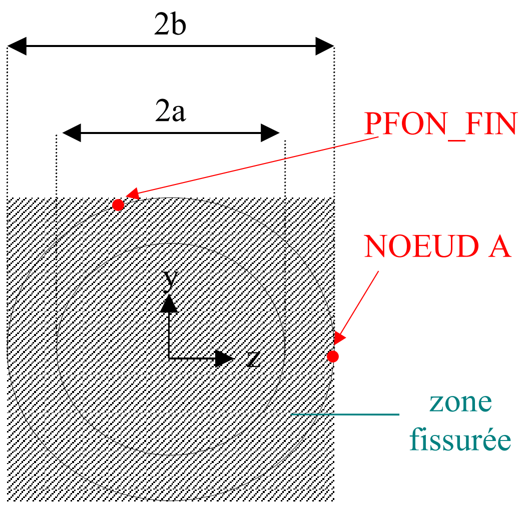


Figure 1.1-b : Plane of cracking

1.2 Material properties

Modulus Young: $E=205000\,MPa$

Poisson's ratio: $\nu=0.3$

1.3 Boundary conditions and loadings

Three loadings will be applied in order to calculate the stress intensity factors $K1$ and $K3$ in $3\mathrm{D}$ by means of operator POST_K1_K2_K3.

Loading 1 tests $K1$ and $K3$.
Loading 2 tests $K2$ without taking into account of the contact.
Loading 3 tests $K2$ with taking into account of the contact.

One expects that $K1$ and $K3$ are constant along the crack tip and so that $K2$ varies.

Note: the cases of tension and torsion can be treated indifferently with or without contact (here, without contact) because there is never closing of crack.

	Case 1: tension and torsion	Case 2: bending without contact	Case 3: bending with contact
Upper face	$N_x=6\,MN$ $T_x=3\,MN$	$M_y=1.5\,MN$	$M_y=1.5\,MN$

Table 1.3 - 1 : Cases of loadings

the preceding forces are applied to structure via discrete elements $3\mathrm{D}$ located at the center of the upper face. It is noted that the point of maximum opening due to imposed bending (next moment Oy) will be the node A (see [Figure 1.1 - B].

Motions of rigid bodies are blocked by the same process with fixed support of the center of the lower face.

2 Reference solution

2.1 Method of calculating used for the reference solution

For an axisymmetric crack in a cylinder infinite length, the method of the Singular Integral equations and Asymptotic Developments [biberon1] makes it possible to calculate the values of the stress intensity factors.

1) Case 1: Tension and Torsion

the tension induces an opening in mode 1. $K1$ is given by the following formula:

$$K_I = \frac{P}{\pi a^2} \sqrt{\pi a} F_1(a/b)$$

where P is the force applied to the upper face and lower and F_1 a given function [Figure 2.1-a].

Torsion induces an opening in mode 3. $K3$ is given by the following formula:

$$K_{III} = \frac{2\mathrm{T}}{\pi a^3} \sqrt{\pi a} F_3(a/b)$$

where T is the applied moment on the upper face and lower and F_3 a given function [Figure 2.1 - has].

1) Case 2: Bending without contact

bending induces an opening in mode 1. The value of $K1$ at the point of maximum opening A is given by the following formula:

$$K_{I_A} = \frac{4\mathrm{M}}{\pi a^3} \sqrt{\pi a} F_2(a/b)$$

where M is the applied moment on the upper face and lower and F_2 a given function [Figure 2.1 - has].

1) Case 3: Bending with analytical

contact It does not exist of solution to this problem. One expects on the one hand that $K1$ is close to the case without contact on the part of crack in opening, and on the other hand that $K1$ is null on the part of crack in closing.

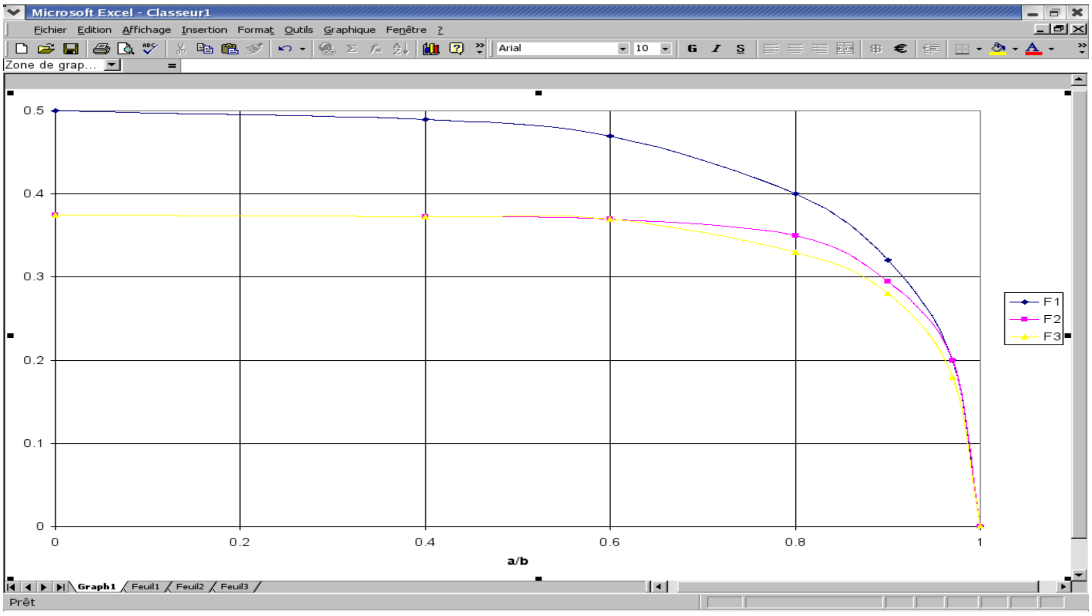


Figure 2.1 - a: Functions $F1$, $F2$ and $F3$

These three functions come from [biberon1].

2.2 Results of Numerical

reference Application:

Except contrary mention, in the continuation of this document, the parameters retained for a and b are:

$$a=0.4\,m$$
$$b=0.5\,m$$

Case 1: Tension and torsion	Case 2: Bending
$K1=5.35\ MPa.m^{1/2}$ $K3=11.22\ MPa.m^{1/2}$	$K1_A=11.71\ MPa.m^{1/2}$

Table 2.2 - 1 : Values of reference

2.3 bibliographical References

•TADA, PARIS, IRWIN: The Stress Analysis Of Cracks Handbook, Del Research Corporation, Hellertoxn, Pennsylvania (1973).
•Computation of the factors of intensity of the stresses by extrapolation of the field of displacements, Handbook of reference of *the Code_Aster*, R7.02.08
•CORNELIU: Quarter-point elements for curved ace fronts, Computers & Structures vol. 17, No 2, pp. 227-231, 1983

3 linear Modelization a: Mesh, classical formulation

3.1 Characteristics of the modelization

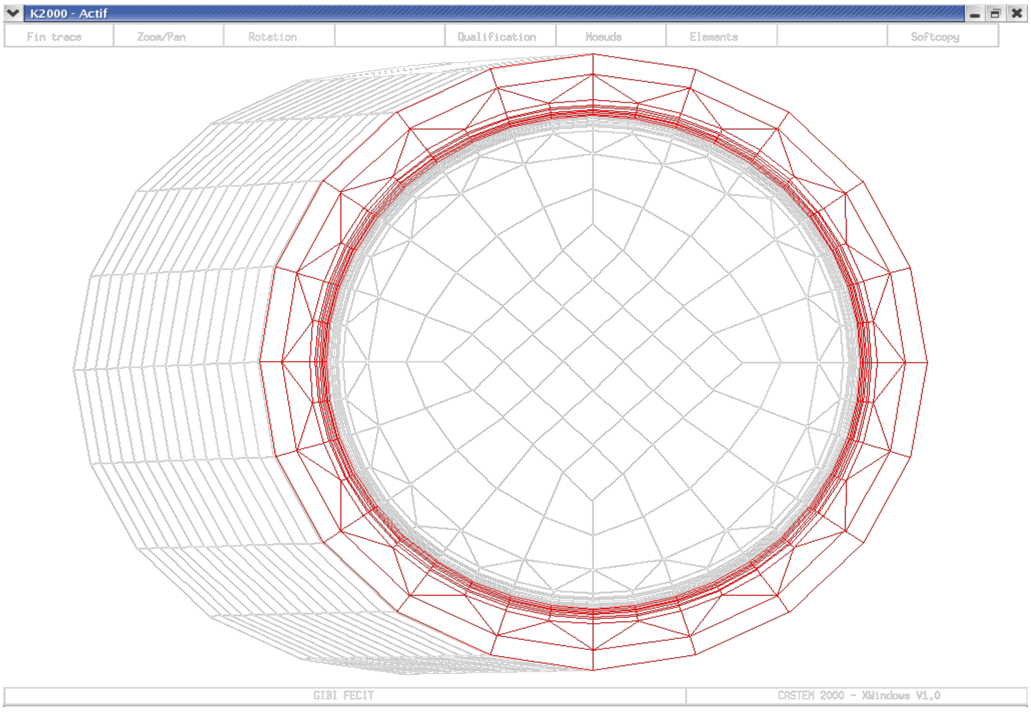


Figure 3.1 - a: the plane of the crack Cuts mesh in

the elements are all of order 1.

The interest of this modelization is to be used as a basis for more evolved formulations, and thus, to be able to note the contribution and the improvements of the other methods.

3.2 Characteristics of the mesh

Many nodes: 11310
Number of meshes: 14453

Type of meshes	Number of meshes
POI1	4
SEG2	39
TRIA3	360
QUAD4	930
PENTA6	5440
HEXA8	7680

Table 3.2 - 1: Characteristics of meshes

3.3 the Remark

The computation of the stress intensity factors is done using `POST_K1_K2_K3` (method of extrapolation of displacements on the lips of crack) [biberon2].

3.4 Values tested and Results of the modelization To

procedure `POST_K1_K2_K3` makes it possible to identify the values of the stress intensity factors except for a coefficient. It is pointed out that this method identifies the stress intensity factor $K1$ (respectively $K2$, $K3$) starting from the jump of displacement by a method of the least squares.

3.4.1 Results in the case of a loading in tension (K1) and torsion (K3)

Identification	Reference	*Aster*	% difference
$K1$ to the node $PFON_{FIN}$	5.35 106	4.52 106	15
$K3$ with the node $PFON_{FIN}$	-11.22 106	-9,54 the 106	15

values of $K1$ and $K3$ must be identical [Figure 4.2-a] for all the nodes of the crack tip because there is an axisymmetric configuration. Here, we test only the values with the node $PFON_{FIN}$.

3.4.2 Results in the case of a loading in bending (K1) without contact

Identification	Reference	*Aster*	% difference
$K1$ to the node A	11.71 106	9,18 106	22

One compare the value of $K1$ with the reference solution only to the point of maximum opening (node A) because it is the only analytical value available in the literature.

3.4.3 Results in the case of a loading in bending (K1) with contact

Identification	Reference	*Aster*	% difference
$K1$ to the node A	10.17 106	8,38 106	20

One compares result obtained with that obtained by *Code_Aster* without taking into account of the contact (NON-regression). This taking into account is carried out by the method of the active stresses.

3.4.4 Evolutions of K1, K2, K3 along the crack tip

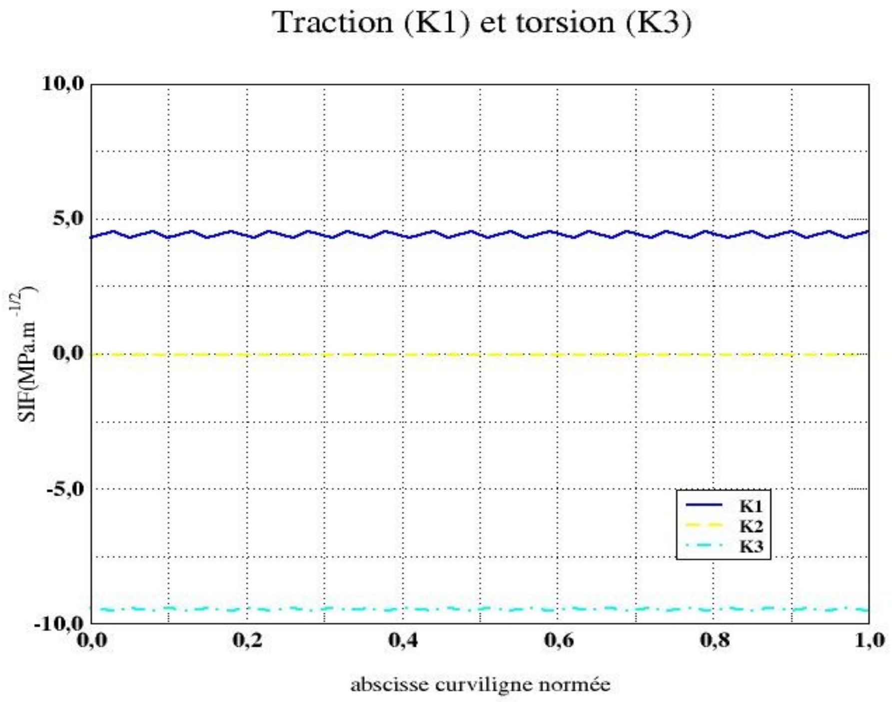


Figure 4.2 - a: $K1$ **,** $K2$ **and** $K3$ **along the crack tip (in** $MPa.m^{1/2}$ **)**

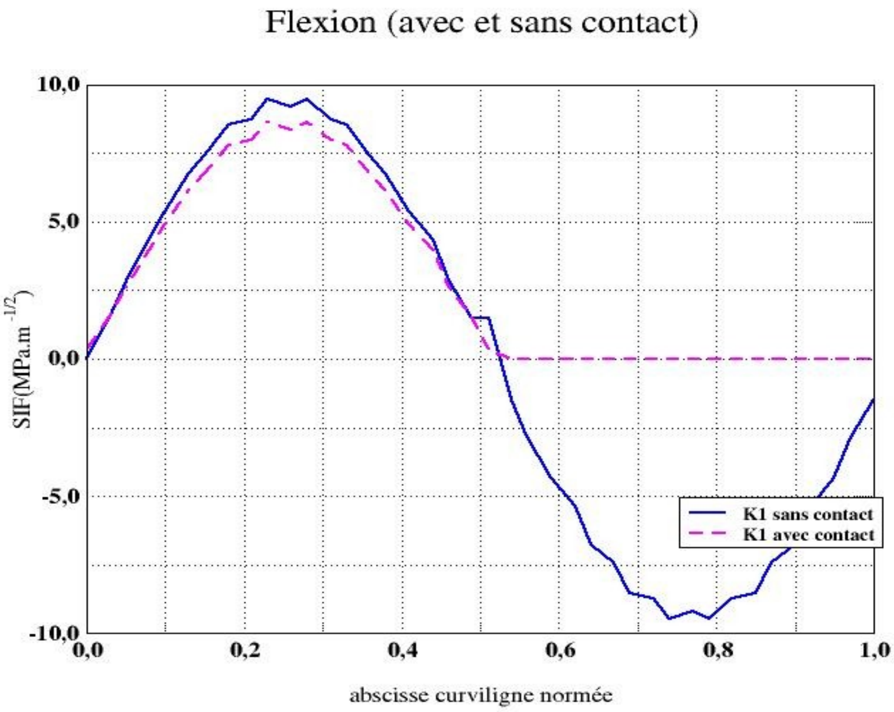


Figure 4.2 - b: $K1$ **along the crack tip (in** $MPa.m^{1/2}$ **)**

Comments on the results:

[Figure 4.2-a] the watch evolution of the factors of intensity of the stresses along the crack tip of axisymmetric crack of depth $100\,mm$ subjected to tension and torsion. One observes many axisymmetric results (with the miscalculations near). Moreover, it is noted that the crack is not requested in mode II.
On [Figure 4.2-b], one of the contact highlights the taking into account. On half of crack in opening, $K1$ of the contact has weaker values with taking into account, because the contact rigidifies structure. On half in closing, $K1$ is null.
In fact, the contact does not take place on all the higher half of crack [Figure 4.2 - C] but on a surface a little smaller. On [Figure 4.2-c] the zone in red represents the contact zone and the zone in blue that of noncontact.

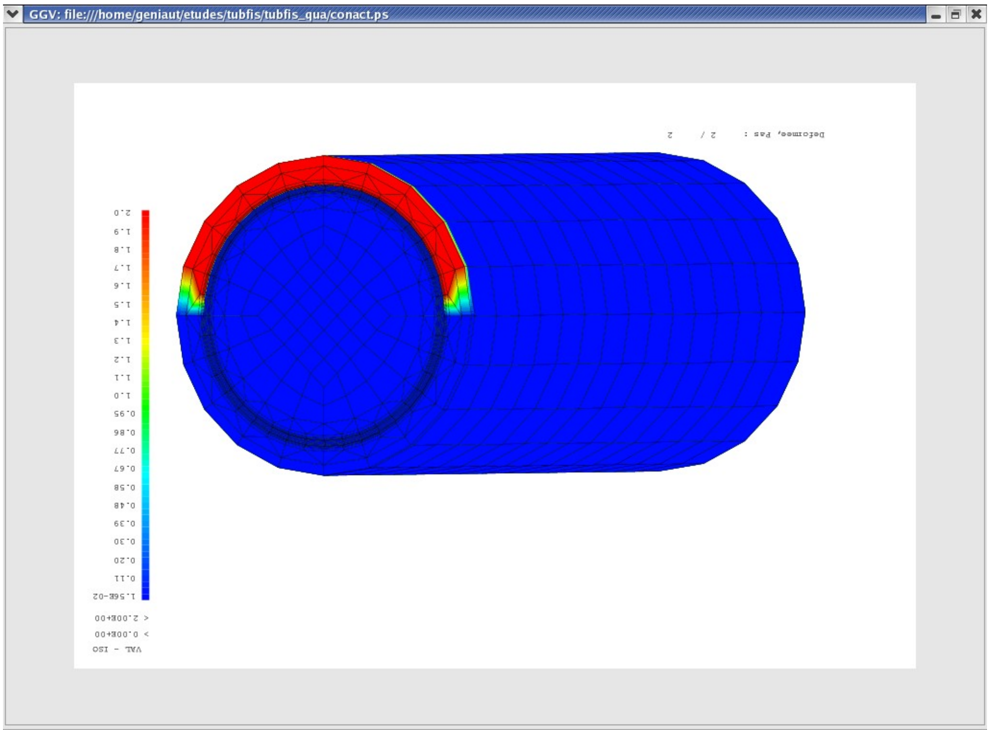
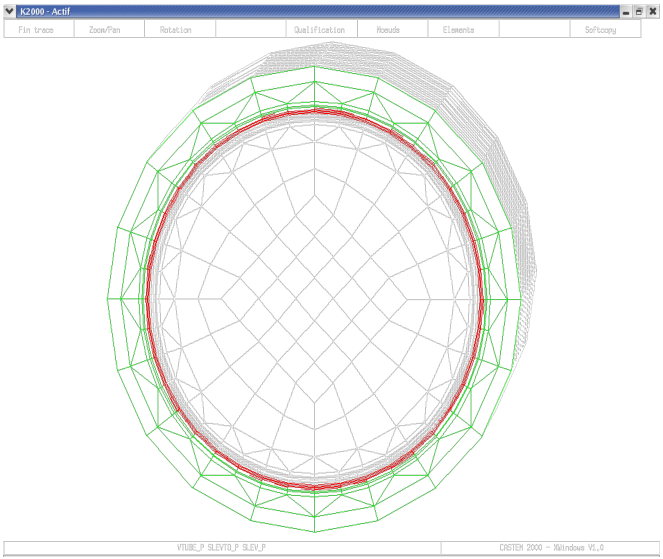


Figure 4.2 - C : Contact

4 quadratic Modelization b: Mesh, classical formulation

4.1 Characteristics of the modelization



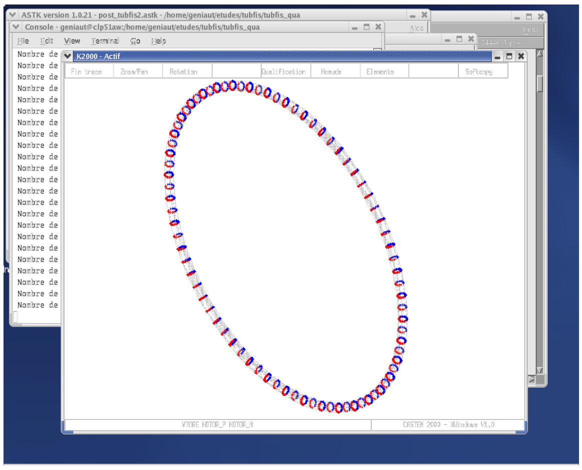


Figure 5.1 - a: Mesh and torus

a torus is created around crack. The elements of the torus are quadratic elements. The elements apart from the torus are linear. Moreover, one uses elements of BARSOUM (nodes mediums moved with the quarter) for meshes having an edge belonging to crack tip [biberon3].

The interest of the use of a mesh of the type BARSOUM is obtaining of results more precise.

4.2 Characteristics of the mesh

Many nodes: 20030
Number of meshes: 16449

Type of meshes	Number of meshes
POI1	2000
SEG3	39
TRIA3	360
QUAD4	610
QUAD8	320
PENTA6	4800
PENTA15	640
HEXA8	5760
HEXA20	1920

Table 5.2 - 1: Characteristics of meshes

the nodes mediums of the edges of the elements touching the crack tip are moved with the quarter of these edges, to obtain a better accuracy.

4.3 Values tested and results of the modelization B

4.3.1 Results in the case of a loading in tension ($K1$) and torsion ($K3$)

Identification	Reference	*Aster*	% difference
$K1$ to the node $PFON_{FIN}$	5,35 106	5.04 106.5,7	5.7
$K3$ with node $PFON_{FIN}$	-11,22 the 106	-10.80 106.3,8	3.8

values of $K1$ and $K3$ must be identical [Figure 6.2 - has] for all the nodes of the crack tip because there is an axisymmetric configuration. Here, we test only the values with the last node of crack ($PFON_{FIN}$).

4.3.2 Results in the case of a loading in bending ($K1$) without contact

Identification	Reference	*Aster*	% difference
$K1$ to the node A	11,71 106	10,29 106	12

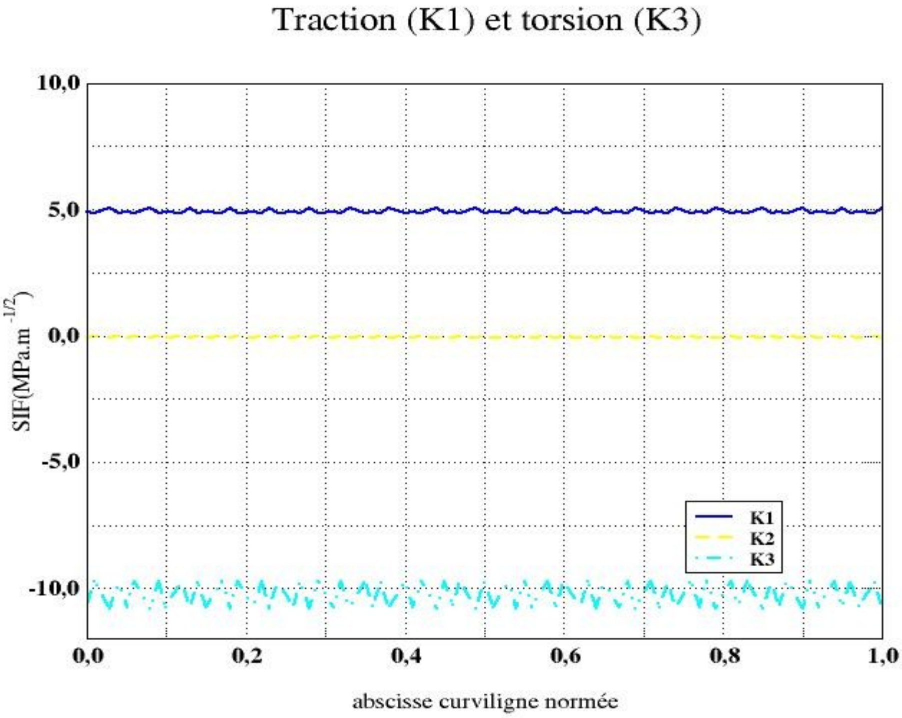
One compares the value of $K1$ with the reference solution only to the point of maximum opening (Node A) because it is the only analytical value available in the literature.

4.3.3 Results in the case of a loading in bending ($K1$) with contact

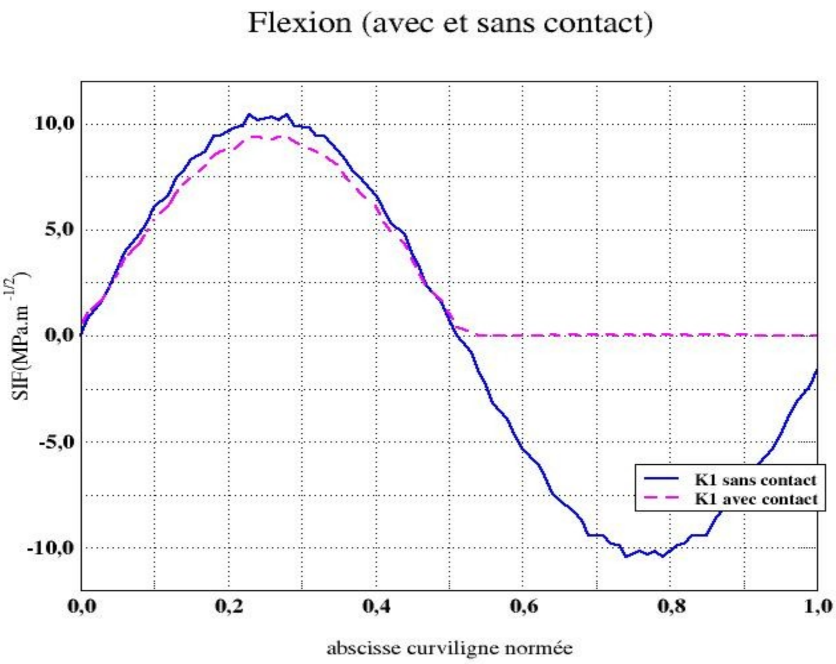
Identification	Reference	*Aster*	% difference
$K1$ to the node A	10,59 106	9,24 106	13

One compares result obtained with that obtained by Aster computation without taking into account of the contact (not - regression). The method of resolution of the contact is that of the active stresses.

4.4 Evolutions of K1, K2, K3 along the crack tip



Appears 4.4-a : $K1$, $K2$ and $K3$ along the crack tip (in $MPa.m^{1/2}$)



4.4-b 4.4-b : $K1$ along the crack tip (in $MPa.m^{1/2}$)

5 Modelization C: Linear mesh, classical formulation and energy method

5.1 Characteristics of the modelization

The modelization of the problem is the same one as that used in A . All the elements are of order 1.

5.2 Characteristics of the mesh

The mesh is similar to that used in A .

Many nodes: 13630
Number of meshes: 17013

Type of meshes	Number of meshes
POI1	4
SEG2	39
TRIA3	360
QUAD4	1090
PENTA6	5760
HEXA8	9760

Table 7.2 - 1 : Characteristics of meshes

5.3 the Values tested and results of the modelization C

One calculates rate of energy restitution and the factors of intensity of the stresses with the command CALC_G, option CALC_K_G. This method is more general than the method of extrapolation of displacements (POST_K1_K2_K3) because it can be used an unspecified crack in the case of (NON-plane crack, with bottom NON-right).

5.3.1 Results in the case of a loading in tension (K1) and torsion (K3)

Identification	Reference	*Aster*	% difference
$Max(K1)$	the 5.35 106	5.11 106	4.47
$Max(K3)$	11.22 106	10.52 106	6.24

values of $K1$ and $K3$ must be identical [Appears 5.4-a] for all the nodes of the crack tip because there is an axisymmetric configuration. Here, we test the maximum of $K1$ and $K3$ for all the points of the crack tip.

5.3.2 Results in the case of a loading in bending (K1) without contact

Identification	Reference	*Aster*	% difference
$K1$ to the node A	11.71 106	10.32 106	11.88

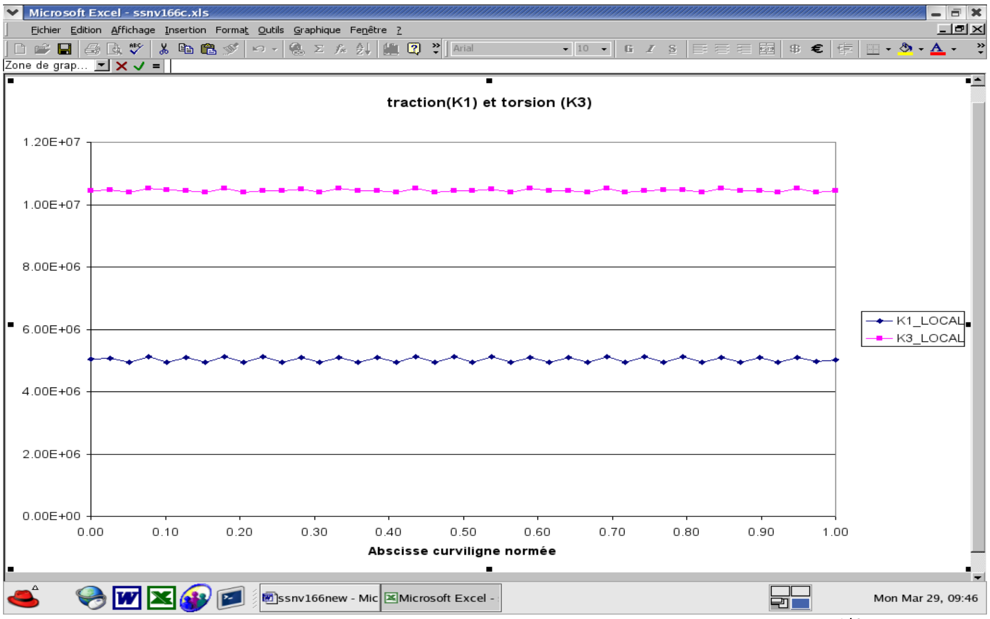
One compares the value of $K1$ with the reference solution only to the point of maximum opening because it is the only analytical value available in the literature. This point is not any more one "node" but a "point" of the crack tip, it should then be located by its number in the list of the points of the crack tip. It is the point located by NUM_PT=11.

5.3.3 Results in the case of a loading in bending (K1) with contact

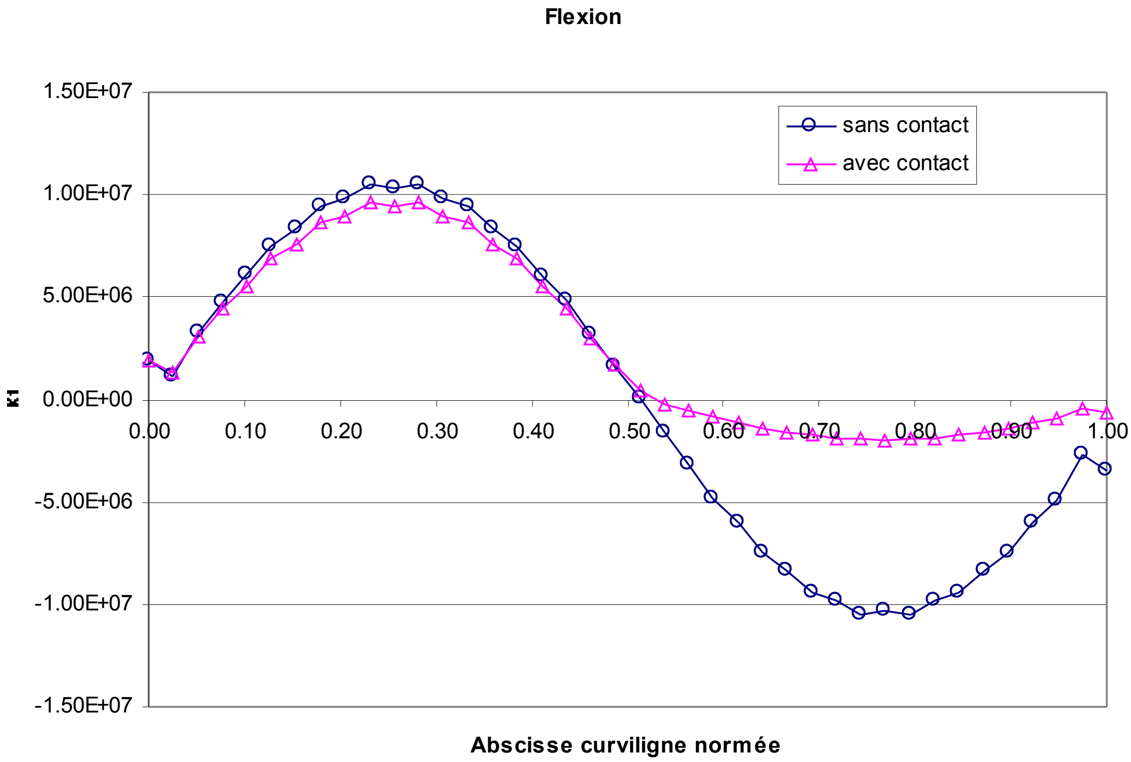
Identification	Reference	*Aster*	% difference
$K1$ to the node A	10.32 106	9.43 106	8.57

One compares result obtained with that obtained by *Code_Aster* without taking into account of the contact (not - regression). This taking into account is carried out by the method of the active stresses. [5.4-b 5.4-b] compares the values of $K1$ along the crack tip in the case of bending with or without contact.

5.4 Evolutions of K1 and K3 along the crack tip



Appears 5.4-a : $K1$ and $K3$ along the crack tip (in $MPa.m^{1/2}$)



5.4-b 5.4-b : $K1$ along crack (in $MPa.m^{1/2}$)

Note:

It is noted that if the contact is taken into account (see [5.4-b 5.4-b]), $K1$ is not really null on the segments of the crack tip where there is closing. That comes owing to the fact that the energy method of computation of K projects the field of solution displacement on the singular auxiliary fields of displacement of an infinitely long crack in opening. However these auxiliary fields are not compatible with the mode of closing present.

6 Summary of the results

the goals of this test are achieved:

- It is a question of the contact of validating the taking into account on the lips of crack with quadratic elements (and elements of Barsoum). The results better, are compared with those obtained with a linear mesh.
- This test shows the interest of the method *« G−thêta »* for computation of the stress intensity factors. This energy method has the advantage of being more general than that using the jump of displacements (`POST_K1_K2_K3`) because it can apply to cracks of unspecified geometry, whereas `POST_K1_K2_K3` is restricted with plane cracks. Moreover, the method *« G−thêta »* gives better results (compared with the analytical solution) that `POST_K1_K2_K3` for the same linear mesh.