Elastic SSNV152- Tension. Computation of the stresses of Cauchy

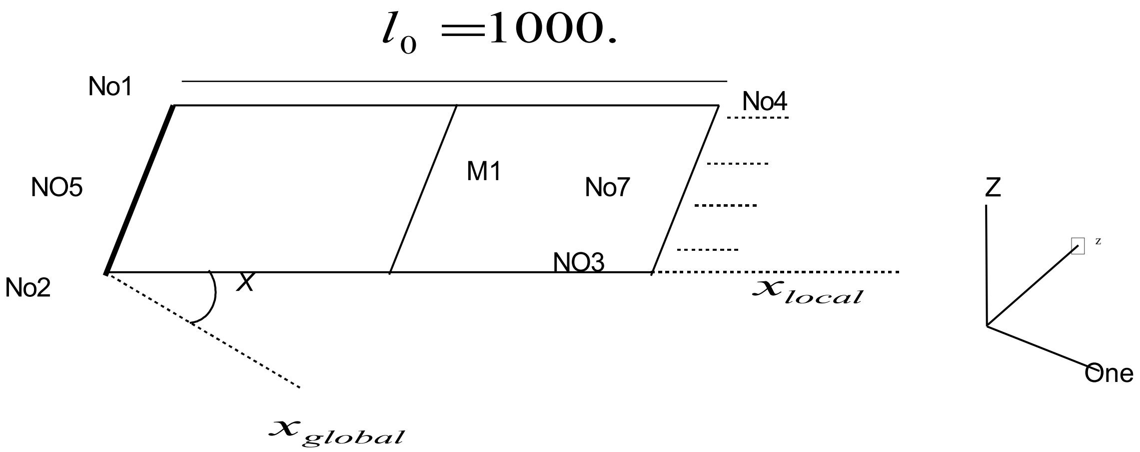
Summarized

the goal of this test is to validate the computation of the stresses of Cauchy by option SIGM_ELNO.

1 Problem of reference

1.1 Geometry

the geometry of this test is a square plate in the plane (x, y) turned from $30°$ ratio with x around z.



calls l the length of the deformed plate there, one will note x, y, z the coordinates of the deformed configuration and the X, Y, Z coordinated initial configuration

1.2 Properties of the materials

One takes $E = 200\,000.MPa$ et $\nu = 0$

1.3 Boundary conditions and mechanical loadings

the nodes are blocked $No1$, $No5$ and $No2$ so that $DX = DY = DZ = DRX = DRY = DRZ = 0$, and one imposes a local displacement $Dx = 100.$ on the nodes $No3$, $No4$ and $No7$.

2 Reference solution

2.1 Method of calculating used for the reference solution

the reference solution is analytical.

Transition of the initial state in a deformed state:

$$x=\frac{1}{l_0}X,\ \ y=\frac{a}{a_0}Y,\ \ z=\frac{b}{b_0}Z$$

where
- a is the length of the deformed shape of the plate according to Y,
- a_0 is the initial length of the plate,
- b is the thickness of the deformed plate,
- b_0 is the initial thickness of the plate.

Owing to the fact that $\nu=0$ and of the assumptions of shell, one has $a=a_0,\ \ b=b_0$

Tensor Green-Lagrange:

By definition of the tensor of Green-Lagrange, one has $E_{ij}=\frac{1}{2}\left\{\frac{\partial u_i}{\partial X_j}+\frac{\partial u_j}{\partial X_i}+\left(\frac{\partial u_k}{\partial X_i}\right)\left(\frac{\partial u_k}{\partial X_j}\right)\right\}$

With $u=x-X=\frac{l-l_0}{l_0}X$, one thus has $E_{11}=\frac{1}{2}\left\{\frac{l-l_0}{l}+\frac{l-l_0}{l}+\frac{(l-l_0)^2}{l_0^2}\right\}=\frac{1}{2}\left(\frac{l^2-l_0^2}{l_0^2}\right)$

While replacing, one has $E_{11}=\frac{1}{2}\left(\frac{1100^2-1000^2}{1000^2}\right)=0.105$

Deformation gradient:

By definition:

$$F=\begin{bmatrix}\frac{dx}{dX} & \frac{dx}{dY} & \frac{dx}{dZ}\\ \frac{dy}{dX} & \frac{dy}{dY} & \frac{dy}{dZ}\\ \frac{dz}{dX} & \frac{dz}{dY} & \frac{dz}{dZ}\end{bmatrix}=\begin{bmatrix}\frac{l}{l_0} & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

That is to say $J=\det F=\frac{l}{l_0}$

Piola-Kirchhoff stresses of second species:

Either S the stress of $PK2$, in our case, $S_{11}=E.E_{11}=200000\times 0.105=21000$

Stress of Cauchy

Or s the stress tensor of Cauchy, one has the relation $s=\left(\frac{1}{\det F}\right)\left(F.S.F^{T}\right)$, one from of deduced

whereas $s_{xx}=\left(\frac{1}{\frac{l}{l_0}}\right)\frac{l}{l_0}.S_{11}.\frac{l}{l_0}=\frac{l}{l_0}.S_{11}=\frac{1100}{1000}.21000=23100$

2.2 Results of reference

One calculates displacements DX and DY with the node $NO3$, the stresses of $PK2$ and the forced of Cauchy on the mesh $M1$.

2.3 Uncertainty on the solution

Result analytical.

2.4 Bibliographical references

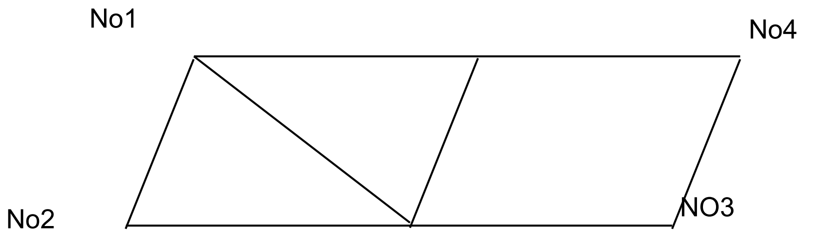
Nothing.

3 Modelization A

3.1 Characteristic of the modelization

One uses elements `COQUE_3D`

3.2 Characteristics of the mesh



the coordinates of the principal nodes:

Node	$Coor_x$	$Coor_y$	$Coor_z$
$N01$	– 500	866.025	0.
$N02$	0	0	0.
$N03$	866.025	500	0.
$N04$	366.025	1366.025	0.

Meshes used are:
1 mesh `QUAD9`
2 meshes `TRIA7`

3.3 Quantities tested and results

Identification	Reference	*Aster*	Difference
DX ($No4$)	8.66025 E+01	8.66025 E+01	4.66 E-05%
DY ($No4$)	50.0	50.0	0%
$PK2$ - $SIXX$ ($M1$)	21000.	21000.	2.04 E-08%
Cauchy- $SIXX$ ($M1$)	23100.	23100.	2.14 E-08%

4 Summary of the results

the found results is in agreement with the analytical solution.