SSNV150 - Triaxial tension with constitutive law BETON_DOUBLE_DP

Summarized

This case of validation is intended to the model check behavior 3D BETON_DOUBLE_DP formulated in the frame of thermoplasticity, for the description of the nonlinear behavior of the concrete, in tension, and compression, with the taking into account of the irreversible variations of the thermal and mechanical characteristics of the concrete, particularly sensitive at high temperature.

The description of cracking is treated in the frame of plasticity, using an energy equivalence, by identifying the density of energy of cracking in mode I, with the plastic work of a homogeneous medium are equivalent, where the plastic strain is uniformly distributed, in an “elementary” zone. This approach preserves the continuity of the formulation of the model, on the group of its behavior, and contributes to avoid the possible numerical difficulties during the change of state of the material.
The pathological sensitivity of the numerical solution to the spatial discretization (mesh), generated by the introduction of a softening behavior of the concrete in tension and compression, is partially solved by introducing an energy of cracking or fracture, dependant a characteristic length l_c, bound in keeping with elements. The resolution of the constitutive equations of the model is carried out by an implicit scheme.

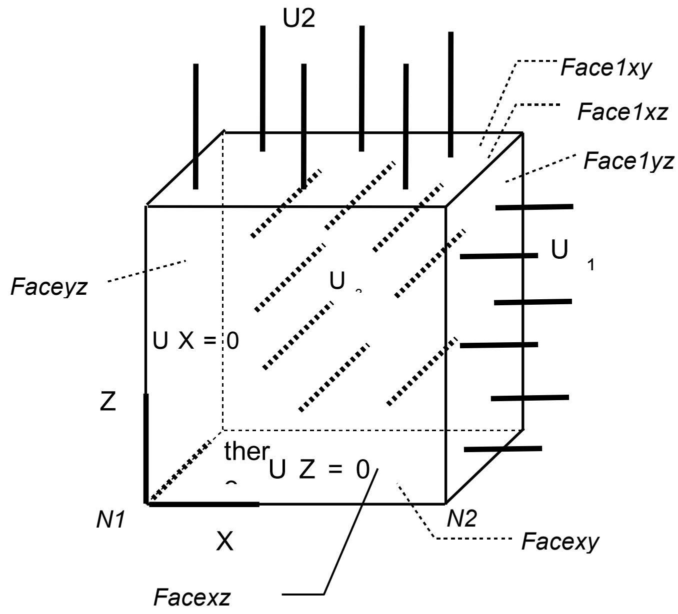
It is about a cube with 8 nodes subjected to a triaxial tension, in imposed displacement. This loading led to the typical case of a hydrostatic stress state, solved by projection at the top of the cone of tension, when one places oneself in a hydrostatic diagram forced equivalent stress/. It is about a case test with analytical solution.

1 Problem of reference

1.1 Geometry

It acts of a cube with 8 nodes, whose three sides have a normal displacement and the no one three opposite sides an imposed and identical normal displacement.
The made cube $1\,mm$ on side. In the modelization A, the cube is directed according to the reference $Oxyz$.

Modelization A



$U_2 = U_1 = U_3$

1.2 Material properties

to test the irreversible evolution of the mechanical characteristics with the temperature, one applies a field of temperature decreasing. Certain variables depend on the temperature, others of drying. Lastly, one applies a coefficient of shrinkage of desiccation non-zero, equal to the thermal coefficient of thermal expansion, to test “data-processing” operation. The thermal strains thus equal and will be opposed to the strains of shrinkage of desiccation. These dependences intervene only for purely data-processing checks, the mechanical characteristics can be regarded as constants.

For the usual linear mechanical characteristics:

Young's modulus:	$E=32\,000\,MPa$	of	$0°C$ with $20°C$
	$E=15\,000\,MPa$		$400°C$ (linear decrease)
	$E=5\,000\,MPa$	with	$800°C$ (linear decrease)
Poisson's ratio:	$\nu=0.18$		
Thermal coefficient of thermal expansion:	$a=10^{-5}/°C$		
Coefficient of shrinkage of desiccation:	$k=10^{-5}$		

For the nonlinear mechanical characteristics of model `BETON_DOUBLE_DP` :

Strength in uniaxial pressing:	$f'c=40\,N/mm^2$	of	$0°C$ with $400°C$
	$f'c=15\,N/mm^2$		$800°C$ (linear decrease)
Strength in uniaxial tension:	$f't=4\,N/mm^2$	of	$0°C$ with $400°C$
	$f't=1.5\,N/mm^2$		$800°C$ (linear decrease)
Ratio of strength in biaxial compression/uniaxial pressing:	$b=1.16$		
Energy of fracture in compression:	$G_c=10\,Nmm/mm$		
Energy of fracture in tension:	$G_t=0.1\,Nmm/mm$		
Ratio of the elastic limit to strength in uniaxial pressing:	30%		

1.3 Boundary conditions and mechanical loadings

Field of temperature decreasing of $20°C$ with $0°C$.

Lower face of the cube ($facexy$):	blocked according to oz .
Upper face of the cube ($face1xy$):	face $U_z=0{,}15\,mm$
left displacement of the cube ($faceyz$):	blocked according to ox .
Right face of the cube ($face1yz$):	face $U_x=0{,}15\,mm$
displacement before cube ($facexz$):	blocked according to oy .
Face postpones cube ($face1xz$):	reference solution $U_y=0{,}15\,mm$

2 displacement

2.1 Method of calculating used for the reference solution

the reference solution is calculated in an analytical way, knowing that in tension, only the criterion of tension is activated, and that in the case of a hydrostatic loading, one is projected at the top of the cone of tension. It is thus necessary to solve a linear system of an equation to an unknown, who allows to obtain the plastic strain cumulated in tension. This one makes it possible to calculate strains and stresses then.

2.2 Computation of the reference solution of reference

For more detail on the notations and the setting in equation, one will refer to the reference document [R7.01.03]. Only, the principal equations are pointed out here.
One notes "" a , imposed displacement following the directions x , y and z . The strain tensor is form $(a, a, a, 0., 0., 0.)$ by taking the usual notations of *Code_Aster* (three principal components, three components of shears).
The stress tensor is form $(\sigma, \sigma, \sigma, 0., 0., 0.)$, in modelization A.

general Équations of the model:

The constitutive equations of the model are written by distinguishing the isotropic part of the deviatoric part of the stress tensors and strains.

$$\sigma_H = \frac{1}{3} tr(\sigma) \quad s = \sigma - \frac{1}{3} tr(\sigma) I \quad \varepsilon_H = \frac{1}{3} tr(\varepsilon) \quad \tilde{\varepsilon} = \varepsilon - \frac{1}{3} tr(\varepsilon) I$$

$$\sigma = s + \sigma_H I \quad \text{and} \quad \varepsilon = \tilde{\varepsilon} + \varepsilon_H I$$

the equivalent stress is written then: $\sigma^{eq} = \sqrt{\frac{3}{2} tr(s^2)}$

In the case of an incremental formulation, and from a variable constitutive law, by noting with an exhibitor *"E"* the elastic components of the stress and strain, one obtains:

$$s^e = \frac{\mu^+}{\mu^-} s^- + 2\mu^+ \Delta\tilde{\varepsilon} \quad \text{and the} \quad {\sigma_H}^e = \frac{K^+}{K^-} {\sigma_H}^- + 3K^+ \Delta\varepsilon_H$$

criteria in compression (f_{comp}) and tension (f_{trac}) are expressed in the following way:

$$f_{comp} = \frac{\tau_{oct} + a.\sigma_{oct}}{b} - f_c(\lambda_c) = \frac{\sqrt{2}}{3b} \sigma^{eq} + \frac{a}{b} \sigma_H - f_c(\lambda_c)$$

$$f_{trac} = \frac{\tau_{oct} + c.\sigma_{oct}}{d} - f_t(\lambda_t) = \frac{\sqrt{2}}{3d} \sigma^{eq} + \frac{c}{d} \sigma_H - f_t(\lambda_t)$$

avec $\tau_{oct}=\sqrt{\frac{tr\left(s^2\right)}{3}}$

$\sigma_{oct}=\sqrt{\frac{tr(\sigma)}{3}}$

λ_c : plastic multiplier in compression

λ_T : plastic multiplier in tension

and the a,b,c,d coefficients of the model

plastic strains in tension and compression are expressed:

$$\Delta\tilde{\varepsilon}^p{}_c=\frac{\Delta\lambda_c}{\sqrt{2}b}\frac{s}{\sigma^{eq}} \qquad \Delta\varepsilon^p{}_{H_c}=\Delta\lambda_c\frac{a}{3b}$$

$$\Delta\tilde{\varepsilon}^p{}_t=\frac{\Delta\lambda_t}{\sqrt{2}d}\frac{s}{\sigma^{eq}} \qquad \Delta\varepsilon^p{}_{H_t}=\Delta\lambda_t\frac{c}{3d}$$

One obtains for the stress:

$$s=s^e-2\mu^+\left(\Delta\tilde{\varepsilon}^p{}_c+\Delta\tilde{\varepsilon}^p{}_t\right) \qquad \sigma_H=\sigma_H{}^e-3K^+\left(\Delta\varepsilon_H{}^p{}_c+\Delta\varepsilon_H{}^p{}_t\right)$$

$$s=\left(1-2\mu^+\left(\frac{\Delta\lambda_c}{\sqrt{2}b}+\frac{\Delta\lambda_t}{\sqrt{2}d}\right)\frac{1}{\sigma^{e\,eq}}\right)s^e \qquad \sigma_H=\sigma_H{}^e-3K^+\left(\Delta\lambda_c\frac{a}{3b}+\Delta\lambda_t\frac{c}{3d}\right)$$

for the equivalent stress:

$$\sigma^{eq}=\sigma^{e\,eq}-2\mu^+\left(\frac{\Delta\lambda_c}{\sqrt{2}b}+\frac{\Delta\lambda_t}{\sqrt{2}d}\right)$$

The two criteria lead then to a system of two equations to two unknowns $\Delta\lambda_c$ and $\Delta\lambda_t$ to solve:

$$\begin{cases}\frac{\sqrt{2}}{3b}\sigma^{e\,eq}+\frac{a}{b}\sigma_H{}^e-\Delta\lambda_c\left(\frac{2\mu^+}{3b^2}+\frac{K^+a^2}{b^2}\right)-\Delta\lambda_t\left(\frac{2\mu^+}{3bd}+\frac{K^+ac}{bd}\right)-f_c\left(\lambda_c{}^-+\Delta\lambda_c\right)=0\\ \frac{\sqrt{2}}{3d}\sigma^{e\,eq}+\frac{c}{d}\sigma_H{}^e-\Delta\lambda_c\left(\frac{2\mu^+}{3bd}+\frac{K^+ac}{bd}\right)-\Delta\lambda_t\left(\frac{2\mu^+}{3d^2}+\frac{K^+c^2}{d^2}\right)-f_t\left(\lambda_t{}^-+\Delta\lambda_t\right)=0\end{cases}$$

In a similar way, in the case of the only criterion of tension activated, **configuration of the case test**, one obtains a system of an equation to an unknown $\Delta\lambda_t$ to be solved:

$$\begin{cases}\frac{\sqrt{2}}{3d}\sigma^{e\,eq}+\frac{c}{d}\sigma_H{}^e-\Delta\lambda_t\left(\frac{2\mu^+}{3d^2}+\frac{K^+c^2}{d^2}\right)-f_t\left(\lambda_t{}^-+\Delta\lambda_t\right)=0\end{cases}$$

Resolution with projection at the top of the cone of tension:
One thus seeks to solve this system, by means of the particular form of the stress tensors and strains, uniforms on the structure.
On the basis of $\varepsilon=(a, a, a, 0., 0., 0.)$ and of $\sigma=(\sigma, \sigma, \sigma, 0., 0., 0.)$. one obtains:

The elastic stress tensor $\begin{cases}\sigma_x = a(3\lambda+2\mu)\\ \sigma_y = a(3\lambda+2\mu)\\ \sigma_z = a(3\lambda+2\mu)\end{cases}$

the elastic deviator of stress $\begin{cases}s_x = 0\\ s_y = 0\\ s_z = 0\end{cases}$

the elastic hydrostatic stress $\sigma^e{}_H = (3\lambda+2\mu)(a) = 3aK$

the elastic equivalent stress $\sigma^e{}_{eq} = 0$

In the case of a curve of linear hardening post-peak in tension, the statement of the hardening parameter is the following one:

$$f_t\left(\theta,\left\|\varepsilon_t^p\right\|\right) = \tau(\theta,\kappa) = f_t'(\theta)\left(1-\frac{\left\|\varepsilon_t^p\right\|}{\kappa_u(\theta)}\right) \text{ with } \kappa_u(\theta) = \frac{2.G_f(\theta)}{l_c . f_t'(\theta)}$$

where θ the maximum of temperature during the history of loading indicates, $f_t'(\theta)$ strength in tension.

$$\Delta\tilde{\varepsilon}^p{}_t = 0 \qquad \Delta\varepsilon^p{}_{H_t} = \Delta\lambda_t \frac{c}{3d} \qquad \sigma_H = \sigma_H{}^e - 3K^+\left(\Delta\lambda_t \frac{c}{3d}\right)$$

The equation characterizing projection at the top of the cone of tension is the following one:

$$\left\{\frac{c}{d}\sigma_H{}^e - \Delta\lambda_t \frac{K^+c^2}{d^2} - f_t'\left(1-\Delta\lambda_t \frac{l_c . f_t'}{2.G_t}\right) = 0 \right. \qquad \text{éq 2.2-1}$$

G_t being the energy of fracture in tension (characteristic of the material).

What makes it possible to obtain the plastic multiplier:

$$\Delta\lambda_t = \frac{\frac{c}{d}\sigma_H{}^e - f_t'}{\frac{K^+c^2}{d^2} - \frac{l_c .(f_t')^2}{2.G_t}} = \frac{\frac{c}{d}(3aK) - f_t'}{\frac{K^+c^2}{d^2} - \frac{l_c .(f_t')^2}{2.G_t}}$$

Then the stress: $\sigma_H = \sigma_H{}^e - 3K^+\left(\Delta\lambda_t \frac{c}{3d}\right) = K\left(3.a - \Delta\lambda_t \frac{c}{d}\right)$

Knowing has, imposed displacement, one obtains all the unknowns of the problem.

2.3 Uncertainty on the solution

the solution being analytical, uncertainty is negligible, about the accuracy of the machine.

2.4 Bibliographical references

The model was defined starting from the following theses and is described in the ratio of specification:

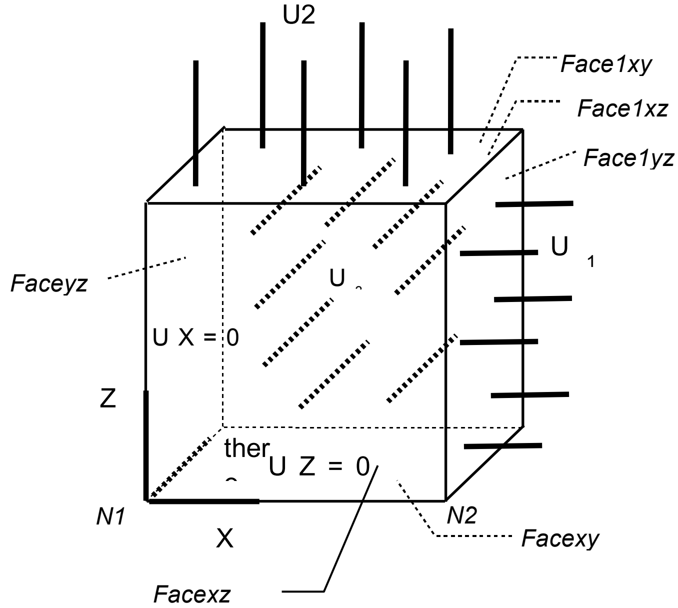
- G. Heinfling, during its thesis "Contribution to the numerical modelization of the behavior of the reinforced concrete concrete and structures under thermomechanical requests at high temperature",
- J.F. Georgin, during its thesis "Contribution to the modelization of the concrete under request of fast dynamics. The taking into account of the effect velocity by viscoplasticity".
- SCSA/128IQ1/RAP/00.034 Version 1.2, Development of a model of behavior 3D concrete with double plasticity criterion in *the Code_Aster* - Specifications".

3 Modelization A

3.1 Characteristic of the modelization

3D (HEXA8)

1 element, stress field and uniform strain.



3.2 Characteristics of the mesh

Many nodes: 8
Number of meshes and type: 1 HEXA8

3.3 Values tested

were tested the components xx and zz of stress field SIGM_ELNO, and the plastic strain cumulated in tension (second local variable, second component of field VARI_ELNO). Displacement being imposed, field EPSI_ELNO is not tested.

Three times correspond to a displacement of 0.005 , 0.01 and $0.015\,mm$.

Field SIGM_ELNO component SIXX

Identification	Reference
For an imposed displacement charges 1.9182065 $U_1=U_2=U_3=0.005$	For
an imposed displacement with them charges 1.161 $U_1=U_2=U_3=0.010$	6770 For
an imposed displacement with them charges with it $U_1=U_2=U_3=0.015$	0.4051470

Field SIGM_ELNO component SIZZ

Identification	Reference
For an imposed displacement charges 1.9182065 $U_1=U_2=U_3=0.005$	For
an imposed displacement with them charges 1.1616770 $U_1=U_2=U_3=0.010$	For
an imposed displacement with them charges with it $U_1=U_2=U_3=0.015$	0.4051470

Field VARI_ELNO component V2 (plastic strain cumulated in tension)

Identification	Reference
For an imposed displacement charges 0.0099232717 $U_1=U_2=U_3=0.005$	For
an imposed displacement with them charges 0.0199535329 $U_1=U_2=U_3=0.010$	For
an imposed displacement with them charges 0.0299837941 $U_1=U_2=U_3=0.015$	Summary

4 of the results with it

This case test compared to the offers very satisfactory results analytical solution, lower than 7.10^{-5} % with a low nombre of iterations (1 or 2 iterations). The solution is obtained from a linear equation in the case of a linear curve of hardening in tension, but the resolution uses an algorithm of Newton in a more general frame.

One can note the hardening of the criterion of tension which takes place during the loading, involving a reduction in the stress (component xx, yy and zz) in addition equalizes with the hydrostatic stress.