SSNV146 - Regularized limit analysis. Spherotoric bottom reserve

Abstract

This test makes it possible to qualify the operators used in regularized limit analysis.

One calculates the Yield-point load by a kinematical approach regularized by the method of Norton-Hoff-Friaâ. Compared to the index A of this document, it is to be noted that the méthode de calcul was modified in the Code_Aster which does not use now any more the Norton-Hoff material but called on the more general resolution with incompressible elements.

The problem of reference is resulting from a European benchmark carried out in the frame of a project Brite EuRam BE97-4547 "LISA", in 1998.

One considers an axisymmetric spherotoric bottom reserve. The constitutive material checks the criterion of Von Mises and the structure is subjected to an internal pressure.

The structure is modelled by incompressible elements.

The resolution by the regularized method of Norton-Hoff-Friaâ is carried out in command `STAT_NON_LINE`. A postprocessing in command `POST_ELEM` makes it possible to obtain the estimates of the hight delimiters and lower of the Yield-point load.

The reference solution is numerical and the results are in perfect agreement with the values of reference. Certain details of implementation of this benchmark are presented in the document of assistance to the use [U2.05.04].

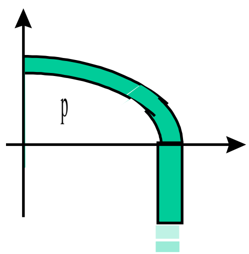
1 Problem of reference

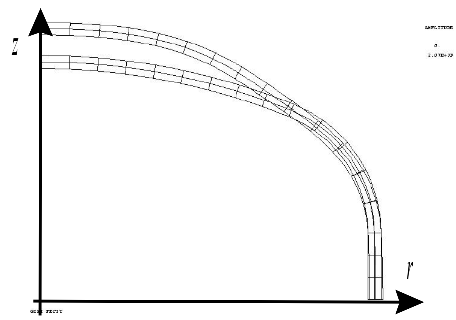
the test is benchmark LA6 of a benchmark of the European project Brite EuRam BE97-4547 “LISA” studied by three organizations:

- EDF (Norton-Hoff method with three values of the parameter of regularization: the Norton-Hoff coefficient n);
- LTAS in Liege (regularized kinematical method of the university software dedicated “**ELSA** ”);
- ForschungZentrum de Jülich (static method approached by finite elements in displacements, reduced representation of the fields of auto-contraintes and algorithm of optimization of the Permas **code**).

One considers an axisymmetric spherotoric bottom reserve subjected to an internal pressure.

The same mesh is used by the three participating organizations. It contains two Q8 elements in the thickness, and in all there are 34 elements, and 141 nodes. The constitutive material is homogeneous and checks the criterion of Von Mises with for threshold the elastic limit (σ_y) of $100\,MPa$.





Results EDF LISA: initial mesh and deformed shape obtained for the coefficient Norton-Hoff *n=31*

The computation of the Yield-point load by the Norton-Hoff-Friaâ method are detailed in the reference document [R7.07.01].

Briefly let us recall that one aims at seeking the Yield-point load λ_{lim} , for which the structure, made up of a material of the perfect elastoplastic type with threshold of von Mises, can support the surface loadings $\lambda_{lim} F$ and the loadings of volume $\lambda_{lim} f$ to which one can possibly add constant loadings (F_0 and f_0). Code_Aster makes it possible to calculate, for each time of computation, i.e. for increasingly weak regularizations, two parameters:

•an estimate of a higher limit of the Yield-point load::

$$\hat{\lambda}_m=\int_{\Omega} \sigma_y \sqrt{\frac{2}{3}\varepsilon(u_m)\cdot\varepsilon(u_m)}\,d\Omega - L_0(u_m)$$

•and, in the absence of constant loading, an estimate of a lower limit $\underline{\lambda}_m$:

$$\underline{\lambda}_m=\int_{\Omega} \frac{A(m)}{m}\cdot\left(\sqrt{\varepsilon(u_m)\cdot\varepsilon(u_m)}\right)^m d\Omega\cdot\left(\operatorname*{Sup}_{x\in\Omega}\left(\frac{\sqrt{\frac{3}{2}\sigma^D(u_m)\cdot\sigma^D(u_m)}}{\sigma_y}\right)\right)^{-1}\leq\hat{\lambda}_m$$

if a constant loading is present, there is the power of the constant loading in the velocity field solution of the problem.

It is noted that the tensor deviatoric of the stresses checks the behavior model of Norton-Hoff:

$$\sigma^D(u)=A(m).\left(\sqrt{\varepsilon^D(u).\varepsilon^D(u)}\right)^{m-2}.\varepsilon^D(u)\quad\Leftrightarrow\sigma^D(u)=A(m)^n.\left(\sqrt{\sigma^D(u).\sigma^D(u)}\right)^{1-n}.\varepsilon^D(u)$$

with $\mathrm{tr}\,\varepsilon(u)=0$, and: $A(m)=k^{1-m}\left(\frac{2}{3}\right)^{m/2}\sigma_y{}^m$ and $n=\dfrac{1}{m-1}$.

1.1 Given for the modelization

Geometry	the axisymmetric spherotoric bottom reserve has the following characteristics: • radius interns cylindrical part: $49\,mm$; • thickness: $2\,mm$; • radius of the spherical part to the apex: $98\,mm$; • radius of the torus of connection: $20\,mm$.
• Material properties	Elastic limit: $\sigma_y=100\,MPa$
Boundary conditions	axial Displacement DY no one on the end $BORD_{INF}$ of the cylindrical part (conditions of symmetry)
Loadings	internal Pressure of $1\,MPa$ applied to the internal wall B_D

1.2 Results of reference

For this case test, one does not have of results analytical but only the numerical values resulting from the computations carried out in the frame of the benchmark LISA and pointed out Table 1.2-1.

The regularized kinematical methods of EDF and the University of Liege give very nearby results. The lower limit provided by FZJ is more important than the preceding hight delimiters, which is an anomaly.

The results selected as values of reference are thus those provided by Ulg and EDF with the old method of calculating of the Yield-point load established in the Code_Aster (using the Norton-Hoff material) which we will identify below like “EDF LISA”.

Modelization			estimated Value higher	estimated lower Value
EDF (1)	$m=1{,}0476$	$n=21$	3.9514	3,6049
	$m=1{,}0322$	$n=31$	3.9456	3,7090
	$m=1{,}0141$	$n=71$	3.9404	3,8372
	$m=1{,}0099$ (2)	$n=101$ (2)	3.9396	3.8673
Univ. of Liège/LTAS			3,931	nothing
Research center FZJ			nothing	3.997

Table 1.2-1 : Results of the benchmark LISA

Foot-note (1) : *the results provided by EDF were got with an old version of Code_Aster which used, for the calculde the Yield-point load, the material Norton-Hoff* $n=(m-1)^{-1}$

Foot-note (2) : *value obtained later on with the benchmark*

1.3 bibliographical References

[1] Voldoire F.: Computation of Yield-point load with *Code_Aster* and benchmark of Brite EuRam "LISA". Note HI-74/98/026/A.

[2] Heitzer Mr. "Traglast- und Einspielanalyse zur Bewertung der Sicherheit to passivate Komponenten." Thesis., RWTH Aachen (1999).

[3] Direct Yan A.M. "Contributions to the limit state analysis of plastified and cracked structures". Thesis, Univ. Liege, (1999).

2 Modelization A

For the limit analysis, Code_Aster uses:

- of the finite elements quasi-incompressible;
- a kinematical approach regularized (method of regularization of Norton-Hoff-Friaâ, cf [R7.07.01]) for the strength criterion of Von Mises (adjustment by a coefficient of regularization of which the limiting value led to convergence);
- a nonlinear static resolution by parametric control;
- postprocessing to obtain an estimate of the values higher $\lambda_{\lim}^{\sup}$ and lower $\lambda_{\lim}^{\inf}$ which frame the limiting value $\lambda_{\lim}$.

The list of times is used to control the method of regularization as Norton-Hoff via a coefficient t, ($m=1+10^{1-t}$), and not the evolution of the loading like during an ordinary computation.

2.1 Characteristics of the modelization

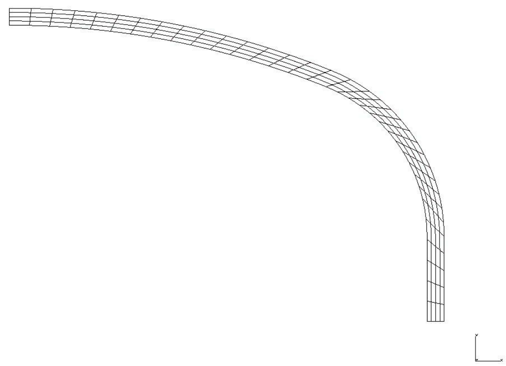
One considers a cylinder modelled by incompressible axisymmetric elements of type `QUAD8`.

2.2 Characteristics of the mesh

Many nodes: 485

Number of meshes: 212

Type of meshes:
136 meshes of incompressible `type` QUAD8;
76 meshes of type `SEG3` for the application of the pressure.



2.3 Quantities tested and results

With the mesh considered, the computation of the benchmark is stopped with $t=2{,}85126\,s$. Additional details on computation are indicated in the document of assistance to the use [U2.05.04].

The Norton-Hoff coefficient corresponding to this time is $m=1{,}0141 \quad n=71$.

Identification	Reference	*Aster*	% difference
	EDF LISA N = 71		
higher Yield-point load	3,9404	3,9351	-0,133
estimated Yield-point load	3,8372	3,8245	-0,331
	EDF LISA N = 101		
higher Yield-point load	3,9396	3,9351	-0,1142

estimated Yield-point load	3,8673	3,8245	-1,0861
	Univ, of Liège/LTAS		
higher Yield-point load	3,931	3,9351	0,106

Table 2.3-1 **: Results of the comparison**

3 Summary of the results

the numerical results of *Code_Aster* are in concord with the values of the numerical references.