SSNV143 - Biaxial tension with constitutive law BETON_DOUBLE_DP

Summarized:

This case of validation is intended to the model check behavior $3D$ BETON_DOUBLE_DP formulated in the frame of thermoplasticity, for the description of the nonlinear behavior of the concrete, in tension, and compression, with the taking into account of the irreversible variations of the thermal and mechanical characteristics of the concrete, particularly sensitive at high temperature.
The description of cracking is treated in the frame of plasticity, using an energy equivalence, by identifying the density of energy of cracking in mode I, with the plastic work of a homogeneous medium are equivalent, where the plastic strain is uniformly distributed, in an "elementary" zone. This approach preserves the continuity of the formulation of the model, on the group of its behavior, and contributes to avoid the possible numerical difficulties during the change of state of the material.
The pathological sensitivity of the numerical solution to the spatial discretization (mesh), generated by the introduction of a softening behavior of the concrete in tension and compression, is partially solved by introducing an energy of cracking or fracture, dependant a characteristic length l_c, bound in keeping with elements.

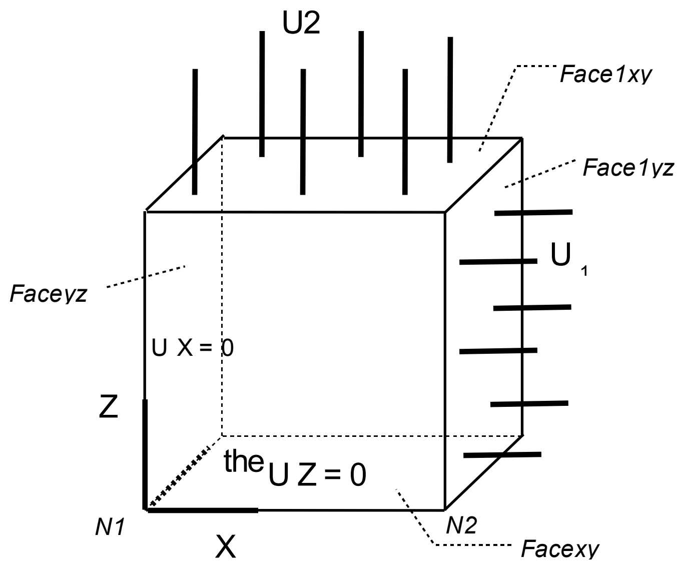
The case test understands two modelizations 3D, the loading consists of a load followed by a discharge.

1 Problem of reference

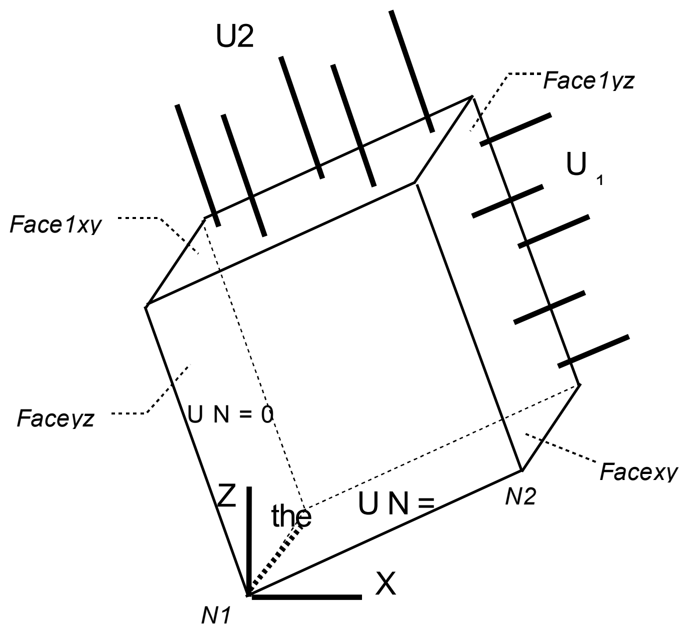
1.1 Geometry

It acts of a cube with 8 nodes, whose two sides have a normal displacement no one, and the two opposite sides have an imposed normal displacement, different one from the other of a coefficient 2. The made cube $1\,mm$ on side. The cases tests are composed of a load, followed by a discharge. In the modelization A, the cube is directed according to the reference $Oxyz$. In the modelization B, it is turned from $30°$ around the axis Oy.

Modelization A



Modelization B



$U_2 = 2.\mathrm{U}_1$

1.2 Material properties

to test the evolution of the mechanical characteristics in an irreversible way with the temperature, one applies a field of temperature decreasing. Certain variables depend on the temperature, others of drying. Lastly, one applies a coefficient of shrinkage of desiccation non-zero, equal to the thermal coefficient of thermal expansion, to test “data-processing” operation. The thermal strains thus equal and will be opposed to the strains of shrinkage of desiccation. These dependences intervene only for checks purely data-processing, the mechanical characteristics can be regarded as constants.

For the usual linear mechanical characteristics:

Young modulus:	$E=32\,000\,MPa$	*of*	$0\,°C$ *with* $20\,°C$
	$E=15\,000\,MPa$		$400\,°C$ *(linear decrease)*
	$E=5\,000\,MPa$	*with*	$800\,°C$ *(linear decrease)*
Poisson's ratio:	$\nu=0.18$		
Thermal coefficient of thermal expansion:	$\alpha=10^{-5}$		
Coefficient of shrinkage of desiccation:	$\kappa=10^{-5}$		

For the nonlinear mechanical characteristics of model `BETON_DOUBLE_DP` :

Strength in uniaxial pressing:	$f'c=40\,N/mm^2$	*of*	$0\,°C$ *with* $400\,°C$
	f' C = 15 N/mm^2	*with*	$800\,°C$ *(linear decrease)*
Strength in uniaxial tension:	$f't=4\,N/mm^2$	*of*	$0\,°C$ *with* $400\,°C$
	$f't=1.5\,N/mm^2$		$800\,°C$ *(linear decrease)*
Ratio of strength in biaxial compression/uniaxial pressing:	$\beta=1.16$		
Energy of fracture in compression:	$Gc=10\,Nmm/mm$		
Energy of fracture in tension:	$\mathrm{Gt}=0.1\,Nmm/mm$		
Ratio of the elastic limit to strength in uniaxial pressing:	*30%*		

1.3 Boundary conditions and mechanical loadings

Field of temperature decreasing of $20\,°C$ with $0\,°C$.

Lower face of the cube ($facexy$):	blocked according to oz .
Upper face of the cube ($face1xy$):	imposed $0.30\,mm$ displacement followed by a left discharge $0.1\,mm$
of Face of the cube ($faceyz$):	blocked according to ox .
Right face of the cube ($face1yz$):	imposed $0.15\,mm$ displacement followed by a discharge of $0.05\,mm$
lower Nodes front face (N_1 , N_2):	blocked according to oy (Suppression of motions of solid body).

2 Reference solution

2.1 Method of calculating used for the reference solution

the reference solution is calculated in an semi-analytical way, knowing that in tension, only the criterion of tension is activated. It is thus necessary to solve a system of an equation to an unknown, who allows to obtain by dichotomy for example, the plastic strain cumulated in tension. This one makes it possible to calculate strains and stresses then. This is possible, knowing displacement, and thus the strain in the two imposed directions. Displacement in the third direction is then an unknown of the problem.
The reference solution is calculated only in tension. The solution is determined by a programme of resolution per dichotomy in independent FORTRAN. In compression, in discharge, the exact solution was not recomputed, and constitutes a solution of non regression code, been dependant on version 5.02.14.
For the modelization B, the results result by rotation from the stress tensor from the modelization A, of the intrinsic reference of the cube to the reference user, the stress field of the two configurations being identical in the intrinsic reference of the cube.

2.2 Computation of the reference solution of reference

For more details on the notations and the setting in equation, one will refer to the reference document. Only, the principal equations are pointed out here.
One notes a, the displacement imposed according to the direction x, and $2.\mathrm{a}$ imposed displacement following the direction z. The strain tensor is form $(a, \varepsilon_y, 2.\mathrm{a}, 0., 0., 0.)$ by taking the usual notations of *Code_Aster* (three principal components, three components of shears).
The stress tensor is form $(\sigma_x, 0., \sigma_z, 0., 0., 0.)$, in modelization A.
the criterion of tension is expressed in the form:

$$f_{trac} = \frac{\tau_{oct} + c.\sigma_{oct}}{d} - f_t(\lambda_t) = \frac{\sqrt{2}}{3d}\sigma^{eq} + \frac{c}{d}\sigma_H - f_t(\lambda_t)$$

The constitutive equations are written by distinguishing the isotropic part of the deviatoric part of the stress tensors and strains.

$$\sigma_H = \frac{1}{3}tr(\sigma) \quad s = \sigma - \frac{1}{3}tr(\sigma)I \quad \varepsilon_H = \frac{1}{3}tr(\varepsilon) \quad \tilde{\varepsilon} = \varepsilon - \frac{1}{3}tr(\varepsilon)I$$

$$\sigma = s + \sigma_H I$$

$$\varepsilon = \tilde{\varepsilon} + \varepsilon_H I$$

The equivalent stress is written then: $\sigma^{eq} = \sqrt{\frac{3}{2}tr(s)}$

In the case of an incremental formulation, and from a variable constitutive law, by noting with an exhibitor e the elastic components of the stress and strain, one obtains:

$s^e = \frac{\mu^+}{\mu^-}s^- + 2\mu^+\Delta\tilde{\varepsilon}$ and the $\sigma_H{}^e = \frac{K^+}{K^-}\sigma_H{}^- + 3K^+\Delta\varepsilon_H$

criteria in compression and tension are expressed in the following way:

$$f_{comp}=\frac{\tau_{oct}+a.\sigma_{oct}}{b}-f_c\left(\lambda_c\right)=\frac{\sqrt{2}}{3b}\sigma^{eq}+\frac{a}{b}\sigma_H-f_c\left(\lambda_c\right)$$

$$f_{trac}=\frac{\tau_{oct}+c.\sigma_{oct}}{d}-f_t\left(\lambda_t\right)=\frac{\sqrt{2}}{3d}\sigma^{eq}+\frac{c}{d}\sigma_H-f_t\left(\lambda_t\right)$$

Plastic strains in tension and compression express themselves:

$$\Delta\tilde{\varepsilon}^p{}_c=\frac{\Delta\lambda_c}{\sqrt{2}b}\frac{s}{\sigma^{eq}} \qquad \Delta\varepsilon^p{}_{H_c}=\Delta\lambda_c\frac{a}{3b}$$

$$\Delta\tilde{\varepsilon}^p{}_t=\frac{\Delta\lambda_t}{\sqrt{2}d}\frac{s}{\sigma^{eq}} \qquad \Delta\varepsilon^p{}_{H_t}=\Delta\lambda_t\frac{c}{3d}$$

One obtains for the stress:

$$s=s^e-2\mu^+\left(\Delta\tilde{\varepsilon}^p{}_c+\Delta\tilde{\varepsilon}^p{}_t\right) \qquad \sigma_H=\sigma_H{}^e-3K^+\left(\Delta\varepsilon_H{}^p{}_c+\Delta\varepsilon_H{}^p{}_t\right)$$

$$s=\left(1-2\mu^+\left(\frac{\Delta\lambda_c}{\sqrt{2}b}+\frac{\Delta\lambda_t}{\sqrt{2}d}\right)\frac{1}{\sigma^{e\,eq}}\right)s^e \qquad \sigma_H=\sigma_H{}^e-3K^+\left(\Delta\lambda_c\frac{a}{3b}+\Delta\lambda_t\frac{c}{3d}\right)$$

for the equivalent stress:

$$\sigma^{eq}=\sigma^{e\,eq}-2\mu^+\left(\frac{\Delta\lambda_c}{\sqrt{2}b}+\frac{\Delta\lambda_t}{\sqrt{2}d}\right)$$

The two criteria lead then to a system of two equations to two unknowns $\Delta\lambda_c$ and $\Delta\lambda_t$ to solve:

$$\begin{cases}\frac{\sqrt{2}}{3b}\sigma^{e\,eq}+\frac{a}{b}\sigma_H{}^e-\Delta\lambda_c\left(\frac{2\mu^+}{3b^2}+\frac{K^+a^2}{b^2}\right)-\Delta\lambda_t\left(\frac{2\mu^+}{3bd}+\frac{K^+ac}{bd}\right)-f_c\left(\lambda_c{}^-+\Delta\lambda_c\right)=0\\ \frac{\sqrt{2}}{3d}\sigma^{e\,eq}+\frac{c}{d}\sigma_H{}^e-\Delta\lambda_c\left(\frac{2\mu^+}{3bd}+\frac{K^+ac}{bd}\right)-\Delta\lambda_t\left(\frac{2\mu^+}{3d^2}+\frac{K^+c^2}{d^2}\right)-f_t\left(\lambda_t{}^-+\Delta\lambda_t\right)=0\end{cases}$$

In a similar way, in the case of the only criterion of tension activated, **configuration of the case test** , one obtains a system of an equation to an unknown $\Delta\lambda_t$ to be solved:

$$\left\{\frac{\sqrt{2}}{3d}\sigma^{e\,eq}+\frac{c}{d}\sigma_H{}^e-\Delta\lambda_t\left(\frac{2\mu^+}{3d^2}+\frac{K^+c^2}{d^2}\right)-f_t\left(\lambda_t{}^-+\Delta\lambda_t\right)=0\right.$$

One thus seeks to solve this system, by means of the particular form of the stress tensors and strains, uniforms on the structure.
On the basis of $\varepsilon = (a, \varepsilon_y, 2.a, 0., 0., 0.)\ et\ de\ \sigma = (\sigma x, 0., \sigma z, 0., 0., 0.)$. one obtains:

The elastic stress tensor $\begin{cases} \sigma_x = a(\lambda + 2\mu) + \varepsilon_y \lambda + 2\ a.\lambda \\ \sigma_y = a.\lambda + \varepsilon_y(\lambda + 2\mu) + 2\,a.\lambda \\ \sigma_z = a.\lambda + \varepsilon_y \lambda + 2\,a.(\lambda + 2\mu) \end{cases}$

the elastic deviator of stress $\begin{cases} s_x = -\frac{2}{3}.\mu.\varepsilon_y \\ s_y = -2.\mu.a + \frac{4}{3}.\mu.\varepsilon_y \\ s_z = 2.\mu.a - \frac{2}{3}.\mu.\varepsilon_y \end{cases}$

the elastic hydrostatic stress $\sigma^e{}_H = (3\lambda + 2\mu)\left(a + \frac{1}{3}.\varepsilon_y\right)$

the elastic equivalent stress $\sigma^e{}_{eq} = \mu\sqrt{4\varepsilon_y{}^2 - 12.a.\varepsilon_y + 12.a^2}$

In the case of a curve of linear hardening post-peak in tension, the statement of the hardening parameter is the following:

$f_t\left(\theta, \left\|\varepsilon_t^p\right\|\right) = \tau(\theta, \kappa) = f_t'(\theta)\left(1 - \frac{\left\|\varepsilon_t^p\right\|}{\kappa_u(\theta)}\right)$ with $\kappa_u(\theta) = \frac{2.G_f(\theta)}{l_c.f_t'(\theta)}$

One thus seeks to solve the equation:

$$\left\{\frac{\sqrt{2}}{3d}\sigma^{eeq} + \frac{c}{d}\sigma_H{}^e - \Delta\lambda_t\left(\frac{2\mu^+}{3d^2} + \frac{K^+c^2}{d^2}\right) - f_t'\left(1 - \Delta\lambda_t\,\frac{l_c.f_t'}{2.G_t}\right) = 0\right. \qquad \text{éq 2.2-1}$$

Knowing that the stress in the direction y is null, one obtains one second equation:

$$\sigma_y = s_y + \sigma_H = 0 = \left(1 - \sqrt{2}\mu^+\frac{\Delta\lambda_t}{d}\frac{1}{\sigma^{eeq}}\right)s^e{}_y + \sigma^e{}_H$$

$$\sigma_y = 0 = \left(1 - \sqrt{2}\mu^+\frac{\Delta\lambda_t}{d}\frac{1}{\sigma^{eeq}}\right)\left(\frac{4}{3}\mu\varepsilon_y - 2.\mu.a\right) + \sigma^e{}_H - K\frac{c.\Delta\lambda_t}{d}$$

From where: $\Delta\lambda_t = \dfrac{\left(\frac{4}{3}\mu\varepsilon_y - 2.\mu.a\right) + \sigma^e{}_H}{\frac{\sqrt{2}\mu}{d.\sigma^{eeq}}\left(\frac{4}{3}\mu\varepsilon_y - 2.\mu.a\right) + K\frac{c}{d}}$ that one can substitute in the statement of the criterion [éq 2.2-1].

Knowing a , imposed displacement, one obtains a nonlinear equation with an unknown, whom one can simply solve by dichotomy, and which makes it possible to calculate the strain ε_y , then all the unknowns of the system.

2.3 Uncertainty on the solution

It is negligible, about the machine accuracy.

2.4 Bibliographical references

The model was defined starting from the following theses:

1) J.F. GEORGIN, during its thesis “Contribution to the numerical modelization of the behavior of the reinforced concrete concrete and structures under thermomechanical requests at high temperature”,

2) G. HEINFLING, during its thesis “Contribution to the modelization of the concrete under request of fast dynamics. The taking into account of the effect velocity by viscoplasticity”, and is described in the ratio of specification:

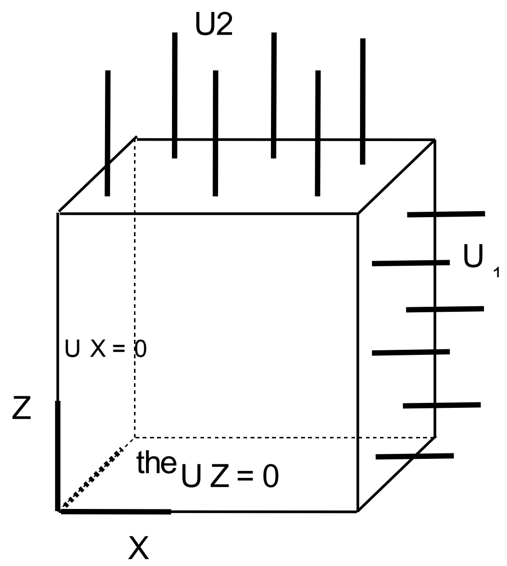
3) SCSA/128IQ1/RAP/00.034 Version 1.2, Development of a model of behavior 3D concrete with double plasticity criterion in *the Code_Aster* - Specifications”.

3 Modelization A

3.1 Characteristic of the modelization

3D (HEXA8)

1 element, stress field and uniform strain.



3.2 Characteristics of the mesh

Many nodes: 8
Number of meshes and type: 1 HEXA8

3.3 Quantities tested and results

were tested the non-zero components of stress field SIGM_ELNO (component xx and zz), the component yy of the strain field EPSI_ELNO , which constitutes an unknown of the system (strains in the two other directions being imposed), the plastic strain cumulated in tension (second local variable, second component of field VARI_ELNO), and finally, only for the fourth case of loading (discharge), the plastic strain cumulated in compression, (first local variable, first component of field VARI_ELNO).

The first three loadings correspond to the load, and have results of reference. The fourth loading corresponds to the discharge, and constitutes result non regression code.

Field SIGM_ELNO component SIXX

Identification	Reference	*Aster*	% difference
For a displacement imposed in load $U_1=0.1$ and $U_2=0.05$	0.1235611	0.1235380	0.019
For a displacement imposed in load $U_1=0.2$ and $U_2=0.10$	$6.882374.10^{-2}$	$6.878218.10^{-2}$	0.060
For a displacement imposed in load $U_1=0.3$ and $U_2=0.15$	$1.408764.10^{-2}$	$1.402639.10^{-2}$	0.435
For a displacement imposed in discharge $U_1=0.1$ and $U_2=0.05$	(*)	$-4.195092.10^{-5}$	-

(*) discharges some, one carries out a test of non regression. There is no calculated analytical solution.

Field SIGM_ELNO component SIZZ

Identification	Reference	*Aster*	% difference
For a displacement imposed in load $U_1=0.1$ and $U_2=0.05$	0.239212	0.239174	0.016
For a displacement imposed in load $U_1=0.2$ and $U_2=0.10$	0.133243	0.133165	0.059
For a displacement imposed in load $U_1=0.3$ and $U_2=0.15$	$2.727403.10^{-2}$	$2.725569.10^{-2}$	0.434
For a displacement imposed in discharge $U_1=0.1$ and $U_2=0.05$	(*)	$-4.959258.10^{-5}$	-

(*) discharges some, one carries out a test of non regression. There is no calculated analytical solution.

Field `EPSI_ELNO` component `EPYY`

Identification	Reference	Aster	% difference
For a displacement imposed in load $U_1=0.1$ and $U_2=0.05$	$-3.419463.10^{-3}$	$-3.419464.10^{-3}$	2.10^{-7}
For a displacement imposed in load $U_1=0.2$ and $U_2=0.10$	$-6.835813.10^{-3}$	$-6.835815.10^{-3}$	2.10^{-7}
For a displacement imposed in load $U_1=0.3$ and $U_2=0.15$	$-1.025216.10^{-2}$	$-1.025216.10^{-2}$	2.10^{-7}
For a displacement imposed in discharge $U_1=0.1$ and $U_2=0.05$	(*)	$-4.357498.10^{-1}$	-

(*) discharges some, one carries out a test of non regression. There is no calculated analytical solution.

Field `VARI_ELNO` component `VARI_2` (plastic strain cumulated in tension)

Identification	Reference	Aster	% difference
For an imposed displacement $U_1=0.1$ and $U_2=0.05$	$1.085728.10^{-2}$	$1.085728.10^{-2}$	5.10^{-9}
For an imposed displacement $U_1=0.2$ and $U_2=0.10$	$2.171556.10^{-2}$	$2.171556.10^{-2}$	5.10^{-9}
For an imposed displacement $U_1=0.3$ and $U_2=0.15$	$3.257385.10^{-2}$	$3.257385.10^{-2}$	4.10^{-9}
For an imposed displacement $U_1=0.1$ and $U_2=0.05$	$3.257385.10^{-2}$	$3.257385.10^{-2}$	4.10^{-9}

(*) discharges some, one carries out a test of non regression. There is no calculated analytical solution.

Field `VARI_ELNO` component `VARI_1` (plastic strain cumulated in compression)

Identification	Reference	Aster	% difference
For a displacement imposed in discharge $U_1=0.1$ and $U_2=0.05$	(*)	$3.528401.10^{-1}$	-

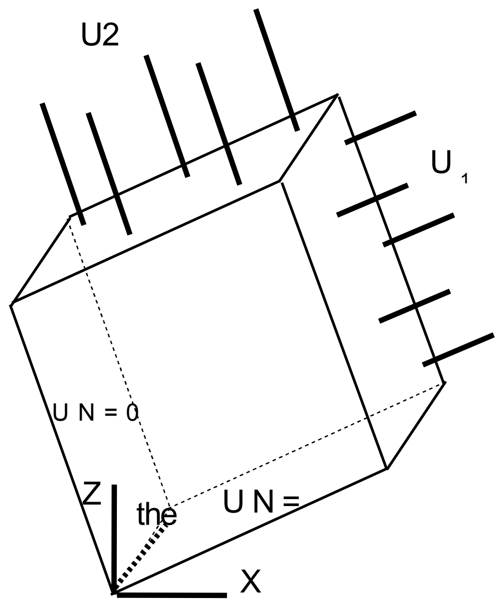
(*) discharges some, one carries out a test of non regression. There is no calculated analytical solution.

4 Modelization B

4.1 Characteristic of the modelization

3D (HEXA8)

1 element, stress field and uniform strain.



4.2 Characteristics of the mesh

Many nodes: 8
Number of meshes and type: 1 `HEXA8`

4.3 Quantities tested and results

were tested the non-zero components of stress field SIGM_ELNO (component xx , zz and xz), the plastic strain cumulated in tension (second local variable, second component of field VARI_ELNO), and finally, only for the fourth case of loading (discharge), the plastic strain cumulated in compression, (first local variable, first component of field VARI_ELNO).

The first three loadings correspond to the load, and have results of reference. The fourth loading corresponds to the discharge, and constitutes result non regression code.

Field SIGM_ELNO component SIXX

Identification	Reference	*Aster*	% difference
For a displacement imposed in load $U_1=0.1$ and $U_2=0.05$	0.152474	0.1524472	0.018
For a displacement imposed in load $U_1=0.2$ and $U_2=0.10$	$8.492877.10^{-2}$	$8.487797.10^{-2}$	0.060
For a displacement imposed in load $U_1=0.3$ and $U_2=0.15$	$1.732484.10^{-2}$	$1.730871.10^{-2}$	0.434
For a displacement imposed in discharge $U_1=0.1$ and $U_2=0.05$	(*)	$-4.386134.10^{-5}$	-

(*) discharges some, one carries out a test of non regression. There is no calculated analytical solution.

Field SIGM_ELNO component SIZZ

Identification	Reference	*Aster*	% difference
For a displacement imposed in load $U_1=0.1$ and $U_2=0.05$	0.210300	0.210265	0.016
For a displacement imposed in load $U_1=0.2$ and $U_2=0.10$	0.117138	0.117069	0.059
For a displacement imposed in load $U_1=0.3$ and $U_2=0.15$	$2.397743.10^{-2}$	$2.387336.10^{-2}$	0.434
For a displacement imposed in discharge $U_1=0.1$ and $U_2=0.05$	(*)	$-4.768217.10^{-5}$	-

(*) discharges some, one carries out a test of non regression. There is no calculated analytical solution.

Field SIGM_ELNO component SIXZ

Identification	Reference	Aster	% difference
For a displacement imposed in load $U_1=0.1$ and $U_2=0.05$	– 5.007871.10 $^{-2}$	– 5.007226.10 $^{-2}$	0.013
For a displacement imposed in load $U_1=0.2$ and $U_2=0.10$	– 2.789472.10 $^{-2}$	– 2.787871.10 $^{-2}$	0.057
For a displacement imposed in load $U_1=0.3$ and $U_2=0.15$	– 5.709873.10 $^{-3}$	– 5.685155.10 $^{-3}$	0.433
For a displacement imposed in discharge $U_1=0.1$ and $U_2=0.05$	(*)	– 3.308936.10 $^{-6}$	-

(*) discharges some, one carries out a test of non regression. There is no analytical solution.

Field VARI_ELNO component VARI_2 (plastic strain cumulated in tension)

Identification	Reference	Aster	% difference
For a displacement imposed in load $U_1=0.1$ and $U_2=0.05$	1.085728.10 $^{-2}$	1.085728.10 $^{-2}$	5.10 $^{-9}$
For a displacement imposed in load $U_1=0.2$ and $U_2=0.10$	2.171556.10 $^{-2}$	2.171556.10 $^{-2}$	5.10 $^{-9}$
For a displacement imposed in load $U_1=0.3$ and $U_2=0.15$	3.257385.10 $^{-2}$	3.257385.10 $^{-2}$	4.10 $^{-9}$
For a displacement imposed in discharge $U_1=0.1$ and $U_2=0.05$	3.257385.10 $^{-2}$	3.257385.10 $^{-2}$	4.10 $^{-9}$

(*) discharges some, one carries out a test of non regression. There is no calculated analytical solution.

Field VARI_ELNO component VARI_1 (plastic strain cumulated in compression)

Identification	Reference	Aster	% difference
For a displacement imposed in discharge $U_1=0.1$ and $U_2=0.05$	(*)	3.528401.10 $^{-1}$	-

(*) discharges some, one carries out a test of non regression. There is no calculated analytical solution.

5 Summary of the results

This case test offers satisfactory results compared to the results of reference, lower than 0.06% for the first two cases of loading, more important for the third, which is explained by a level of relatively low stress (one reaches the end of the curve of hardening in tension).

The test discharges some (fourth loading) makes it possible to check non regression code.

The nombre of iterations is relatively important with the first computation step, about 13, then drops to 7,4 and 1, which is explained by the transition of the plastic threshold to the first computation step, to reach a quasi linear behavior thereafter (curved post-peak linear).
One obtains also a more significant number of iterations to step 31 (beginning of the fourth case of loading), then a nombre of iterations dropping up to 1, because of the transition in discharge, with a behavioral change, follow-up of a quasi linear behavior thereafter (curved post-peak linear).