SSNV138 - Plate Cantilever in large rotations subjected to one Summarized

moment:

Quasi-static computation of an elastic plate embedded on a side and subjected to one bending moment at the other side, leading to large rotations of the plate.

Interest:

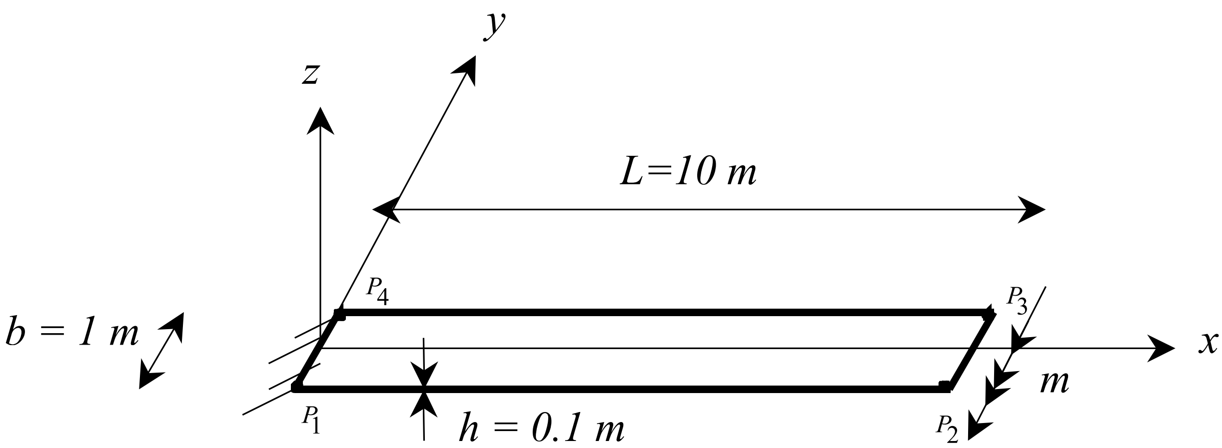
To test nonlinear finite elements geometrical COQUE_3D (modelizations A and C) and POU_D_T_GD (modelization B) using the algorithm of update of large rotations $3D$ GROT_GDEP in STAT_NON_LINE.
To also compare the results with the finite element of beam multifibre POU_D_TGM which makes it possible to treat large rotations under the assumption of small increments of loading.

Note:

> *This test is the version plates case test of beam SSNL103. The mechanical characteristics were modified in order to support a surface modelization.*

1 Problem of reference

1.1 Geometry



Plates rectangular clamped in P_1P_4 and P_1P_4 subjected in P_2P_3 to a linear couple:

$$m=-me_y \;\; ; \;\; m>0$$

1.2 Material properties and characteristic of section

Behavior elastic:

$$E=12\times 10^6\, Pa \;\; ; \;\; \nu=0$$

The fact that the Poisson's ratio (ν) is null makes the solution of plate identical to that of beam.
I_y is the inertia of the section with a model of beam:

$$I_y=\frac{b\ h^3}{12}=\frac{1}{12}\times 10^{-3}$$

1.3 Boundary conditions and loading

Fixed support in P_1P_4 . One seeks the successive states of equilibrium under the loading made up of the linear couple in P_2P_3 :

$m(t)=100\ t$; t pseudo-TEMPS.

One is interested particularly in displacements horizontal and vertical and rotation of line P_2P_3 .

2 Reference solution

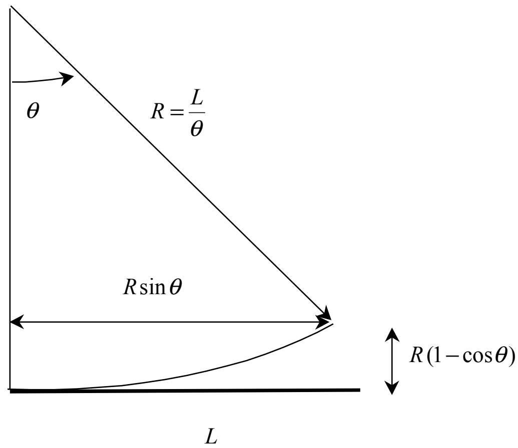
2.1 Method of calculating used for the reference solution

With a kinematics of beam and a model in resulting forces, the curvature (in large rotations) of the cantilever subjected to the bending moment $M=mb$ is, with the preceding numerical data:

$$\frac{d\theta}{dx}=\frac{mb}{EI_y}=\frac{t}{L}$$

It is the solution of Eulerian.

2.2 Results of reference



According to the solution of Eulerian, the deformed shape is an arc of a circle. With the section $P_2\ P_3\quad(x=L)$, rotation is worth:

$$\theta(x=L)=t\,.$$

In the absence of normal force, mean surface remains inextensible and the radius of curvature is given by:

$$R=\left(\frac{d\theta}{dx}\right)^{-1}=\frac{L}{t}$$

Horizontal displacement is then

$$u=R\sin\theta-L=L\left(\frac{\sin t}{t}-1\right)$$

and vertical displacement is

$$v=R(1-\cos\theta)=\frac{L}{t}(1-\cos t)$$

2.3 bibliographical References

[1] Mr. AL MIKDAD: Static and Dynamics of the Beams in Large rotations and Resolution of the Problems of Nonlinear Instability. Doctorate, University of Technology of Compiegne (1998).

[2] J.C. SIMO and L. CONSIDERING QUOC: A Three-dimensional Finite Strain Rod Model. Leaves II: Computational Aspects. Comput. Meth. Appl. Mech. Engrg. 58,79 - 116 (1986).

[3] J.C. SIMO, D.D. FOX TERRIER and M.S. RIFAI: There are Resulting Stress Exact Geometrically Shell Model. Leaves III: Computational Aspects of the Nonlinear Theory. Comput. Meth. Appl. Mech. Engrg. 79,21 - 70 (1990).

3 Modelization A

3.1 Characteristic of the modelization

Modelization COQUE_3D

3.2 Characteristics of the mesh

Many nodes: 54
Number of meshes and type: 10 QUAD9 and 1 SEG3

3.3 Functionalities tested

The modelization COQUE_3D in nonlinear geometrical.
The static algorithm of update of large rotations GROT_GDEP of STAT_NON_LINE.

3.4 Quantities tested and Values

3.4.1 results tested

the incremental analysis is carried out in the interval of pseudo-TEMPS $[0:2.4]$ in fourteen steps of load.

3.4.1.1 History of horizontal rotation DRY to the nodes charged

Time	Couples m	Reference DRY (radians)
0.6	60.	– 0.6000E+00
1.2.120		– 1.2000E+00
1.8.180		– 1.8000E+00
2.4.240		– 2.4000E+00

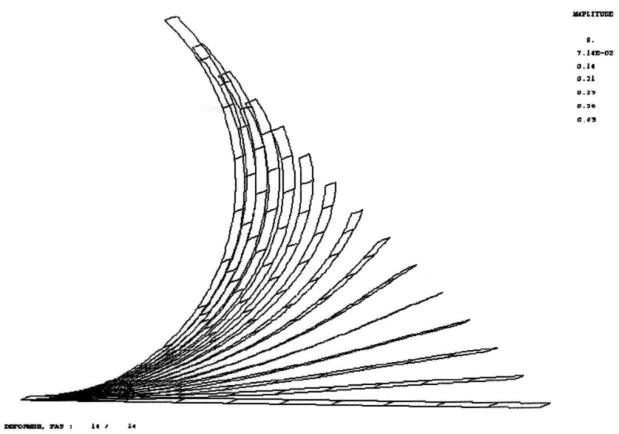
3.4.1.2 History of horizontal displacement DX to the nodes charged

Time	Couples m	Reference
0.6	60.	– 5.8929E-01
1.2.120		– 2.23300E+00
1.8.180		– 4.58973E+00
2.4.240		– 7.18557E+00

3.4.1.3 History of vertical displacement DZ to the nodes charged

Time	Couples m	Reference
0.6	60.	2.91107E+00
1.2.120		5.31368E+00
1.8.180		6.81778E+00
2.4.240		7.23914E+00

We present hereafter a visualization of the deformed shape during 14 step of load:



3.4.2 Remarks

One uses `COEF_RIGI_DRZ` = 0.001. The value of the angle swing reached is of 135 degrees.

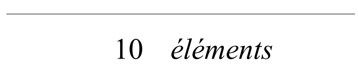
4 Modelization B

4.1 Characteristic of modelization

POU_D_T_GD (beam $3D$ in large rotations).

modelization POU_D_T_GD.

4.2 Characteristics of the mesh



Many nodes: 11
Number of meshes and type: 10 SEG2

4.3 Functionalities tested

nonlinear element geometrical POU_D_T_GD.
The static algorithm of update of large rotations ELAS_POUTRE_GD of STAT_NON_LINE.

4.4 Quantities tested and Values

4.4.1 results tested

the incremental analysis is carried out in the interval of pseudo-TEMPS $[0:6]$ in 60 steps of load.

4.4.1.1 History of horizontal rotation DRY (radians) to the nodes charged

Time	Moment m	Reference
0.3	30.	– 0.3000E+00
0.6	60.	– 0.6000E+00
3.0.300	.	– 3.0000E+00
6.600		-6

4.4.1.2 History of horizontal displacement $DX(m)$ to the nodes charged

Time	Moment m	Reference
0.3	30.	– 1.4932E-01
0.6	60.	– 5.8934E+01
3.0.300	.	– 9.5296
6.600		– 10.4657

4.4.1.3 History of vertical displacement $DZ(m)$ to the nodes charged

Time	Moment m	Reference
0.3	30.	1.4887E+00
0.6	60.	2.9110E+00
3.0.300	.	6.6333
6.600		6.638286E-02

5 Modelization C

5.1 Characteristic of the modelization

Modelization COQUE_3D

5.2 Characteristics of the mesh

Many nodes: 64
Number of meshes and type: 20 TRIA7 and 1 SEG3

5.3 Functionalities tested

The modelization COQUE_3D in nonlinear geometrical.
The static algorithm of update of large rotations GROT_GDEP of STAT_NON_LINE.

5.4 Quantities tested and Values

5.4.1 results tested

the incremental analysis is carried out in the interval of pseudo-TEMPS $[0:2.2]$ in eight step of load.

5.4.1.1 History of horizontal rotation DRY to the nodes charged

Time	Couples m	Reference
0.6	60	– 0.6000E+00
1.2.120		– 1.2000E+00
1.8.180		– 1.8000E+00
2.2.220		– 2.1728E+00

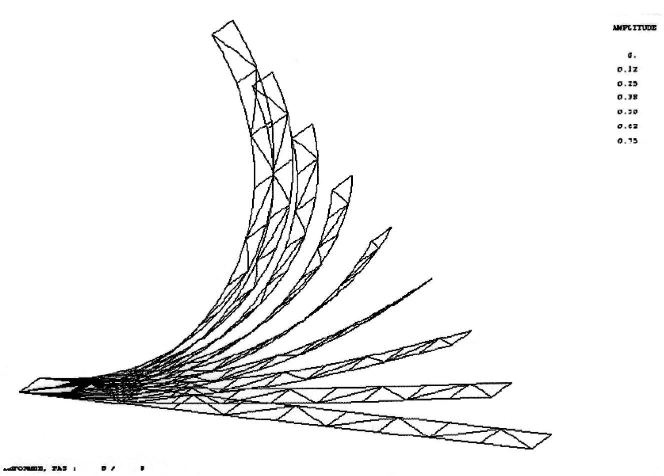
5.4.1.2 History of horizontal displacement DX to the nodes charged

Time	Couples m	Reference
0.6	60	– 5.8929E-01
1.2.120		– 2.23300E+00
1.8.180		– 4.58973E+00
2.2.220		– 6,3250163463

5.4.1.3 History of vertical displacement DZ to the nodes charged

Time	Couples m	Reference
0.6	60	2.91107E+00
1.2.120		5.31368E+00
1.8.180		6.81778E+00
2.2.220		7,22046E+00

We present hereafter a visualization of the deformed shape during 8 steps of load:



5.4.2 Remarks

One uses `COEF_RIGI_DRZ` = 0.001. The value of the angle swing reached is of 125 degrees.

6 Modelization D

6.1 Characteristic of modelization

POU_D_TGM (beam 3D multifibre for the geometrical nonlinear analysis and material).

6.2 Characteristics of the mesh

10 *éléments*

Many nodes: 11
Number of meshes and type: 10 SEG2

6.3 Characteristics of the mesh of the cross-sectional area

Many fibers: 160 (40 in the thickness and 4 in the width)
Number of meshes and type: 160 QUAD4

6.4 Functionalities tested

- nonlinear element multifibre POU_D_TGM.
- The computation strains PETIT_REAC with reactualization of the geometry and the taking into account of large rotations in STAT_NON_LINE.

6.5 Quantities tested and Values

6.5.1 results tested

the incremental analysis is carried out in the interval of pseudo-TEMPS $[0:6]$ in 1200 not of load.

6.5.1.1 History of horizontal rotation DRY (radians) to the nodes charged

Time	Moment m	Reference
0.3	30.	– 0.3000E+00
0.6	60.	– 0.6000E+00
3.0.300	.	– 3.0000E+00
6.600		-6

6.5.1.2 History of horizontal displacement DX(m) to the nodes charged

Time	Moment m	Reference
0.3	30.	– 1.4932E-01
0.6	60.	– 5.8934E-01
3.0.300	.	– 9.5296
6.600		– 10.4657

6.5.1.3 History of vertical displacement DZ(m) to the nodes charged

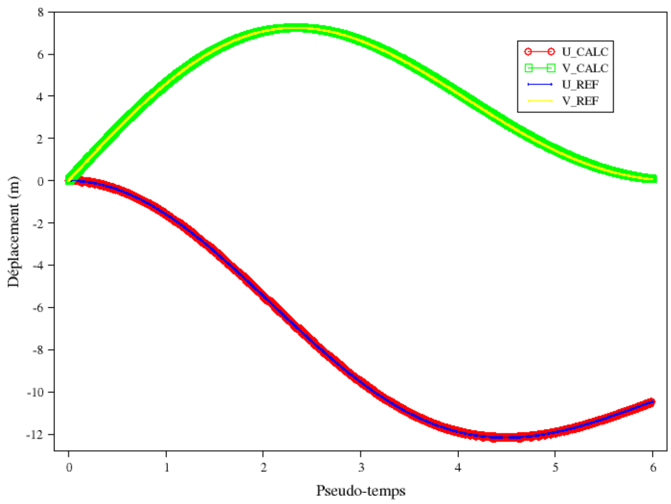
Time	Moment m	Reference
0.3	30.	1.4887E+00
0.6	60.	2.9110E+00
3.0.300	.	6.6333
6.600		6.638286E-02

With the last time step, displacements vertical very weak compared to maximum is reached during the way of loading (DZ maximum around $7\,m$). A comparison with the reference solution into relative is not very relevant (there would be then almost 30% of relative error). One would prefer a test into relative compared to maximum displacement: $\frac{valeur\ calculée - valeur\ de\ référence}{DZ_{max}} < tol^{relative}$.

With this intention, one tests in absolute, with a tolerance in being worth absolute $DZ_{max}.tol^{relative}$.

6.5.2 Graphic results of the modelization D

Déplacements U et V en fonction du pseudo-temps



7 Modelization E

7.1 Characteristic of the modelization

This modelization is identical to modelization D. the only difference is at the level of the management of time step.

Management of the under-cutting of time step by vent-driven: So with convergence, the displacement increment is such that $max(DX, DY) > 5.\mathrm{e}-2$ on a node of the mesh, then one Re-cutting time step.

7.2 Quantities tested and results

One tests the same values as those of the modelization D, with the same tolerances.

8 Summary of the results

One notices difficulties of convergence which disappear by multiplying the thickness by 3 or 4.

It is necessary to increase the value of the COEF_RIGI_DRZ which allots a stiffness around the norm of the shell elements which is worth by default 10^{-5} (smallest flexural rigidity around the directions in the plane of the shell) in order to be able to increase the value of the swing angle that one can reach. Values of this coefficient until 10^{-3} remain licit.

During the iterations of Newton, strains of membrane appear and are cancelled with convergence.

The velocities of convergence of the algorithms of NEWTON are comparable for modelizations POU_D_T_GD and COQUE_3D.

The speed of convergence of the algorithm of NEWTON in the case of modelization POU_D_TGM is much lower than the two others because this modelization requires in this case to make very small increments of loading for describing the geometrical transformation well and remaining on the assumption of the small strains. The cost in tempss' CPU feels some since computation is meadows of 10 times longer than that of modelization POU_D_T_GD. Of course the element multifibre has the advantage of being able to treat several types of behavior and not only one elastic behavior like element POU_D_T_GD. If the accuracy necessary is not about 1%, one can allow oneself to use less time step.
In addition it is important for a problem like this one, where the inertia of the section plays a dominating part, to take care to discretize the section with sufficient fibers when POU_D_TGM is used, to obtain inertia nearest possible to the theoretical value (this is why one has with a grid with nearly 40 fibers in the thickness).