SSNV137 - Cable of prestressed in a Summarized concrete straight beam

One considers a concrete straight beam, of square section, crossed over its length by a cable of prestressed out of steel. With at-rest state, the cable is parallel to fiber average of the beam and excentré compared to the two principal planes. The beam and the cable are embed-free. The cable is put in tension at its loose lead, in order to prestress the beam in bending-compression. The losses of tension along the cable are neglected.

The goal of this benchmark is to validate the méthode de calcul of the state of equilibrium of a prestressed concrete structure, when this structure is modelled by elements 3D, associated with the basic elements representing the cable of prestressing.

The features particular to test are the following ones:

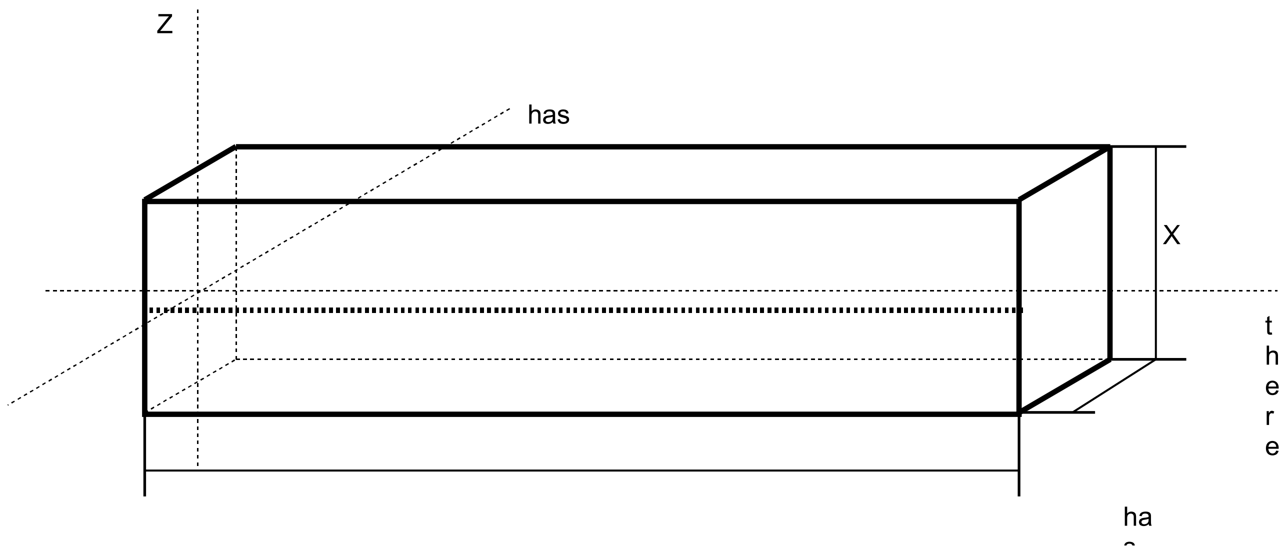
- operator `DEFI_CABLE_BP` : determination of the kinematic relations between the degrees of freedom of the nodes of a cable and the degrees of freedom of the nodes “close” to a concrete structure modelled by elements 3D;
- operator `STAT_NON_LINE`, option `COMP_INCR` : computation of the state of equilibrium.

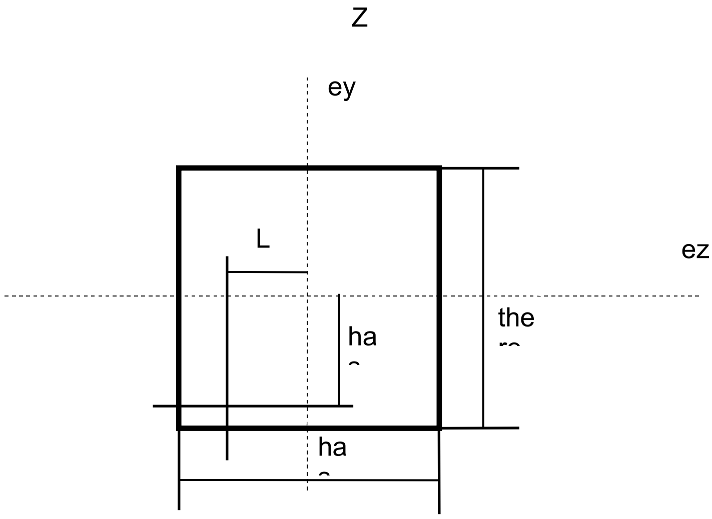
The got results are validated by comparison with an analytical solution of reference.

1 Problem of reference

1.1 Geometry

the beam out of concrete is right, of square section.
Its dimensions are $L\times a\times a=3\,m\times 0{,}4\,m\times 0{,}4\,m$.
The cable crosses the beam parallel with average fiber and it is excentré compared to the two principal planes. Eccentricities according to the directions y and z are worth respectively $e_y=-0{,}12\,m$ and $e_z=-0{,}16\,m$.
The area of the straight section cable is worth $S_a=2{,}5.10^{-3}\,m^2$.





1.2 Concrete material Properties of

the materials constituting beam: Young modulus $E_b=4{,}5.10^{10}\,Pa$
Material steel constituting the cable: Young modulus $E_a=1{,}85.10^{11}\,Pa$
the Poisson's ratio is taken equal to 0 for the two materials. One thus cancels the effects Fish in the directions y and z .

The losses of tension in the cable being neglected, the various parameters being used for their estimate are fixed at 0.

1.3 Boundary conditions and loadings

the nodes of the beam located on the face $x=0$ are blocked in translation according to the three directions. Among these nodes the “neighbors” of ending node left of the cable are, which is thus blocked in translation by the kinematic relations. One thus should not impose additional boundary conditions in this node, which would be redundant with the kinematic relations and would make impossible the resolution in displacements (singular matrix).

One ending node applies to right cable a normal force of tension $(F_0;0\ ;0)$, with $F_0=10^6 N$.

2 Reference solution

the analytical solution of reference is determined by the theory of the beams. A embed-free beam is considered. The geometrical characteristics are those defined in paragraph [§2.1]. One applies at the loose lead a normal force of compression $(-F\ ;0\ ;0)$ and a bending moment $(0\ ;e_z.F\ ;-e_y.F)$.

The solution of this problem is the following one:

Tensor of the stresses:

$$\overset{\Rightarrow}{\sigma}=\begin{bmatrix}\sigma_{xx} & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix} \text{ with } \sigma_{xx}=-\frac{F}{a^2}\left(1+\frac{12e_y}{a^2}y+\frac{12e_z}{a^2}z\right) \qquad \text{éq 2-1}$$

Displacements : by neglecting the effects Fish one obtains

$$\begin{cases} u(x,y,z)=-\dfrac{F}{E_b a^2}\left(1+\dfrac{12e_y}{a^2}y+\dfrac{12e_z}{a^2}z\right)x\\ v(x,y,z)=\dfrac{6Fe_y}{E_b a^4}x^2\\ w(x,y,z)=\dfrac{6Fe_z}{E_b a^4}x^2 \end{cases} \qquad \text{éq 2-2}$$

with the boundary conditions $\begin{cases} u=v=w=0\\ \dfrac{\partial v}{\partial x}=\dfrac{\partial w}{\partial x}=0 \end{cases}$ en $x=0$

In the statements above, F indicates the residual normal force in the cable after shortening elastic of the beam, which can be clarified according to the initial tension F_0 .

The axial strain rate of the concrete on the level of the cable is written

$$\varepsilon_{xx}^{béton}=\frac{\sigma_{xx}}{E_b}=-\frac{F}{E_b a^2}\left[1+\frac{12\,e_y^2}{a^2}+\frac{12\,e_z^2}{a^2}\right]$$

the residual normal force in the cable results from the initial tension F_0 by the relation $\varepsilon_{xx}^{béton}=\varepsilon_{xx}^{acier}$ and $\varepsilon_{xx}^{acier}=\dfrac{F-F_0}{E_a S_a}$; from where:

$$F\ =F_0+E_a S_a \varepsilon_{xx} \text{ that is to say } F=\frac{F_0}{1+\dfrac{E_a S_a}{E_b a^2}\left(1+\dfrac{12e_y^2}{a^2}+\dfrac{12e_z^2}{a^2}\right)}$$

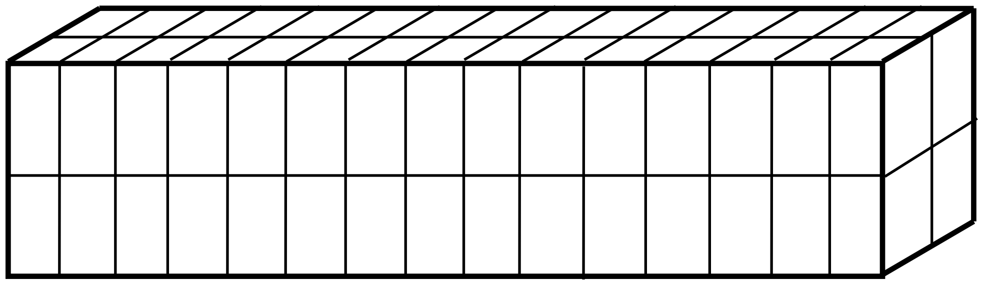
éq 2-3

the numerical values of reference are calculated using the formulas [éq 2-1], [éq 2-2] and [éq 2-3].

3 Modelization A

3.1 Characteristic of the modelization

the beam out of concrete is represented by 60 elements MECA_HEXA20, supported per as many meshes hexahedrons with 20 nodes. The figure below gives a simplified representation of the mesh of the beam.



Concrete material is affected with the elements, for which behaviors ELAS (Young modulus) are $E_b=4,5.10^{10} Pa$ defined and BPEL_BETON : the parameters characteristic of this relation are fixed at 0 because one neglects the losses of tension along the cable of prestressing.

The degrees of freedom DX DY , and DZ of the nodes of the face $x=0$ are blocked.

The cable is represented by 30 elements MECA_BARRE, supported per as many meshes segments with 2 nodes. The ends left and right-hand side are respectively the nodes $NC000001$ and $NC000031$.

An area of cross-section $S_a=2,5.10^{-3} m^2$ is assigned to the elements, as well as a material steel for which behaviors ELAS (Young modulus) are $E_a=1,85.10^{11} Pa$ defined and BPEL_ACIER : the parameters characteristic of this relation are fixed at 0 (neglected losses of tension), except for the stress yield stress for which a value zero is illicit ($f_{prg}=1,77.10^9 Pa$).

To avoid any redundancy with the kinematic relations, no blocking is forced on the node $NC000001$ (cf notices paragraph [§2.3]).

The tension $F_0=10^6 N$ is applied to the node $NC000031$. This value of tension is coherent with the values of section and yield stress, for a cable of prestressed of standard strand.

The computation of the state of equilibrium of the beam group and cable is carried out in only one step, the behavior being elastic. One carries out then a complementary computation allowing to determine the nodal stresses of the elements of the beam.

3.2 Stages of computation and features tested

the main steps of computation correspond to the features which one wishes to validate:

- operator DEFI_MATERIAU : definition of behavior models BPEL_BETON and BPEL_ACIER, in the cas particulier where the losses of tension along the cable of prestressing are neglected (default values of the parameters);

- operator `DEFI_CABLE_BP` : determination of a constant profile of tension along the cable of prestressing, losses being neglected; computation of the coefficients of the kinematic relations between the degrees of freedom of the nodes of the cable and the degrees of freedom of the nodes "close" to the concrete beam, in the case of a beam modelled by elements 3D;
- operator `AFFE_CHAR_MECA` : definition of a loading of the type `RELA_CINE_BP` ;
- operator `STAT_NON_LINE`, option `COMP_INCR` : computation of the state of equilibrium by taking account of the loading of the type `RELA_CINE_BP`, in the case of a beam modelled by elements 3D.

One uses finally operator `CALC_CHAMP` option `SIGM_ELNO` in order to calculate the nodal stresses of the elements of the beam.

3.3 Results of the modelization A

3.3.1 Displacements of the nodes of the beam

One compares the values extracted field `DEPL` resulting from `STAT_NON_LINE` with the theoretical values from reference. The tolerance of relative variation compared to the reference is worth:

- 3% for the node $NB010527$;
- 1% for the nodes $NB030127$, $NB050127$ and $NB050527$;
- 0,1% for the other nodes.

Node	Component	Value of reference	Computed value	relative Variation
NB010105	DX	– 2,298342.10-4 m	– 2,298342.10-4 m	+2,35.10^{-7}%
NB010305	DX	– 1,237569.10-4 m	– 1,237569.10-4 m	+4,91.10^{-8}%
NB010505	DX	– 1,767956.10-5 m	– 1,767956.10-5 m	– 1,13.10^{-7}%
NB030105	DX	– 1,502762.10-4 m	– 1,502762.10-4 m	+2,87.10^{-7}%
NB030305	DX	– 4,419890.10-5 m	– 4,419890.10-5 m	– 1,12.10^{-7}%
NB030305	DY	– 7,955801.10-5 m	– 7,955801.10-5 m	+1,31.10^{-8}%
NB030305	DZ	– 1,060773.10-4 m	– 1,060773.10-4 m	+4,53.10^{-7}%
NB030505	DX	+6,187845.10-5 m	+6,187845.10-5 m	+4,91.10^{-8}%
NB050105	DX	– 7,071823.10-5 m	– 7,071823.10-5 m	+2,84.10^{-8}%
NB050305	DX	+3,535912.10-5 m	+3,535912.10-5 m	– 1,13.10^{-7}%
NB050505	DX	+1,414365.10-4 m	+1,414365.10-4 m	– 2,54.10^{-7}%
NB010116	DX	–8,618785.10-4 m	– 8,618783.10-4 m	– 1,87.10^{-7}%
NB010316	DX	– 4,640884.10-4 m	– 4,640884.10-4 m	+5,86.10^{-8}%
NB010516	DX	– 6,629834.10-5 m	– 6,629837.10-5 m	+4,12.10^{-7}%
NB030116	DX	– 5,635359.10-4 m	– 5,635360.10-4 m	+1,15.10^{-7}%
NB030316	DX	– 1,657459.10-4 m	– 1,657459.10-4 m	– 8,23.10^{-8}%
NB030316	DY	– 1,118785.10-3 m	– 1,118785.10-3 m	– 4,18.10^{-7}%
NB030316	DZ	– 1,491713.10-3 m	– 1,491713.10-3 m	– 1,95.10^{-7}%
NB030516	DX	+2,320442.10-4 m	+2,320442.10-4 m	+5,66.10^{-8}%
NB050116	DX	– 2,651934.10-4 m	– 2,651934.10-4 m	– 5,31.10^{-8}%
NB050316	DX	+1,325967.10-4 m	+1,325967.10-4 m	+3,21.10^{-8}%
NB050516	DX	+5,303867.10-4 m	+5,303869.10-4 m	+2,95.10^{-7}%
NB010127	DX	– 1,493923.10-3 m	– 1,494742.10-3 m	+ 0,055%
NB010327	DX	– 8,044199.10-4 m	– 8,039511.10-4 m	– 0,058%
NB010527	DX	– 1,149171.10-4 m	– 1,123172.10-4 m	– 2,262%
NB030127	DX	– 9,767956.10-4 m	– 9,755085.10-4 m	– 0,132%
NB030327	DX	– 2,872928.10-4 m	– 2,870992.10-4 m	– 0,067%
NB030327	DY	– 3,361326.10-3 m	– 3,361041.10-3 m	– 0,008%
NB030327	DZ	– 4,481768.10-3 m	– 4,481603.10-3 m	– 0,004%

NB030527	DX	+4,022099.10-4 m	+4,021519.10-4 m	– 0,014%
NB050127	DX	– 4,596685.10-4 m	– 4,599190.10-4 m	– 0,598%
NB050327	DX	+2,298343.10-4 m	+2,296287.10-4 m	– 0,089%
NB050527	DX	+9,193370.10-4 m	+9,167311.10-4 m	– 0,283%

3.3.2 Normal stress in the beam

One compare the values extracted field SIGM_ELNO resulting from CALC_CHAMP with the theoretical values from reference.
The component to which the tests relate is SIXX.
The tolerance of relative variation compared to the reference is worth 0,1%.

Node	Nets	Value of reference	Computed value	relative Variation
NB010116	HX010115	– 2,585635.107 Pa	– 2,585622.107 Pa	– 4,97.10- 6%
NB010316	HX010115	– 1,392265.107 Pa	– 1,392266.107 Pa	+9,60.10- 7%
NB010516	HX010315	– 1,988950.106 Pa	– 1,989086.106 Pa	+ 0,007%
NB030116	HX010115	– 1,690608.107 Pa	– 1,690605.107 Pa	– 1,66.10- 6%
NB030316	HX010115	– 4,972376.106 Pa	– 4,972387.106 Pa	+2,39.10- 6%
NB030516	HX010315	+6,961326.106 Pa	+6,961321.106 Pa	– 6,61.10- 7%
NB050116	HX030115	– 7,955801.106 Pa	– 7,955959.106 Pa	+ 0,002%
NB050316	HX030115	+3,977901.106 Pa	+3,977883.106 Pa	– 4,46.10- 6%
NB050516	HX030315	+1,591160.107 Pa	+1,591176.107 Pa	+ 0,001%

3.3.3 Displacements of the nodes of the cable of prestressed

One compares the values extracted field DEPL resulting from STAT_NON_LINE with the theoretical values from reference. The tolerance of relative variation compared to the reference is worth:

- 1% for the node $NC000031$, component DZ ;
- 0,1% for the other nodes.

Node	Component	Value of reference	Computed value	relative Variation
NC000006	DY	– 1,243094.10-4 m	– 1,243094.10-4 m	– 6,24.10- 8%
NC000006	DZ	– 1,657459.10-4 m	– 1,657459.10-4 m	– 2,64.10- 7%
NC000011	DY	– 4,972376.10-4 m	– 4,972376.10-4 m	– 5,90.10- 8%
NC000011	DZ	– 6,629834.10-4 m	– 6,629834.10-4 m	+3,99.10- 8%
NC000016	DY	– 1,118785.10-3 m	– 1,118785.10-3 m	– 3,13.10- 7%
NC000016	DZ	– 1,491713.10-3 m	– 1,491713.10-3 m	– 7,49.10- 8%
NC000021	DY	– 1,988950.10-3 m	– 1,988946.10-3 m	– 1,96.10- 6%
NC000021	DZ	– 2,651934.10-3 m	– 2,651929.10-3 m	– 1,74.10- 6%
NC000026	DY	– 3,107735.10-3 m	– 3,107026.10-3 m	– 0,023%
NC000026	DZ	– 4,143646.10-3 m	– 4,142654.10-3 m	– 0,024%
NC000031	DY	– 4,475138.10-3 m	– 4,475186.10-3 m	+ 0,001%
NC000031	DZ	– 5,966851.10-3 m	– 6,010387.10-3 m	+ 0,730%

3.3.4 normal Force in the cable of prestressed

One compare the value extracted field SIEF_ELNO resulting from STAT_NON_LINE with the theoretical value from reference.
The component to which the test relates is N .
The tolerance of relative variation compared to the reference is worth 0,1%.

Node	Nets	Value of reference	Computed value	relative Variation
$NC000016$	$SG000015$	$+7{,}955801.10^5 N$	$+7{,}955805.10^5 N$	$+5{,}42.10^{-7}\%$

4 Summary of the results

the computed values correspond indeed to those theoretically expected. One obtains well a state of bending-compression for the concrete beam.

The more important variations observed in some nodes closer to the loose lead can be explained by the more or less good adequacy of a modelization 3D for a structure of type beam. Thus the mesh remains enough coarse not to increase the cost of computation. It is pointed out finally that the reference solution is established under the assumptions of the theory of the beams.