SSNV127 - Roll in a boring with contact and Summarized

friction:

This problem corresponds to a quasi-static analysis of a problem of mechanics with contact and friction. A cylinder is compressed in a cylindrical boring of slightly higher diameter by a concentrated force applied to its axis.

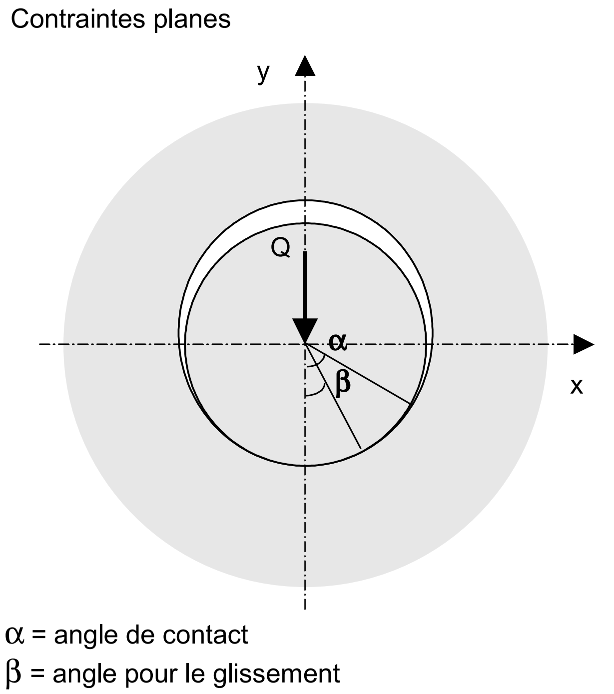
This test, named "problem of Klang", is rather largely used in the literature to validate modelizations of contact with friction and was in particular used by P. Alart and A. Curnier [bib1] to validate their finite elements of contact and friction.

Five modelizations 2D are proposed (A, B, C and D with linear elements and E with quadratic elements):
The modelization A the "LAGRANGIAN" algorithm of the method with master-slave pairing tests.
The modelization B the "LAGRANGIAN" algorithm of the method with nodal pairing tests.
The modelization C the algorithm of the method "PENALIZATION" with master-slave pairing tests, the penalization relates only to friction.
The modelization D the algorithm of the method "PENALIZATION" with master-slave pairing tests, the penalization relates to the contact and friction.
The modelization E the "LAGRANGIAN" algorithm of the method with master-slave pairing tests.
The modelizations F and H test the algorithm of the method "CONTINUE " with quadratic elements (SEG3) and linear (SEG2) of contact, respectively.

The results are compared with an analytical solution given by Klang [bib2].
Instead of reproducing this relatively complicated analytical solution we will use the values of the pressures obtained from this one [bib1].

1 Problem of reference

1.1 Geometry



Radius of the cylinder: $r=5.999\,cm$.

Radius of boring: $R=6.\,cm$.

Position of the points of reference on contact surface: a point all three degrees of angle starting from bottom, until 60°.

1.2 Material properties

Rolls and boring:

Young modulus:	$E=2.1\,10^{11}\,N/m^2$
Poisson's ratio:	$\nu=0.3$
Coefficient of friction:	$\mu=0.4$

1.3 Boundary conditions and loadings

the solid mass containing boring is supposed infinite being, its displacements will be blocked (according to X and there) on a concentric circle with boring.

The cylinder is subjected to a distributed force Q according to the thickness (z) being worth: $Q=-1875\,10^3\,N/m$.
This force is applied in an increment.

2 Reference solution

2.1 Method of calculating used for the reference solution

the reference solution is analytical [bib2].

2.2 Results of reference

Prediction on the contact zone: 60 degrees.
Prediction on the beginning of the zone of sliding: 26,2 degrees.

Forces of pressure on the points of contact surface:

Identification		Reference
SIXX for an angle of	0°	– 1.7813E+07
SIXX	3°	– 1.7813E+07
SIXX	6°	– 1.7750E+07
SIXX	9°	– 1.7688E+07
SIXX	12°	– 1.7594E+07
SIXX	15°	– 1.7470E+07
SIXX	18°	– 1.7312E+07
SIXX	21°	– 1.7125E+07
SIXX	24°	– 1.6906E+07
SIXX	27°	– 1.6656E+07
SIXX	30°	– 1.6343E+07
SIXX	33°	– 1.5937E+07
SIXX	36°	– 1.5406E+07
SIXX	39°	– 1.4781E+07
SIXX	42°	– 1.4031E+07
SIXX	45°	– 1.3094E+07
SIXX	48°	– 1.1169E+07
SIXX	51°	– 1.0593E+07

2.3 bibliographical References

1) P. Alart, A. Curnier "A mixed formulation for frictional contact problems" Methods Computer in Applied Mechanics and Engineering (1991) p. 353-375

2) Mr. Klang "One interior contact under friction between cylindrical elastic bodies in contact" Thesis, Linköping University, Linköping, 1979.

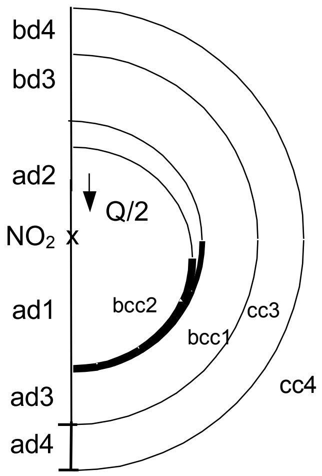
3 Modelization A

3.1 Characteristic of the modelization

the symmetry of the problem makes it possible to model only half of it ($X \geq 0$).

60 finite elements SEG2 are laid out regularly on initial contact surface (3 degrees of angle for each one).

The cylinder and circular volume surrounding boring are with a grid with elements QUA4 and TRIA3.

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CONTACT
METHODE = "LAGRANGIAN"
COULOMB = 0.4

Main APPARIEMENT='
MAIT_ESCL': bcc1
Slave: bcc2
```

Boundary condition:

on the GROUP_MA: $CC4$ $DX=0$ $DY=0$.
on the GROUP_MA $AD1$ $AD2$ $AD3$ $AD4$ $BD3$, $DX=0$.
and $BD4$:

Loadings:

The symmetry of the problem compared to the plane $x=0$ makes it possible to model the concentrated force by a nodal force $Fy=-937.5\,10^{3}\,N$, equivalent to $Q/2$ for a cylinder length unit, applied to the nodes group O_2 , centers cylinder.
This force is applied into 1 increment.

3.2 Characteristics of the mesh

Many nodes:	1281
Number of meshes and types:	128 SEG2
	156 TRIA3
	1108 QUAD4

3.3 Values tested: SIGM_ELNO

One tests the forces of pressure norm generated by contact-friction. These forces defined in polar coordinates are expressed in MPa .

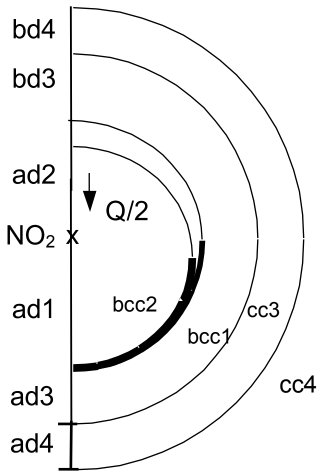
Identification		Reference	*Aster*	% difference
SIXX for an angle of	0°	– 1.7813E+07	– 1.79080E+07	0.534
SIXX	3°	– 1.7813E+07	– 1.88485E+07	5.814
SIXX	6°	– 1.7750E+07	– 1.89858E+07	6.962
SIXX	9°	– 1.7688E+07	– 1.88134E+07	6.363

SIXX	12°	– 1.7594E+07	– 1.85821E+07	5.616
SIXX	15°	– 1.7470E+07	– 1.83315E+07	4.931
SIXX	18°	– 1.7312E+07	– 1.78831E+07	3.299
SIXX	21°	– 1.7125E+07	– 1.74723E+07	2.028
SIXX	24°	– 1.6906E+07	– 1.67279E+07	– 1.053
SIXX	27°	– 1.6656E+07	– 1.60957E+07	– 3.364
SIXX	30°	– 1.6343E+07	– 1.56801E+07	– 4.056
SIXX	33°	– 1.5937E+07	– 1.57565E+07	– 1.132
SIXX	36°	– 1.5406E+07	– 1.55705E+07	1.068
SIXX	39°	– 1.4781E+07	– 1.51262E+07	2.336
SIXX	42°	– 1.4031E+07	– 1.43093E+07	1.984
SIXX	45°	– 1.3094E+07	– 1.33902E+07	2.262
SIXX	48°	– 1.1169E+07	– 1.24478E+07	11.450
SIXX	51°	– 1.0593E+07	– 1.11500E+07	5.259

4 Modelization B

4.1 Characteristic of the modelization

This modelization is identical to the modelization A (and in particular mesh). Only the method of pairing differs which here is “NODAL ”.

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CONTACT
METHODE = "LAGRANGIAN"
COULOMB = 0.4

APPARIEMENT=' NODAL'
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4.2 Values tested: SIGM_ELNO

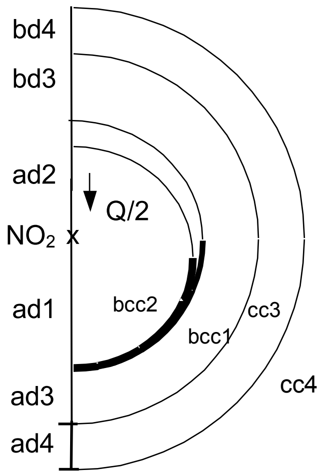
One tests the forces of pressure norm generated by contact-friction. These forces defined in polar coordinates are expressed in MPa .

Identification		Reference	*Aster*	% difference
SIXX for an angle of	0°	– 1.7813E+07	– 1.86019E+07	4.429
SIXX	3°	– 1.7813E+07	– 1.93157E+07	8.436
SIXX	6°	– 1.7750E+07	– 1.93512E+07	9.021
SIXX	9°	– 1.7688E+07	– 1.91026E+07	7.998
SIXX	12°	– 1.7594E+07	– 1.88145E+07	6.937
SIXX	15°	– 1.7470E+07	– 1.85112E+07	5.960
SIXX	18°	– 1.7312E+07	– 1.80078E+07	4.019
SIXX	21°	– 1.7125E+07	– 1.75572E+07	2.524
SIXX	24°	– 1.6906E+07	– 1.67641E+07	– 0.839
SIXX	27°	– 1.6656E+07	– 1.63429E+07	– 1.879
SIXX	30°	– 1.6343E+07	– 1.63762E+07	0.203
SIXX	33°	– 1.5937E+07	– 1.63898E+07	2.842
SIXX	36°	– 1.5406E+07	– 1.59991E+07	3.850
SIXX	39°	– 1.4781E+07	– 1.53865E+07	4.097
SIXX	42°	– 1.4031E+07	– 1.44452E+07	2.952
SIXX	45°	– 1.3094E+07	– 1.34146E+07	2.449
SIXX	48°	– 1.1169E+07	– 1.23898E+07	10.931
SIXX	51°	– 1.0593E+07	– 1.08630E+07	2.549

5 Modelization C

5.1 Characteristic of the modelization

This modelization is identical to the modelization A (and in particular mesh). Only the method of resolution differs which is here “PENALIZATION ” with penalization only on friction.

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CONTACT
METHODE = "PENALIZATION
COULOMB = 0.4

Main APPARIEMENT="
MAIT_ESCL': bcc1
Slave: bcc2
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5.2 Values tested: SIGM_ELNO

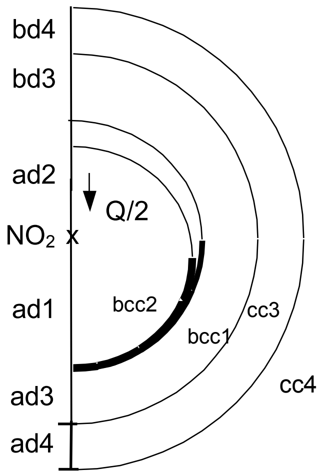
One tests the forces of pressure norm generated by contact-friction. These forces defined in polar coordinates are expressed in MPa .

Identification		Reference	*Aster*	% difference
SIXX for an angle of	0°	– 1.7813E+07	– 1.86714E+07	4.819
SIXX	3°	– 1.7813E+07	– 1.97269E+07	10.745
SIXX	6°	– 1.7750E+07	– 1.99928E+07	12.636
SIXX	9°	– 1.7688E+07	– 1.98849E+07	12.421
SIXX	12°	– 1.7594E+07	– 1.97193E+07	12.080
SIXX	15°	– 1.7470E+07	– 1.95408E+07	11.854
SIXX	18°	– 1.7312E+07	– 1.91611E+07	10.681
SIXX	21°	– 1.7125E+07	– 1.88051E+07	9.811
SIXX	24°	– 1.6906E+07	– 1.82459E+07	7.926
SIXX	27°	– 1.6656E+07	– 1.78217E+07	6.999
SIXX	30°	– 1.6343E+07	– 1.71305E+07	4.819
SIXX	33°	– 1.5937E+07	– 1.64093E+07	2.964
SIXX	36°	– 1.5406E+07	– 1.55834E+07	1.152
SIXX	39°	– 1.4781E+07	– 1.46776E+07	– 0.699
SIXX	42°	– 1.4031E+07	– 1.36676E+07	– 2.589
SIXX	45°	– 1.3094E+07	– 1.27603E+07	– 2.548
SIXX	48°	– 1.1169E+07	– 1.24156E+07	11.161
SIXX	51°	– 1.0593E+07	– 1.08692E+07	2.608

6 Modelization D

6.1 Characteristic of the modelization

This modelization are identical to the modelization A (and in particular mesh). Only the method of resolution differs which is here “PENALIZATION ” with penalization on the contact and friction.

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CONTACT
METHODE = “PENALIZATION
COULOMB = 0.4

Main APPARIEMENT=”
MAIT_ESCL': bcc1
Slave: bcc2
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6.2 Values tested: SIGM_ELNO

One tests the forces of pressure norm generated by contact-friction. These forces defined in polar coordinates are expressed in MPa .

Identification		Reference	*Aster*	% difference
SIXX for an angle of	0°	– 1.7813E+07	– 1.867141E+07	4.819
SIXX	3°	– 1.7813E+07	– 1.972695E+07	10.745
SIXX	6°	– 1.7750E+07	– 1.999288E+07	12.636
SIXX	9°	– 1.7688E+07	– 1.988495E+07	12.421
SIXX	12°	– 1.7594E+07	– 1.971935E+07	12.080
SIXX	15°	– 1.7470E+07	– 1.954087E+07	11.854
SIXX	18°	– 1.7312E+07	– 1.916115E+07	10.681
SIXX	21°	– 1.7125E+07	– 1.880513E+07	9.811
SIXX	24°	– 1.6906E+07	– 1.824592E+07	7.926
SIXX	27°	– 1.6656E+07	– 1.782171E+07	6.999
SIXX	30°	– 1.6343E+07	– 1.713057E+07	4.819
SIXX	33°	– 1.5937E+07	– 1.640937E+07	2.964
SIXX	36°	– 1.5406E+07	– 1.558344E+07	1.152
SIXX	39°	– 1.4781E+07	– 1.467761E+07	– 0.699
SIXX	42°	– 1.4031E+07	– 1.366763E+07	– 2.590
SIXX	45°	– 1.3094E+07	– 1.276022E+07	– 2.549
SIXX	48°	– 1.1169E+07	– 1.241550E+07	11.160
SIXX	51°	– 1.0593E+07	– 1.086923E+07	2.608

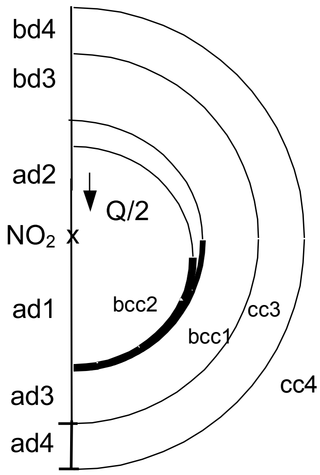
7 Modelization E

7.1 Characteristic of the modelization

the symmetry of the problem make it possible to model only half of it ($X \geq 0$).

30 finite elements SEG3 are laid out regularly on contact surface. Because of curvature of these elements, one uses the method of projection "QUADRATIQUE ".

The cylinder and circular volume surrounding boring are with a grid with elements QUA8 and TRIA6.

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CONTACT
METHODE = "LAGRANGIAN"
COULOMB = 0.4

Main APPARIEMENT='
MAIT_ESCL': bcc1
Slave: bcc2
```

Boundary condition:

on the GROUP_MA:	$CC4$	$DX=0 \quad DY=0$.
on the GROUP_MA	$AD1 \quad AD2 \quad AD3 \quad AD4 \quad BD3$, and $BD4$:	$DX=0$.

Loadings:

The symmetry of the problem compared to the plane $x=0$ makes it possible to model the concentrated force by a nodal force $Fy=-937.5\,10^3\,N$, equivalent to $Q/2$ for a cylinder length unit, applied to the nodes group O_2 , centers cylinder.
This force is applied into 1 increment.

7.2 Characteristics of the mesh

Many nodes:	1603
Number of meshes and types:	88 SEG3
	58 TRIA6
	456 QUAD8

7.3 Values tested: SIGM_ELNO

One tests the forces of pressure norm generated by contact-friction. These forces defined in polar coordinates are expressed in MPa .

Identification		Reference	*Aster*	% difference
SIXX for an angle of	0°	– 1.7813E+07	– 1.87916E+07	5.494
SIXX	3°	– 1.7813E+07	– 1.87072E+07	5.020

SIXX	6°	– 1.7750E+07	– 1.85202E+07	4.340
SIXX	9°	– 1.7688E+07	– 1.83433E+07	3.705
SIXX	12°	– 1.7594E+07	– 1.81013E+07	2.884
SIXX	15°	– 1.7470E+07	– 1.77266E+07	1.469
SIXX	18°	– 1.7312E+07	– 1.74321E+07	0.694
SIXX	21°	– 1.7125E+07	– 1.68102E+07	– 1.838
SIXX	24°	– 1.6906E+07	– 1.64256E+07	– 2.841
SIXX	27°	– 1.6656E+07	– 1.62913E+07	– 2.189
SIXX	30°	– 1.6343E+07	– 1.57932E+07	– 3.364
SIXX	33°	– 1.5937E+07	– 1.55232E+07	– 2.596
SIXX	36°	– 1.5406E+07	– 1.48572E+07	– 3.562
SIXX	39°	– 1.4781E+07	– 1.41428E+07	– 4.317
SIXX	42°	– 1.4031E+07	– 1.28218E+07	– 8.618
SIXX	45°	– 1.3094E+07	– 1.19407E+07	– 8.808
SIXX	48°	– 1.1169E+07	– 1.07287E+07	– 3.942
SIXX	51°	– 1.0593E+07	– 9.70591E+06	– 8.374

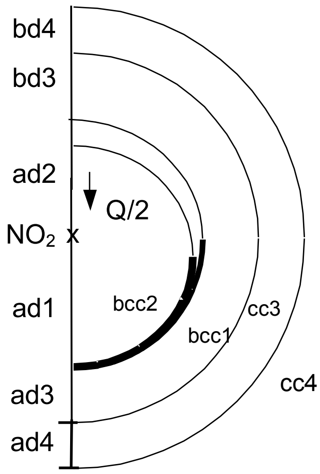
8 Modelization F

8.1 Characteristic of the modelization

the symmetry of the problem makes it possible to model only half of it ($X \geq 0$).

60 finite elements SEG3 are laid out regularly on initial contact surface (3 degrees of angle for each one).

The cylinder and circular volume surrounding boring are with a grid with elements QUA8 and TRIA6.

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CONTACT
METHODE = "CONTINUE"
COULOMB = 0.4

Main APPARIEMENT='
MAIT_ESCL': bcc1
Slave: bcc2
```

Boundary condition:

on the GROUP_MA: $CC4$ $DX=0$ $DY=0$.
on `the GROUP_MA` $AD1$ $AD2$ $AD3$ $AD4$ $BD3$, $DX=0$.
and $BD4$:

Loadings:

The symmetry of the problem compared to the plane $x=0$ makes it possible to model the concentrated force by a nodal force $Fy=-937.5\,10^3\,N$, equivalent to $Q/2$ for a cylinder length unit, applied to the nodes group O_2 , centers cylinder.
This force is applied into 1 increment.

8.2 Characteristics of the mesh

Many nodes:	1603
Number of meshes and types:	88 SEG3
	58 TRIA6
	456 QUAD8

8.3 Values tested: `SIGM_ELNO`

One tests the forces of pressure norm generated by contact-friction. These forces defined in polar coordinates are expressed in MPa .

Identification		Reference	*Aster*	% difference
`SIXX` for an angle of	0°	– 1.7813E+07	– 1.88235E+07	5.673
`SIXX`	3°	– 1.7813E+07	– 1.87386E+07	5.196
`SIXX`	6°	– 1.7750E+07	– 1.85220E+07	4.349
`SIXX`	9°	– 1.7688E+07	– 1.84104E+07	4.084
`SIXX`	12°	– 1.7594E+07	– 1.81071E+07	2.917

SIXX	15°	– 1.7470E+07	– 1.78019E+07	1.900
SIXX	18°	– 1.7312E+07	– 1.73913E+07	0.458
SIXX	21°	– 1.7125E+07	– 1.68233E+07	1.762
SIXX	24°	– 1.6906E+07	– 1.65322E+07	– 2.211
SIXX	27°	– 1.6656E+07	– 1.63678E+07	– 1.730
SIXX	30°	– 1.6343E+07	– 1.60549E+07	– 1.763
SIXX	33°	– 1.5937E+07	– 1.56070E+07	– 2.071
SIXX	36°	– 1.5406E+07	– 1.48829E+07	– 3.395
SIXX	39°	– 1.4781E+07	– 1.41776E+07	– 4.082
SIXX	42°	– 1.4031E+07	– 1.29292E+07	–7.852
SIXX	45°	– 1.3094E+07	– 1.19737E+07	– 8.555
SIXX	48°	– 1.1169E+07	– 1.06929E+07	– 4.263
SIXX	51°	– 1.0593E+07	– 0.96396E+07	– 8.999

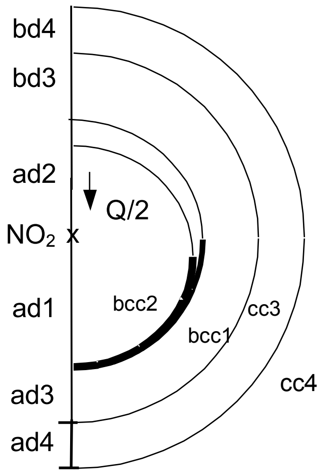
9 Modelization H

9.1 Characteristic of the modelization

the symmetry of the problem makes it possible to model only half of it ($X \geq 0$).

60 finite elements SEG2 are laid out regularly on initial contact surface (3 degrees of angle for each one).

The cylinder and circular volume surrounding boring are with a grid with elements QUA4 and TRIA3.

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CONTACT
METHODE = "CONTINUE"
COULOMB = 0.4

Main APPARIEMENT='
MAIT_ESCL': bcc1
Slave: bcc2
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Boundary condition:

on the GROUP_MA: $CC4$ $DX=0$ $DY=0$ ·
on the GROUP_MA $AD1$ $AD2$ $AD3$ $AD4$ $BD3$, $DX=0$.
and $BD4$:

Loadings:

The symmetry of the problem compared to the plane $x=0$ makes it possible to model the concentrated force by a nodal force $Fy=-937.5\,10^3\,N$, equivalent to $Q/2$ for a cylinder length unit, applied to the nodes group O_2 , centers cylinder.
This force is applied into 1 increment.

9.2 Characteristics of the mesh

Many nodes:	1282
Number of meshes and types:	128 SEG2
	162 TRIA3
	1106 QUAD4

9.3 Values tested: SIGM_ELNO

One tests the forces of pressure norm generated by contact-friction. These forces defined in polar coordinates are expressed in MPa .

Identification		Reference	*Aster*	% difference
SIXX for an angle of	0°	– 1.7813E+07	– 1.79054E+07	0.519
SIXX	3°	– 1.7813E+07	– 1.88551E+07	5.850
SIXX	6°	– 1.7750E+07	– 1.89952E+07	7.015
SIXX	9°	– 1.7688E+07	– 1.88212E+07	6.407
SIXX	12°	– 1.7594E+07	– 1.85900E+07	5.661

SIXX	15°	– 1.7470E+07	– 1.83361E+07	4.958
SIXX	18°	– 1.7312E+07	– 1.78879E+07	3.327
SIXX	21°	– 1.7125E+07	– 1.74676E+07	2.001
SIXX	24°	– 1.6906E+07	– 1.67457E+07	– 0.948
SIXX	27°	– 1.6656E+07	– 1.60969E+07	– 3.357
SIXX	30°	– 1.6343E+07	– 1.56549E+07	– 4.211
SIXX	33°	– 1.5937E+07	– 1.57323E+07	– 1.284
SIXX	36°	– 1.5406E+07	– 1.55859E+07	1.168
SIXX	39°	– 1.4781E+07	– 1.51202E+07	2.295
SIXX	42°	– 1.4031E+07	– 1.42668E+07	1.680
SIXX	45°	– 1.3094E+07	– 1.33861E+07	2.230
SIXX	48°	– 1.1169E+07	– 1.24518E+07	11.485
SIXX	51°	– 1.0593E+07	– 1.11524E+07	5.280

10 Summary of the results

the results are of good quality if it is considered that they are compared with an analytical solution and that the meshes used are not particularly fine.

The best solutions are given by the modelizations E and F (algorithms Lagrangian and continuous, respectively) which use quadratic elements with quadratic projection (without using this projection, the results are false). Then the modelizations come A and H which, like the preceding one, use the algorithms Lagrangian and continuous, respectively, with master-slave pairing. If one uses nodal pairing (modelization B), the solution is good but it should be stressed that this technique of pairing is limited to the small slidings. Finally the modelizations come C and D which use the algorithm of penalization and which give acceptable results. Let us note that convergence with these algorithms is much slower and difficult.

If one examines the extent of contact surface using the printing of data structure `VALE_CONT` (see results file), one notes that it is given very precisely (60°). With regard to adherent surface, the algorithms analytically return an acceptable value about 30° instead of 26.2°.