SSNV122 - Rotation and following tension very-elastic of a Summarized

bar:

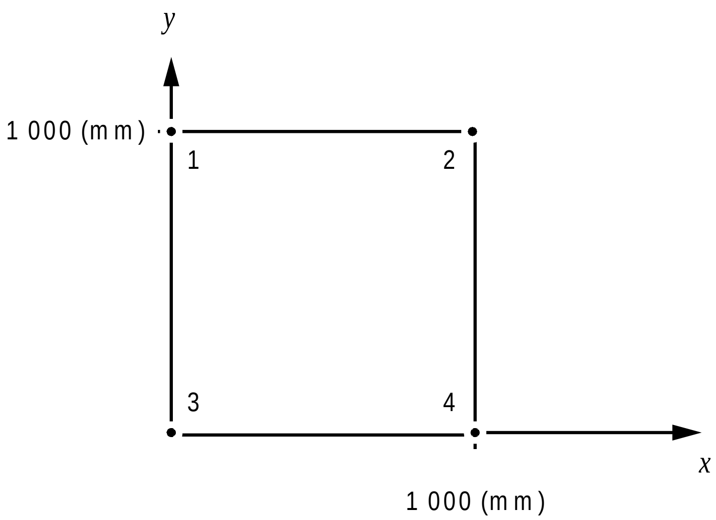
This test of quasi-static mechanics consists in making turn of 90° a parallelepipedic bar and subjecting it to an important tension by means of follower forces. One validates thus the kinematics of the large deformations very-elastics (command STAT_NON_LINE, key word COMP_ELAS), and thus in particular large rotations, for a linear elastic behavior model, as well as the taking into account of follower forces (command STAT_NON_LINE key word TYPE_CHARGE: "SUIV").

The bar is modelled by a voluminal element (HEXA8, modelization A).

The results got by *Code_Aster* do not differ from the theoretical solution.

1 Problem of reference

1.1 Geometry



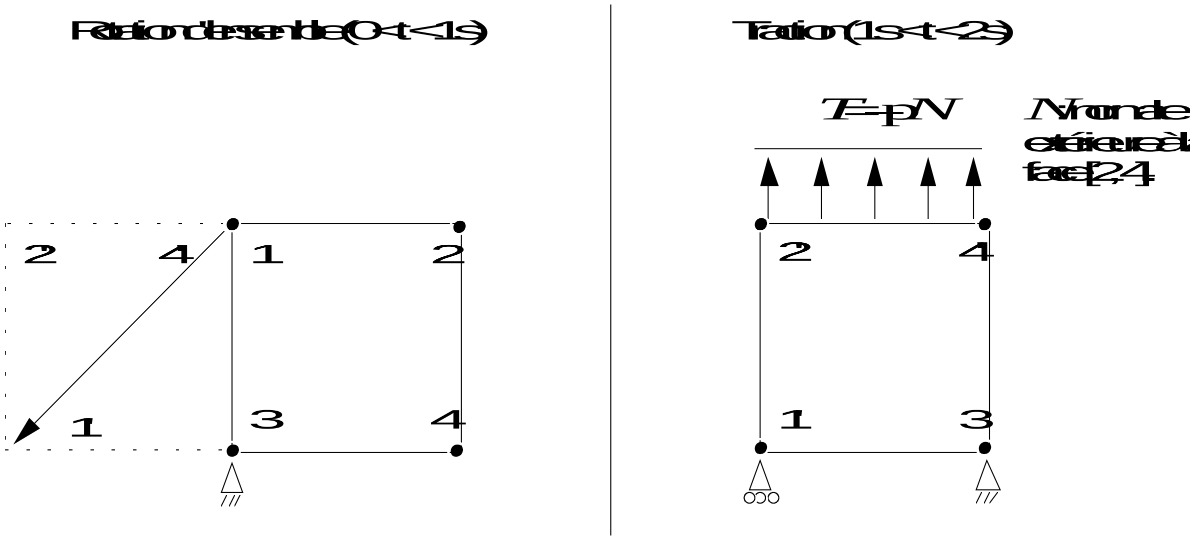
1.2 Material properties

Behavior very-elastic of COMING SAINT - KIRCHHOFF:

$$\mathbf{S} = \frac{\nu E}{(1+\nu)(1-2\nu)} \operatorname{tr}(\mathbf{E})\,\mathbf{1} + \frac{E}{1+\nu}\mathbf{E} \qquad E = 200\,000. \text{ MPa} \quad \nu = 0.3$$

1.3 Boundary conditions and loadings

the loading is applied in two times: first of all, an overall rotation of structure, followed by a tension exerted by follower forces.



2 Reference solution

2.1 Method of calculating used for the reference solution

It acts of a plane problem. One can followed by a seek the solution in the form of a rigid rotation thermal expansion of a factor a in a direction and b the other:

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \xrightarrow{\text{rotation}} \begin{pmatrix} -Y \\ X \\ Z \end{pmatrix} \xrightarrow{\text{traction}} \begin{pmatrix} b(-Y) \\ a\ X \\ Z \end{pmatrix} \quad \text{soit} \quad u = \begin{pmatrix} -X & -bY \\ AX & -Y \\ 0 & \end{pmatrix}$$

The gradient of the transformation and the strain of Green-Lagrange are then:

$$\mathbf{F} = \begin{bmatrix} 0 & -b & 0 \\ a & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \mathbf{E} = \begin{bmatrix} e_x & 0 & 0 \\ 0 & e_y & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{où} \quad \begin{matrix} e_x = \dfrac{a^2-1}{2} \\ e_y = \dfrac{b^2-1}{2} \end{matrix}$$

The behavior model leads to a diagonal stress tensor Lagrangian (with λ and the μ coefficients of Lamé):

$$\begin{aligned} S_{xx} &= (\lambda+2\mu)\ e_x + \lambda e_y \\ S_{yy} &= \lambda e_x + (\lambda+2\mu)\ e_y \\ S_{zz} &= \lambda e_x + \lambda e_y \end{aligned} \qquad \text{où} \qquad \begin{aligned} \lambda &= \frac{\nu E}{(1+\nu)(1-2\nu)} \\ \mu &= \frac{E}{2(1+\nu)} \end{aligned}$$

One from of deduced the tensor from the stresses of Cauchy, so diagonal:

$$\sigma_x = \frac{b}{a} S_y \qquad \sigma_y = \frac{a}{b} S_x \qquad \sigma_z = \frac{1}{ab} S_z$$

Finally the boundary conditions are written:

$$\sigma_x = 0 \quad (\text{bord libre}) \qquad \sigma_y = -p \quad (\text{traction})$$

One can moreover calculate the forces exerted on the sides:

$$\begin{aligned} &[1,3] && F_y = -\sigma_y\, b\, S_{o[1,3]} \\ &[3,4] && F_x = 0 \\ &[1,2,3,4] && F_z = \begin{cases} -\sigma_z\, a b\, S_{o[1,2,3,4]} & \text{sur le côté inférieur de la face} \\ \sigma_z\, a b\, S_{o[1,2,3,4]} & \text{sur le côté supérieur de la face} \end{cases} \end{aligned}$$

where $S_{o[\ ]}$ initial surfaces of the sides represent.

2.2 Results of reference

One adopts like results of reference displacements, the strains of Grenn - Lagrange, the stresses of Cauchy and the forces exerted on the sides $[1,3]$, $[3,4]$ and $[1,2,3,4]$ at the end of the loading ($t=2s$).

One seeks p such as thermal expansion $a=1{,}1$

is $p=-26610.3\,MPa$.

Thermal expansion b and the displacements are then:

$$b=0.9539 \quad e_x=0.105 \quad e_y=-0.045$$

The stresses of Cauchy are worth:

$$\sigma_x=0 \quad \sigma_y=26610.3\text{ MPa} \quad \sigma_z=6597.6\text{ MPa}$$

Lastly, the exerted forces are:

$$\begin{aligned} F_x &= 0 \\ F_y &= -25384\ S_{o[1,3]}\ N \\ F_z &= -6.9228\ 10^9\ N \qquad (\text{côté inférieur}) \end{aligned}$$

2.3 Uncertainty on the analytical

solution Solution.

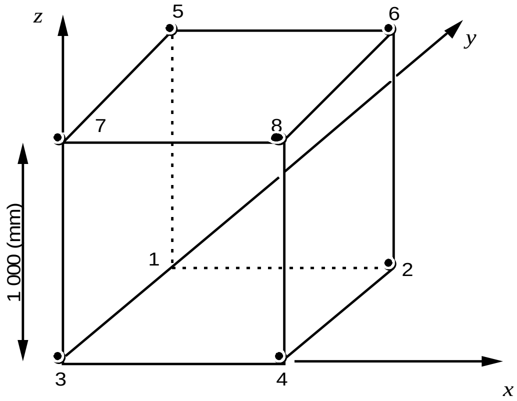
2.4 Bibliographical references

1) Eric LORENTZ “a nonlinear behavior model hyper elastic” Notes intern EDF/DER HI - 74/95/011/0

3 Modelization A

3.1 Characteristic of the voluminal

modelization Modelization: 1 mesh `HEXA 8`
1 mesh `QUAD4`



- rigid phase of rotation $0 \le t \le 1\ s$

[3,7]	$DX = 0$	$DY = 0$	$DZ = 0$
[1,5]	$DX = -1000\ \sin\left(\frac{\pi t}{2}\right)$	$DY = -1000\left(1 - \cos\frac{\pi t}{2}\right)$	$DZ = 0$
[2,6]			$DZ = 0$
[4,8]			$DZ = 0$

- phase of tension: $1\text{s} \le t \le 2\text{s}$
  - boundary conditions (`TYPE_CHARGE: "DIDI"`)

[3,7]	$\Delta DX = 0$	$\Delta DY = 0$	$DZ = 0$
[1,5]		$\Delta DY = 0$	$DZ = 0$
[2,6]			$DZ = 0$
[4,8]			$DZ = 0$

  - loading: pressure (negative) on face $[2,4,8,6]$ (`PRES_REP`): net $[2,4,8,6]$ (`QUAD4`): $PRES = -26610.3(t-1)$.

3.2 Characteristics of the mesh

Many nodes: 8 Number of meshes: 2
1 `HEXA8`
1 `QUAD4`

3.3 Quantities tested and results

the values are tested at the end of the loading ($t = 2\text{s}$)

Identification	Reference	*Aster*	% difference
Displacement DX (NO2)	– 1953.94	– 1953.92	0
Displacement DY (NO2)	100.	100.	0

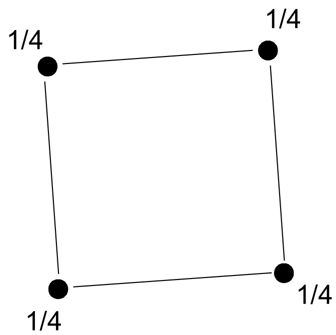
Stresses SIXX (PG1)	0	8. 10-10	
Stresses SIYY (PG1)	26610.3	26610.3	0
Stresses SIZZ (PG1)	6597.6	6597.6	0
Stresses SIXY (PG1)	0	10 -26	
Stresses SIXZ (PG1)	0	10 -11	
Stresses SIYZ (PG1)	0	10 -10	
Strain EPXX (PG1)	0.105	0.105	0
Strain EPYY (PG1)	– 0.045	– 0.045	0
Strain EPZZ (PG1)	0	10 -16	
Strain EPXY (PG1)	0	10 -14	
Strain EPXZ (PG1)	0	10 -14	
Strain EPYZ (PG1)	0	10 -16	
nodal Reaction DX (NO3)	0	10 -3	
nodal Reaction DY (NO3)	– 6.3462 109	– 6.3461 109	– 0.001
nodal Reaction DZ (NO3)	– 1.7307 109	– 1.7307 109	0.004

3.4 Remarks

Computation of the nodal force:

The applied force F on a face described by a linear mesh is distributed by:

$$F_{noeud} = \frac{1}{4} F$$



4 Summary of the results

It appears at the conclusion of this test that the numerical solution coincides remarkably with the analytical solution. One will notice however that the strong not linearity due to large rotations requires a relatively fine discretization in time, without being penalizing on the accuracy since, contrary to an incremental behavior model, the errors time step do not cumulate a one on the other.