SSNV121 - Rotation and tension very-elastic of a Summarized

bar:

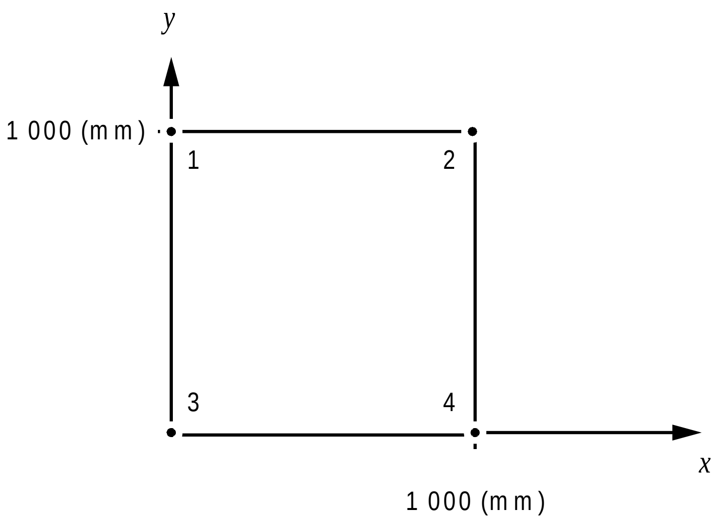
This test of quasi-static mechanics consists in making turn of $90°$ a parallelepipedic bar, to subject it to an important tension for finally letting it return in a discharged state. One validates thus the kinematics of the large deformations very-elastics (command STAT_NON_LINE, key word COMP_ELAS), and thus in particular large rotations, for a linear elastic behavior model.

The bar is modelled by an element voluminal (HEXA8, modelization A) or plane (QUAD4, assumption of plane strains, modelization B).

The results got by *Code_Aster* do not differ from the theoretical solution.

1 Problem of reference

1.1 Geometry



1.2 Material properties

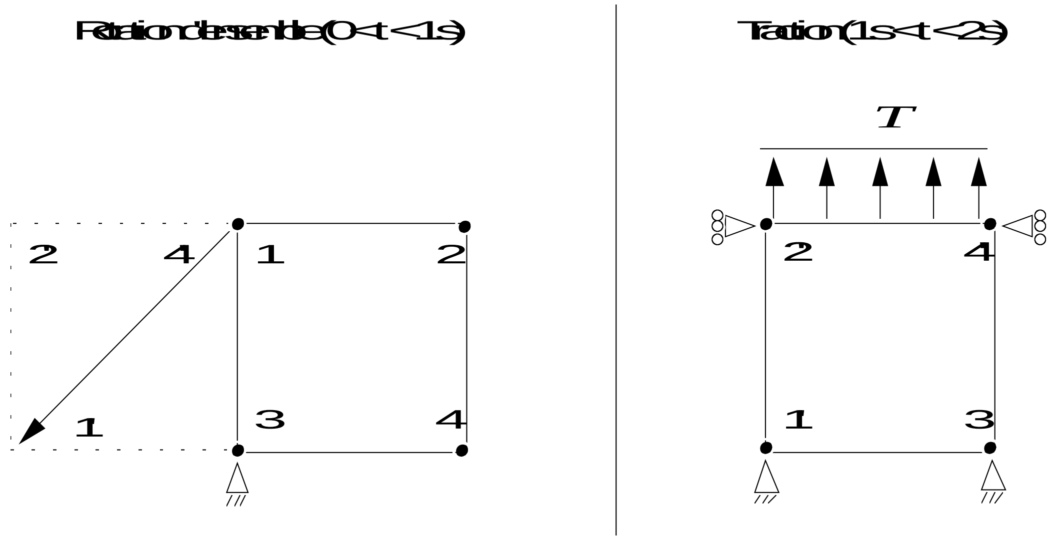
Behavior very-elastic of Coming St - Kirchhoff:

$$\mathbf{S} = \frac{\nu E}{(1+\nu)(1-2\nu)} \operatorname{tr}(\mathbf{E})\, \mathbf{1} + \frac{E}{1+\nu} \mathbf{E}$$

$E = 200\,000.$ MPa
$\nu = 0.3$

1.3 Boundary conditions and loadings

the loading is applied in two times: first of all, an overall rotation of structure, followed by a tension in the new configuration:



2 Reference solution

2.1 Method of calculating used for the reference solution

It acts of a plane problem. One can seek the solution in the form of a rigid rotation and a lengthening of a factor λ in the direction y.

$$\mathbf{U}(X,Y,Z) = \begin{bmatrix} -X-Y \\ (1+\lambda)X-Y \\ 0 \end{bmatrix}$$

The gradient of the transformation and the strain of Green-Lagrange are then:

$$\mathbf{F} = \begin{bmatrix} 0 & -1 & 0 \\ 1+\lambda & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \mathbf{E} = \begin{bmatrix} e & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ avec } e = \frac{\lambda(\lambda+2)}{2}$$

The behavior model leads then to a diagonal stress tensor Lagrangian:

$$\begin{cases} S_{xx} = \dfrac{(1-\nu)\,E}{(1+\nu)(1-2\nu)}\,e \\ S_{yy} = S_{zz} = \dfrac{\nu\,E}{(1+\nu)(1-2\nu)}\,e \end{cases}$$

The boundary condition of the balance equation then enables us to determine the value of lengthening λ :

$$T = (\mathbf{FS})_{yx} = (1+\lambda)S_{xx} \Rightarrow \frac{(1-\nu)\,E}{(1+\nu)(1-2\nu)}\frac{\lambda(\lambda+1)(\lambda+2)}{2} = T$$

The stress of Cauchy is given by:

$$\boldsymbol{\sigma} = \frac{1}{\mathrm{Det}\,\mathbf{F}}\mathbf{F}\,\mathbf{S}\,\mathbf{F}^T \Rightarrow \begin{cases} \sigma_{xx} = \sigma_{zz} = \dfrac{S_{yy}}{1+\lambda} \\ \sigma_{yy} = (1+\lambda)\,S_{xx} \end{cases}$$

Lastly, the force exerted on the sides:

- [2,4]: $F_y = \sigma_{yy}\,S_{[2,4]} = \sigma_{yy}\,S_{o[2,4]}$
- [4,3]: $F_x = \sigma_{xx}\,S_{[4,3]} = \sigma_{xx}(1+\lambda)\,S_{o[4,3]}$
- [1,2,3,4]: $F_z = \sigma_{zz}\,S_{[1,2,3,4]} = \sigma_{zz}(1+\lambda)\,S_{o[1,2,3,4]}$

where $S_{o[.]}$ initial surfaces of the sides represent.

2.2 Results of reference

One adopts like results of reference displacements, the stress of Cauchy and the force exerted on the sides [2,4] and [4,3].

At time $t=2\ s$ **:**

One seeks T such as lengthening $\lambda = 0.1$

is $T=31\ 096.154\ MPa$.

The stress of Cauchy is then:

$$\begin{cases}\sigma_{xx} = \sigma_{zz} = 11\ 013.986 \text{ MPa}\\ \sigma_{yy} = 31\ 096.154 \text{ MPa}\end{cases}$$

The exerted forces are:

$$F_x = 12\ 115.385 \times S_{o[4,3]}\ N$$
$$F_y = 31\ 096.154 \times S_{o[2,4]}\ N$$
$$F_z = 12\ 115.385 \times S_{o[1,2,3,4]}\ N$$

At time $t=3\ s$ **:**

The bar returned in its initial state:

$$\begin{cases}\lambda = 0\\ \sigma = 0\\ F = 0\end{cases}$$

2.3 Uncertainty on the analytical

solution Solution.

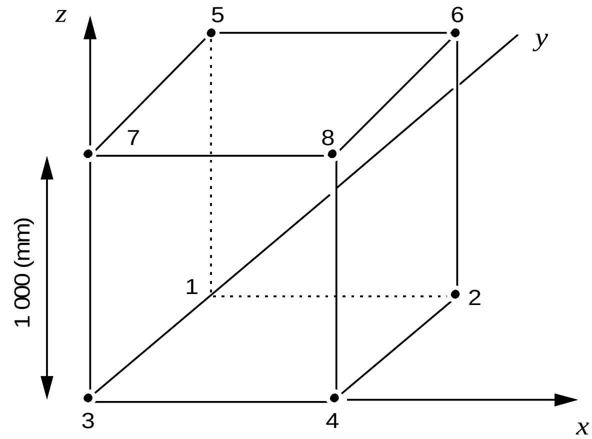
2.4 Bibliographical references

1) Eric LORENTZ "a nonlinear behavior model hyper elastic" Notes intern EDF/DER HI - 74/95/011/0

3 Modelization A

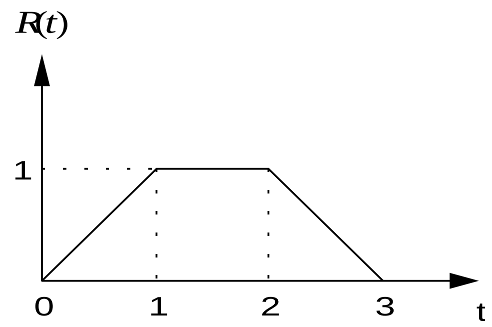
3.1 Characteristic of the voluminal

modelization Modelization: 1 mesh HEXA 8
1 mesh QUAD4



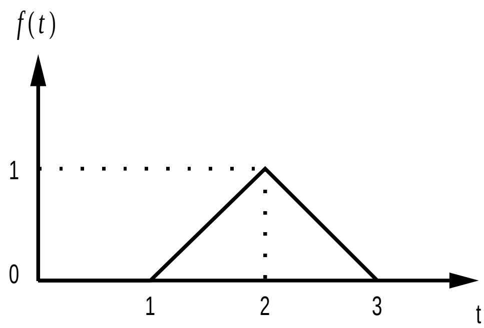
Boundary conditions:

(3,7) : DX=0 DY=0
(1,5) : DX=-1000R(t) DY=-1000R(t)
(2,6) : DX=-2000R(t)
(4,8) : DX=-1000R(t)



Loading: Tension on the face $[2,4,8,6]$

nets $[2,4,8,6]$ (QUAD4): $FY=31\,096.154\, f(t)\, MPa$



3.2 Characteristics of the mesh

Many nodes: 8

Number of meshes: 2
1 HEXA8
1 QUAD4

3.3 Quantities tested and results

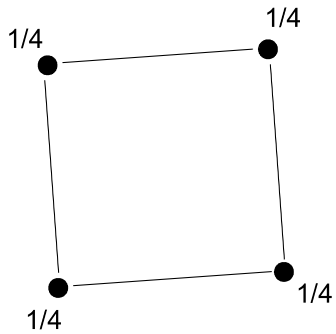
Identification	Reference	*Aster*	% difference
T = 2 Displacement DX (NO2)	100.100		0
T = 2 Displacement DY (NO4)	1100	1100	0
T = 2 Stresses SIGXX (PG1)	11013.986	11013.986	0
T = 2 Stresses SIGYY (PG1)	31096.154	31096.154	0
T = 2 Stresses SIGZZ (PG1)	11013.986	11013.986	0
T = 2 Stresses SIGXY (PG1)	0	□10 -9	/
T = 2 Stresses SIGXZ (PG1)	0	□10 -10	/
T = 2 Stresses SIGYZ (PG1)	0	□10 -10	/
T = 3 Displacement DX10 (NO2)	0	□10 -11	/
T = 3 Displacement DY (NO4)	0	□10 -12	/
T = 3 Stresses SIGXX (PG1)	0	□10 -9	/
T = 3 Stresses SIGYY (PG1)	0	□10 -10	/
T = 3 Stresses SIGZZ (PG1)	0	□10 -9	/
T = 3 Stresses SIGXY (PG1)	0	□10 -11	/
T = 3 Stresses SIGXZ (PG1)	0	□10 -10	/
T = 3 Stresses SIGYZ (PG1)	0	□10 -11	/
T = 2 nodal Force DX (NO8)	3.0289 109	3.0288 109	– 0.002%
T = 2 nodal Force DY (NO8)	7.774 109	7.774 109	0
T = 2 nodal Force DZ (NO8)	3.0289 109	3.0288 109	– 0.002%

3.4 Remarks

Computation of the nodal force:

The applied force F on a face described by a linear mesh is distributed by:

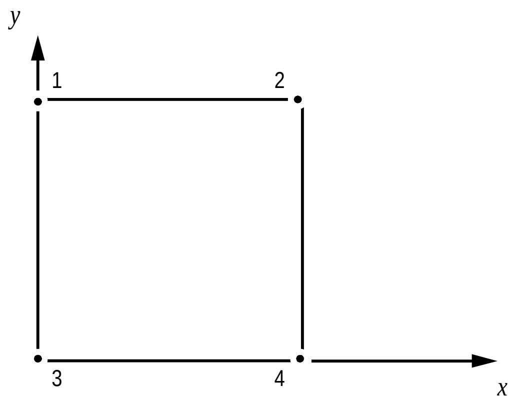
$$F_{noeud} = \frac{1}{4} F$$



4 Modelization B

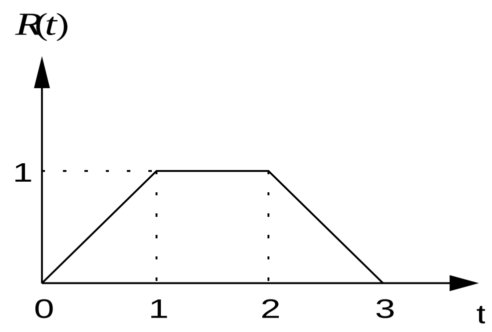
4.1 Characteristic of the modelization

Modelization $2D$ plane strains



Boundary conditions:

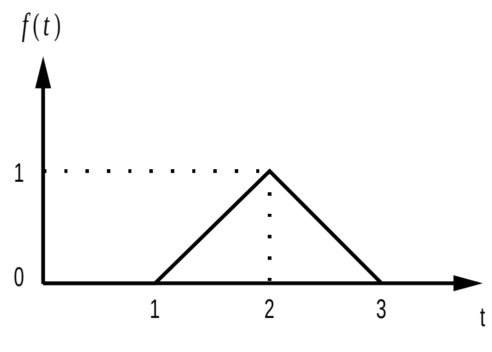
3:	DX=0	DY=0
1:	DX=-1000R(t)	DY=-1000R(t)
2:	DX=-2000R(t)	
4:	DX=-1000R(t)	



Loading:

Tension on the face $[2,4]$

nets $[2,4]$: $FY=31\ 096.154\ f(t)\ MPa$



4.2 Characteristics of the mesh

Many nodes: 4

Number of meshes: 2
1 QUAD4
1 SEG2

4.3 Quantities tested and results

Identification	Reference	*Aster*	% difference
T = 2 Displacement DX (NO2)	100.100		0

T = 2 Displacement DY (NO4)	1100	1100	0
T = 2 Stresses SIGXX (PG1)	11013.986	11013.986	0
T = 2 Stresses SIGYY (PG1)	31096.154	31096.154	0
T = 2 Stresses SIGZZ (PG1)	11013.986	11013.986	0
T = 2 Stresses SIGXY (PG1)	0	□10 -10	/
T = 3 Displacement DX (NO2)	0	□10 -12	/
T = 3 Displacement DY (NO4)	0	□10 -12	/
T = 3 Stresses SIGXX (PG1)	0	□10 -10	/
T = 3 Stresses SIGYY (PG1)	0	□10 -10	/
T = 3 Stresses SIGZZ (PG1)	0	□10 -10	/
T = 3 Stresses SIGXY (PG1)	0	□10 -10	/
T = 2 nodal Force DX (NO4)	6.0577 106	6.0577 106	0
T = 2 nodal Force DY (NO4)	15.5481 106	15.5481 106	0

4.4 Remarks

Computation of the nodal force:

The applied force F on a face described by a linear mesh is distributed by:

$$F_{noeud} = \frac{1}{2} F$$

5 Summary of the results

It appears at the conclusion of this test that the numerical solution coincides remarkably with the analytical solution. One will notice however that the strong not linearity due to large rotations requires a relatively fine discretization in time, without being penalizing on the accuracy since, contrary to an incremental constitutive law, the errors time step do not cumulate a one on the other.