SSNV112 - Hollow roll into incompressible (large deformations)

Summarized:

This test makes it possible to validate the quasi-incompressible elements in large deformations, in static for a three-dimensional, axisymmetric or two-dimensional problem (plane strains). One considers a hollow roll subjected to an internal radial displacement. The material has a Poisson's ratio equal to 0.4999 and one uses quasi-incompressible elements INCO_GD with the strains of SIMO_MIEHE and the quasi-incompressible elements INCO_LOG and INCO_LUP with the strains of GDEF_LOG.

Fifteen modelizations are carried out for this problem:

Modelization a: makes it possible to test modelization 3D_INCO_GD with HEXA20,

Modelization b: makes it possible to test modelization 3D_INCO_GD with TETRA10,

Modelization C: allows to test modelization D_PLAN_INCO_GD with QUAD8 and TRIA6,

Modelization D: allows to test modelization AXIS_INCO_GD with QUAD8 and TRIA6,

Modelization E: allows to test modelization 3D_INCO_GD with PENTA15,

Modelization F: allows to test modelization 3D_INCO_LOG with HEXA20,

Modelization G: allows to test modelization 3D_INCO_LOG with TETRA10,

Modelization H: allows to test modelization D_PLAN_INCO_LOG with QUAD8 and TRIA6,

Modelization I: allows to test modelization AXIS_INCO_LOG with QUAD8 and TRIA6,

Modelization J: allows to test modelization 3D_INCO_LOG with PENTA15,

Modelization K: allows to test modelization 3D_INCO_LUP with HEXA20,

Modelization L: allows to test modelization 3D_INCO_LUP with TETRA10,

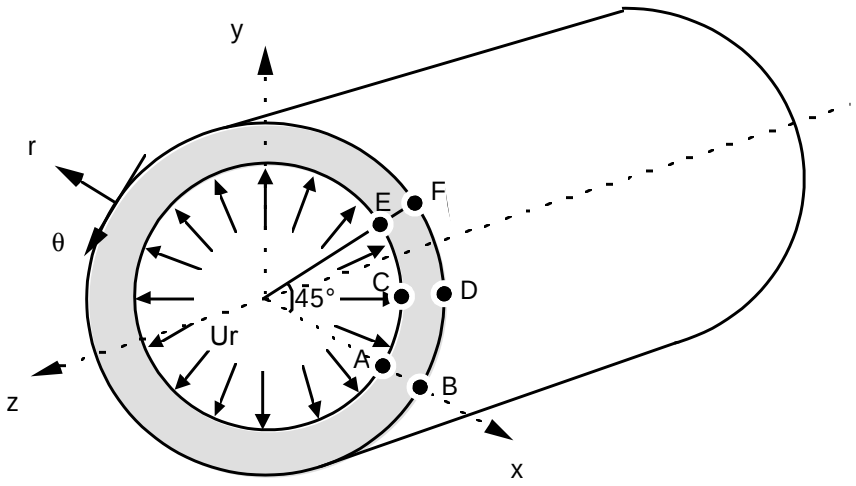
Modelization M: allows to test modelization D_PLAN_INCO_LUP with QUAD8 and TRIA6,

Modelization N: allows to test modelization AXIS_INCO_LUP with QUAD8 and TRIA6,

Modelization O: allows to test modelization 3D_INCO_LUP with PENTA15,

1 Problem of reference

1.1 Geometry



Rayon interne a = 0.1 m
Rayon externe b = 0.2 m

Coordonnées of the points:

	A	B	E	F	C	D
x	0.1	0.2	$0.1\times\cos(45)$	$0.2\times\cos(45)$	$0.1\times\cos(22.5)$	$0.2\times\cos(22.5)$
y	0	0	$0.1\times\sin(45)$	$0.1\times\sin(45)$	$0.1\times\sin(22.5)$	$0.1\times\sin(22.5)$
z	0	0	0	0	0	0

1.2 Properties of the material

$E=2.10^5\ MPa$
$\nu=0.4999$

1.3 Boundary conditions and loadings

Radial displacement $U_0=6.10^{-5}m$ (expansion)

2 Reference solution

2.1 Method of calculating

For the studied problem, displacement $\boldsymbol{u}$ is radial and thus of the form $\boldsymbol{u}=[u,0,0]$.

One from of deduced the general form from the tensor of the strains in large deformations:

$$\boldsymbol{b}=\boldsymbol{F}\,\boldsymbol{F}^{T}=\begin{bmatrix}(1+u')^2 & 0 & 0\\ 0 & \left(1+\frac{u}{r}\right)^2 & 0\\ 0 & 0 & 1\end{bmatrix}$$

as well as the form of the tensor of the stresses, which is written simply if one takes into account the fact that $J=\det \boldsymbol{F}=1$ for an incompressible problem:

$\boldsymbol{\sigma}=-p\,\boldsymbol{I}_{\boldsymbol{d}}+\mu\,\boldsymbol{b}^{\boldsymbol{d}}$, that is to say:

$$\begin{cases}\sigma_{rr}=-p+\mu\left(\frac{2}{3}(1+u')^2-\frac{1}{3}\left(1+\frac{u}{r}\right)^2-\frac{1}{3}\right)\\ \sigma_{\theta}\theta=-p+\mu\left(-\frac{1}{3}(1+u')^2+\frac{2}{3}\left(1+\frac{u}{r}\right)^2-\frac{1}{3}\right)\\ \sigma_{zz}=-p+\mu\left(-\frac{1}{3}(1+u')^2-\frac{1}{3}\left(1+\frac{u}{r}\right)^2+\frac{2}{3}\right)\\ \sigma_{r\theta}=\sigma_{rz}=\sigma_{\theta z}=0\end{cases}$$

The writing of the balance equations leads to the checking of only one equation:

$$\sigma'_{rr}+\frac{\sigma_{rr}-\sigma_{\theta\theta}}{r}=0$$

who allows to determine the pressure p knowing the field of radial displacement u :

$$p'=\mu\left(\frac{4}{3}(1+u')u''-\frac{2}{3}\left(1+\frac{u}{r}\right)\left(\frac{u'}{r}-\frac{u}{r^2}\right)+\frac{(1+u')^2}{r}-\frac{\left(1+\frac{u}{r}\right)^2}{r}\right)$$

2.2 Particularization of the solution

the condition of incompressibility is written $\det F=1$ with $\boldsymbol{F}=\begin{bmatrix}1+u' & 0 & 0\\ 0 & 1+\frac{u}{r} & 0\\ 0 & 0 & 1\end{bmatrix}$. Displacement u thus checks the following differential equation:

$$ru'+u+u'u=0 \qquad \text{éq 2.2-1}$$

the imposed loading is following it $u=U_0$ in $r=a$.

The solution in displacement is thus:

$$\begin{cases} u_r = -r + r\sqrt{r^2 + U_0\left(U_0 + 2\mathrm{a}\right)} \\ u_\theta = u_z = 0 \end{cases}$$

The tensor of the strains thus has as a statement:

$$\begin{cases} b_{rr} = \dfrac{r^2}{r^2 + U_0\left(U_0 + 2\mathrm{a}\right)} \\ b_{\theta\theta} = \dfrac{r^2 + U_0\left(U_0 + 2\mathrm{a}\right)}{r^2} \\ b_{zz} = 1 \\ b_{r\theta} = b_{z\theta} = b_{\theta z} = 0 \end{cases}$$

And the forced are worth:

$$\begin{cases} \sigma_{rr} = -p + \mu\left(\dfrac{2}{3}\dfrac{r^2}{r^2 + U_0\left(U_0 + 2\mathrm{a}\right)} - \dfrac{1}{3}\dfrac{r^2 + U_0\left(U_0 + 2\mathrm{a}\right)}{r^2} - \dfrac{1}{3}\right) \\ \sigma_\theta\theta = -p + \mu\left(-\dfrac{1}{3}\dfrac{r^2}{r^2 + U_0\left(U_0 + 2\mathrm{a}\right)} + \dfrac{2}{3}\dfrac{r^2 + U_0\left(U_0 + 2\mathrm{a}\right)}{r^2} - \dfrac{1}{3}\right) \\ \sigma_{zz} = -p + \mu\left(-\dfrac{1}{3}\dfrac{r^2}{r^2 + U_0\left(U_0 + 2\mathrm{a}\right)} - \dfrac{1}{3}\dfrac{r^2 + U_0\left(U_0 + 2\mathrm{a}\right)}{r^2} + \dfrac{2}{3}\right) \\ \sigma_{r\theta} = \sigma_{z\theta} = \sigma_{\theta z} = 0 \end{cases}$$

with p obtained by integration of [éq 2.2-1] which is worth:

$$p = \mu\left(\frac{U_0\left(U_0 + 2\mathrm{a}\right)}{6r^2} - \frac{2U_0\left(U_0 + 2\mathrm{a}\right)}{3\left(U_0\left(U_0 + 2\mathrm{a}\right) + r^2\right)} - \log(r) + \frac{1}{2}\log\left(U_0\left(U_0 + 2\mathrm{a}\right) + r^2\right)\right) + C$$

where C One is

a constant obtains finally the following numerical values:

in $r = 0.1$:	in $r = 0.2$:
$u_r = 6.10^{-5}$	$u_r = 3.006710^{-5}$
$\sigma_{rr} = -59.9955$	$\sigma_{rr} = 0.$
$\sigma_{\theta\theta} = 99.9566$	$\sigma_{\theta\theta} = 40.006$
$\sigma_{zz} = 19.9326$	$\sigma_{zz} = 20.$

the transition in the Cartesian system is done using the following relations:

$$\sigma_{xx}=\sigma_{rr}\cos^2\theta+\sigma_{\theta\theta}\sin^2\theta-2\sigma_{r\theta}\sin\theta\cos\theta$$
$$\sigma_{\theta\theta}=\sigma_{rr}\sin^2\theta+\sigma_{\theta\theta}\cos^2\theta+2\sigma_{r\theta}\sin\theta\cos\theta$$
$$\sigma_{zz}=\sigma_{rr}\sin\theta\cos\theta-\sigma_{\theta\theta}\sin\theta\cos\theta-2\sigma_{r\theta}(\cos^2\theta-\sin^2\theta)$$

2.3 Quantities and results of reference

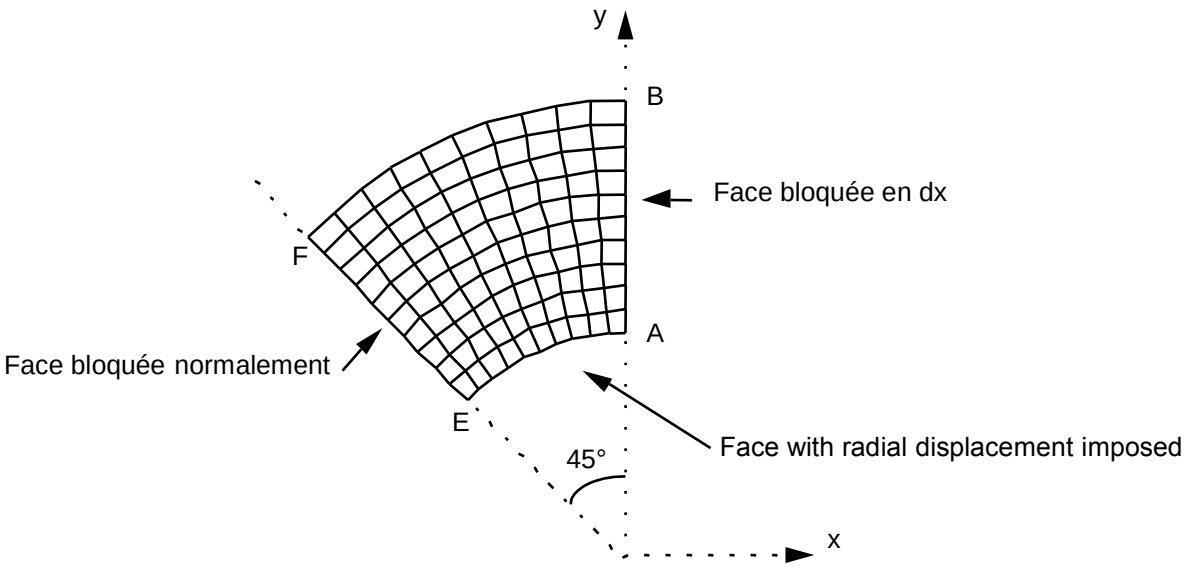
One compared to values of reference:

- displacements (u, v) at the points A and F,
- stresses $(\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \sigma_{xy})$ at the points A and F,
- von Mises stresses and Tresca as well as the eigenvalues of the tensor of the stresses at the point A.

3 Modelization A

3.1 Characteristic of the modelization

Mesh with incompressible elements 3D_INCO_GD of type HEXA20 only



Along the axis z :

- total thickness $e=0.01$
- 2 layers of limiting

elements Conditions:

```
DDL_IMPO=GROUP_NO       = ' FACSUP'   DZ   =0   .
             GROUP_NO = ' FACINF'     DZ   =0   .          sides AEFD ( z=0
and z=0.01 )
              GROUP_NO = ' FACEAB'    DX   =0   .          face AB
FACE_IMPO=GROUP_MA      = ' FACEEF'   DNOR =0   .          face EF
             GROUP_MA = ' FACEAE'     DNOR =-6.10  -5      face AE
```

3.2 Characteristics of the mesh

Many nodes: 1501 nodes
Number of meshes: 240 HEXA20

3.3 Quantities tested and results

displacements and the forced are evaluated at the points A and F. The components of field SIEQ_NOEU are tested at the point A formulates.

Standard		identification of reference	Reference	Tolerance
A	u	ANALYTIQUE	0.	1. 10-5
	v	ANALYTIQUE	6. 10-5	1. 10-3
	σ_{xx}	ANALYTIQUE	99.9566	0.01
	σ_{yy}	ANALYTIQUE	-59.9955	0.02
	σ_{zz}	ANALYTIQUE	19.9326	0.035
	σ_{xy}	ANALYTIQUE	0.	0.012
	VMIS	ANALYTIQUE	138.5226	0.02
	TRESCA	ANALYTIQUE	159.9521	0.02
	PRIN_1	ANALYTIQUE	-59.9955	0.02
	PRIN_2	ANALYTIQUE	19.9326	0.035
	PRIN_3	ANALYTIQUE	99.9566	0.01
	VMIS_SG	ANALYTIQUE	138.5226	0.02

Identification		Standard of reference	Reference	Tolerance
F	u	ANALYTIQUE	-2.1217 10-5	1. 10-3
	v	ANALYTIQUE	2.1217 10-5	1. 10-3
	σ_{xx}	ANALYTIQUE	20.003	0.005
	σ_{yy}	ANALYTIQUE	20.003	0.005
	σ_{zz}	ANALYTIQUE	20.003	0.005
	σ_{xy}	ANALYTIQUE	20.003	0.005

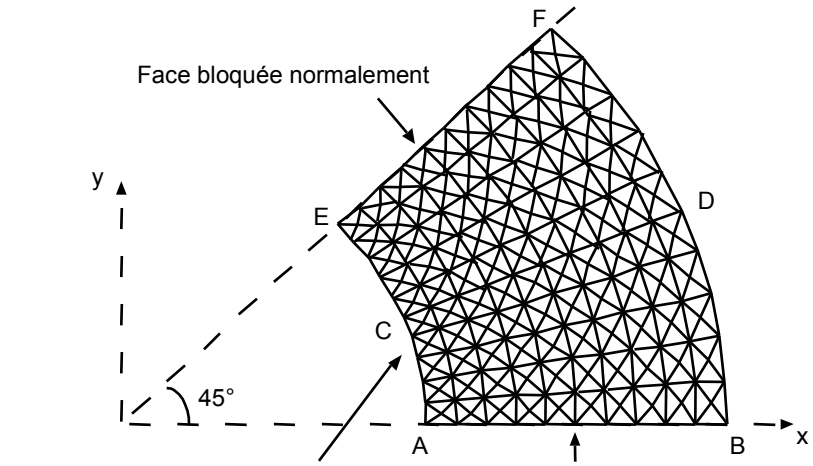
3.4 Remarks

One gets very good results since for all the examined quantities, the difference between the solution obtained with the code and the analytical solution is lower than 0.1% for displacements and lower than $3.5\ \%$ for the stresses.

4 Modelization B

4.1 Characteristic of the modelization

Mesh with incompressible elements 3D_INCO_GD of type TETRA10 only



AB is on the axis OX (contrary to the modelization A).
The mesh was obtained with GMSH for one density of $0,01$.

Limiting conditions:

```
DDL_IMPO=GROUP_NO       = ' FACSUP'   DZ   =0   .
             GROUP_NO = ' FACINF'     DZ   =0   .          sides AEFD ( z=0
and z=0.01 )
              GROUP_NO = ' FACEAB'    DY   =0   .          face AB
FACE_IMPO=GROUP_MA      = ' FACEEF'   DNOR =0   .          face EF
             GROUP_MA = ' FACEAE'     DNOR =-6.10  -5face  AE
```

4.2 Characteristics of the mesh

Many nodes: 2064
Number of meshes: 1121 TETRA10

4.3 Quantities tested and results

displacements and the forced are evaluated at the points A and F. The components of field SIEQ_NOEU are tested at the point A formulates.

Standard		identification of reference	Reference	Tolerance
A	u	ANALYTIQUE	6. 10-5	2. 10-3
	v	ANALYTIQUE	0.	1. 10-5
	σ_{xx}	ANALYTIQUE	-59.9955	0.025
	σ_{yy}	ANALYTIQUE	99.9566	0.02
	σ_{zz}	ANALYTIQUE	19.9326	0.03
	σ_{xy}	ANALYTIQUE	0.	0.03
	VMIS	ANALYTIQUE	138.5226	0.01
	TRESCA	ANALYTIQUE	159.9521	0.01
	PRIN_1	ANALYTIQUE	-59.9955	0.025
	PRIN_2	ANALYTIQUE	19.9326	0.03
	PRIN_3	ANALYTIQUE	99.9566	0.015
	VMIS_SG	ANALYTIQUE	138.5226	0.01

Identification		Standard of reference	Reference	Tolerance
F	u	ANALYTIQUE	2.1217 10-5	1. 10-3
	v	ANALYTIQUE	2.1217 10-5	1. 10-3
	σ_{xx}	ANALYTIQUE	20.003	0.005
	σ_{yy}	ANALYTIQUE	20.003	0.005
	σ_{zz}	ANALYTIQUE	20.003	0.005
	σ_{xy}	ANALYTIQUE	-20.003	0.01

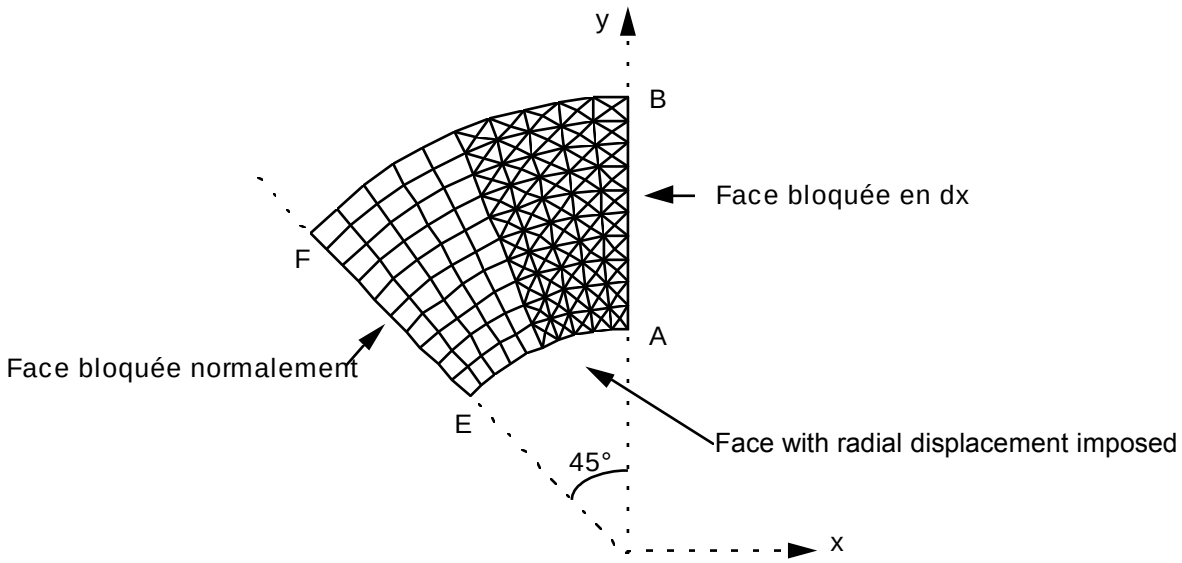
4.4 Remarks

the got results are completely correct since the stresses are obtained with an accuracy lower than $3\,\%$ even $1\,\%$ than the point F. the variation is a little more important here than for the HEXA20, but can be explained by the fact why the loading is imposed here in a way a little less precise since displacement U on the point A, is defined only with one accuracy of 0.158% against 0.077% (evening factor 2, that one finds on the stresses).

5 Modelization C

5.1 Characteristic of the modelization

Mesh with incompressible elements D_PLAN_INCO_GD of type TRIA6 and QUAD8



limiting Conditions:

```
DDL_IMPO  =GROUP_NO  = ' GRNM11'      DX   =0.CÔTÉ          AB
FACE_IMPO=GROUP_MA       = ' GRMA12'      DNOR =0.COTÉ            EF
          =GROUP_MA  = ' GRMA13'      DNOR =-6 . 10-5face    AE
```

Name of the nodes:

$A = N2 \quad B = N361 \quad C = N121 \quad D = N584 \quad E = N155 \; , \; F = N503$

5.2 Characteristic of the mesh

Many nodes: 591
Number of meshes: 200 TRIA6, 50 QUAD8.

5.3 Quantities tested and results

displacements and the forced are evaluated at the points A and F. The components of field SIEQ_NOEU are tested at the point A formulates.

Standard		identification of reference	Reference	Tolerance
A	u	ANALYTIQUE	0.	1. 10-5
	v	ANALYTIQUE	6. 10-5	5. 10-3
	σ_{xx}	ANALYTIQUE	99.9566	5. 10-3
	σ_{yy}	ANALYTIQUE	-59.9955	0.03
	σ_{zz}	ANALYTIQUE	19.9326	0.03
	σ_{xy}	ANALYTIQUE	0.	0.03
	VMIS	ANALYTIQUE	138.5226	0.02
	TRESCA	ANALYTIQUE	159.9521	0.02
	PRIN_1	ANALYTIQUE	-59.9955	0.03
	PRIN_2	ANALYTIQUE	19.9326	0.03
	PRIN_3	ANALYTIQUE	99.9566	0.02
	VMIS_SG	ANALYTIQUE	138.5226	0.02

Identification		Standard of reference	Reference	Tolerance
F	u	ANALYTIQUE	-2.1217 10-5	5. 10-3
	v	ANALYTIQUE	2.1217 10-5	5. 10-3
	σ_{xx}	ANALYTIQUE	20.003	5. 10-3
	σ_{yy}	ANALYTIQUE	20.003	5. 10-3
	σ_{zz}	ANALYTIQUE	20.003	5. 10-3
	σ_{xy}	ANALYTIQUE	20.003	5. 10-3

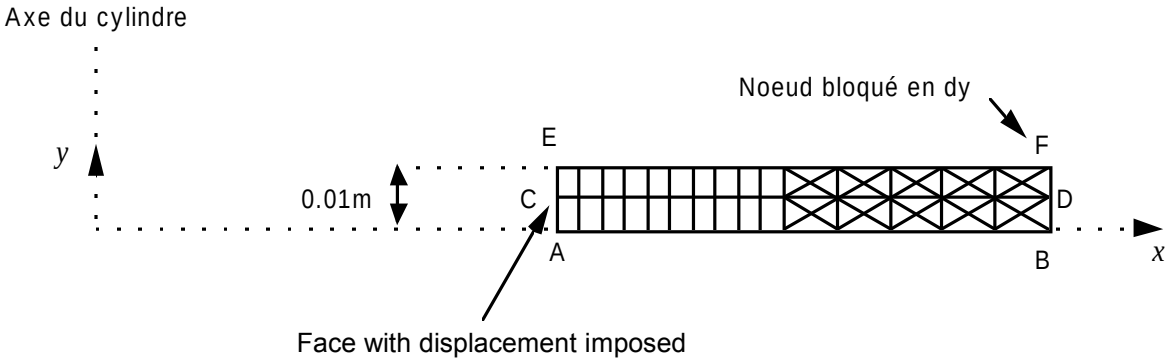
5.4 Remarks

As for the modelization 3D, the got results are completely satisfactory.

6 Modelization D

6.1 Characteristic of the modelization

Mesh with incompressible elements AXIS_INCO_GD of type TRIA6 and QUAD8



limiting Conditions:

```
DDL_IMPO=GROUP_NO        = ' FACSUP'      DY = 0.          side EF
              GROUP_NO = ' FACINF'        DY = 0.          side AB
FACE_IMPO=       GROUP_MA = ' FACEAE'        DX =  6. 10-5face       AE
```

6.2 Characteristic of the mesh

Many nodes: 175.
Number of meshes and types: 20 QUAD8, 40 TRIA6.

6.3 Quantities tested and results

displacements and the forced are evaluated at the points A and F. The components of field SIEQ_NOEU are tested at the point A formulates.

Standard		identification of reference	Reference	Tolerance
A	u	ANALYTIQUE	6. 10-5	1. 10-3
	v	ANALYTIQUE	0.	1. 10-5
	σ_{xx}	ANALYTIQUE	-59.9955	5. 10-3
	σ_{yy}	ANALYTIQUE	19.9326	5. 10-3
	σ_{zz}	ANALYTIQUE	99.9566	5. 10-3
	σ_{xy}	ANALYTIQUE	0.	1. 10-5
	VMIS	ANALYTIQUE	138.5226	5. 10-3
	TRESCA	ANALYTIQUE	159.9521	5. 10-3
	PRIN_1	ANALYTIQUE	-59.9955	5. 10-3
	PRIN_2	ANALYTIQUE	19.9326	5. 10-3
	PRIN_3	ANALYTIQUE	99.9566	5. 10-3
	VMIS_SG	ANALYTIQUE	138.5226	5. 10-3

Standard		Identification of reference	Reference	Tolerance
F	u	ANALYTIQUE	3. 10-5	1. 10-3
	v	ANALYTIQUE	0.	1. 10-5
	σ_{xx}	ANALYTIQUE	0.	0.03
	σ_{yy}	ANALYTIQUE	20.0	5. 10-3
	σ_{zz}	ANALYTIQUE	40.006	5. 10-3
	σ_{xy}	ANALYTIQUE	0.5	. 10-3

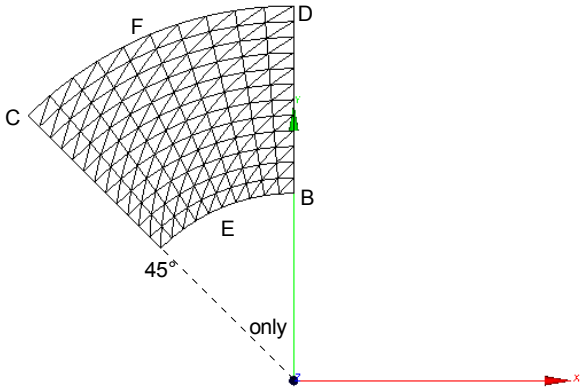
6.4 Remarks

the accuracy obtained is very good since all the stresses are obtained with an accuracy lower than 0.5%.

7 Modelization E

7.1 Characteristic of the modelization

Mesh with incompressible elements 3D_INCO_GD of limiting type PENTA15



A Conditions:

DDL_IMPO=GROUP_NO = ' FACSUP' DZ =0 .
GROUP_NO = ' FACINF' DZ =0 . sides $AEFD$ ($z=0$ and $z=0.01$)
GROUP_NO = ' FACEAB' DX =0 . face AB
FACE_IMPO=GROUP_MA = ' FACEEF' DNOR =0 . face EF
GROUP_MA = ' FACEAE' DNOR =-6.10 -5face AE

7.2 Characteristics of the mesh

Many nodes: 1861
Number of meshes: 480 PENTA15

7.3 Quantities tested and results

displacements and the forced are evaluated at the points A and F. The components of field SIEQ_NOEU are tested at the point A formulates.

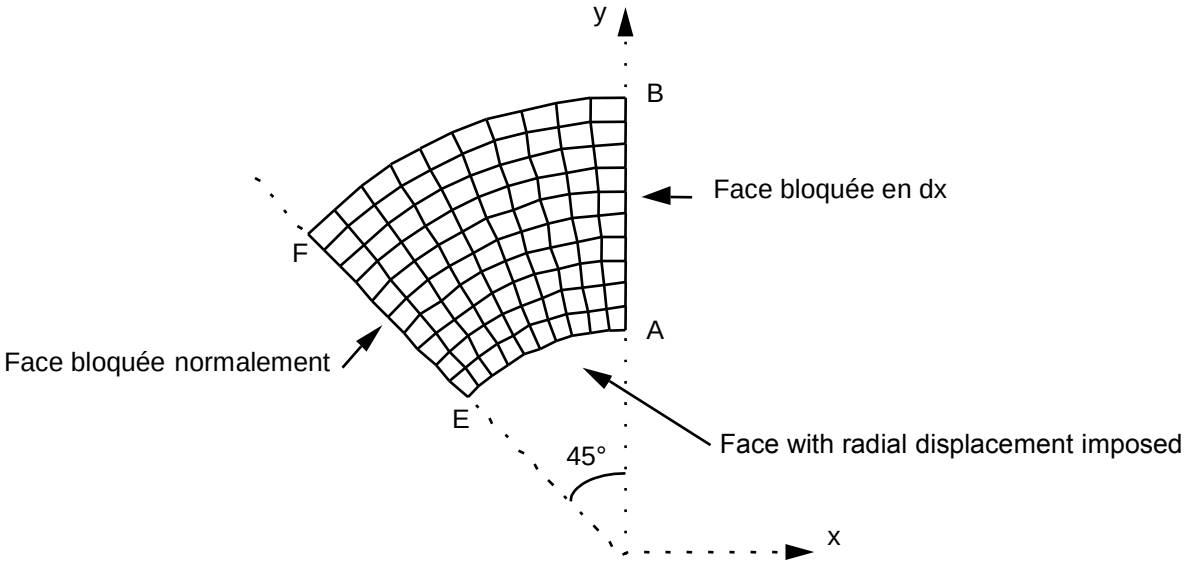
Standard		identification of reference	Reference	Tolerance
A	u	ANALYTIQUE	0.	1. 10-5
	v	ANALYTIQUE	6. 10-5	1. 10-3
	σ_{xx}	ANALYTIQUE	99.9566	0.03
	σ_{yy}	ANALYTIQUE	-59.9955	0.09
	σ_{zz}	ANALYTIQUE	19.9326	0.08
	σ_{xy}	ANALYTIQUE	0.	0.005
	VMIS	ANALYTIQUE	138.5226	0.05
	TRESCA	ANALYTIQUE	159.9521	0.05
	PRIN_1	ANALYTIQUE	-59.9955	0.09
	PRIN_2	ANALYTIQUE	19.9326	0.08
	PRIN_3	ANALYTIQUE	99.9566	0.03
	VMIS_SG	ANALYTIQUE	138.5226	0.05

Identification		Standard of reference	Reference	Tolerance
F	u	ANALYTIQUE	-2.1217 10-5	3. 10-3
	v	ANALYTIQUE	2.1217 10-5	3. 10-3
	σ_{xx}	ANALYTIQUE	20.003	0.04
	σ_{yy}	ANALYTIQUE	20.003	0.17
	σ_{zz}	ANALYTIQUE	20.003	0.07
	σ_{xy}	ANALYTIQUE	20.003	0.008

8 Modelization F

8.1 Characteristic of the modelization

Mesh with incompressible elements 3D_INCO_LOG of type HEXA20 only



Along the axis z :

- total thickness $e=0.01$
- 2 layers of limiting

elements Conditions:

```
DDL_IMPO=GROUP_NO       = ' FACSUP'   DZ   =0   .
             GROUP_NO = ' FACINF'     DZ   =0   .          sides AEFD ( z=0
and z=0.01 )
              GROUP_NO = ' FACEAB'    DX   =0   .          face AB
FACE_IMPO=GROUP_MA      = ' FACEEF'   DNOR =0   .          face EF
             GROUP_MA = ' FACEAE'     DNOR =-6.10  -5      face AE
```

8.2 Characteristic of the mesh

Many nodes: 1501 nodes
Number of meshes: 240 HEXA20

8.3 Quantities tested and results

displacements and the forced are evaluated at the points A and F. The components of field SIEQ_NOEU are tested at the point A formulates.

Standard		identification of reference	Reference	Tolerance
A	u	ANALYTIQUE	0.	1. 10-3
	v	ANALYTIQUE	6. 10-5	1. 10-3
	σ_{xx}	ANALYTIQUE	99.9566	0.01
	σ_{yy}	ANALYTIQUE	-59.9955	0.03
	σ_{zz}	ANALYTIQUE	19.9326	0.05
	σ_{xy}	ANALYTIQUE	0.	0.03
	VMIS	ANALYTIQUE	138.5226	0.001
	TRESCA	ANALYTIQUE	159.9521	0.001
	PRIN_1	ANALYTIQUE	-59.9955	0.0025
	PRIN_2	ANALYTIQUE	19.9326	0.005
	PRIN_3	ANALYTIQUE	99.9566	0.0005
	VMIS_SG	ANALYTIQUE	138.5226	0.001

Identification		Standard of reference	Reference	Tolerance
F	u	ANALYTIQUE	-2.1217 10-5	0.005
	v	ANALYTIQUE	2.1217 10-5	0.005
	σ_{xx}	ANALYTIQUE	20.003	0.002
	σ_{yy}	ANALYTIQUE	20.003	0.002
	σ_{zz}	ANALYTIQUE	20.003	0.0025
	σ_{xy}	ANALYTIQUE	20.003	0.0015

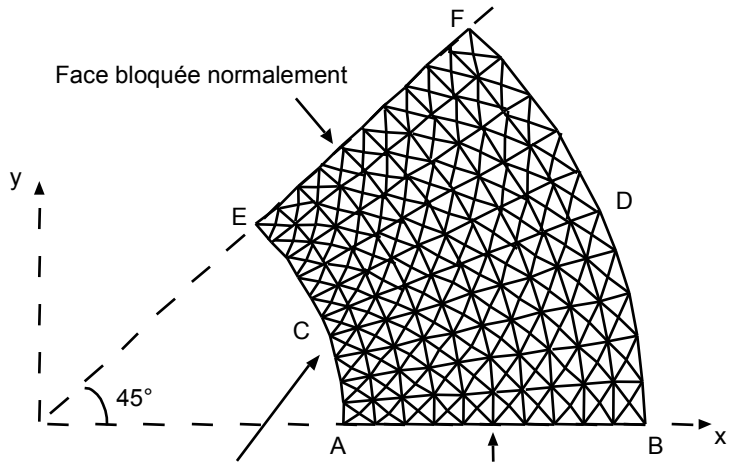
8.4 Remarks

One gets very good results since for all the examined quantities, the difference between the solution obtained with the code and the analytical solution is lower than 0.5% for displacements and lower than $5\ \%$ for the stresses.

9 Modelization G

9.1 Characteristic of the modelization

Mesh with incompressible elements 3D_INCO_LOG of type TETRA10 only



AB is on the axis OX (contrary to the modelization A).
The mesh was obtained with GMSH for one density of $0,01$.

Limiting conditions:

```
DDL_IMPO=GROUP_NO       = ' FACSUP'   DZ   =0   .
             GROUP_NO = ' FACINF'     DZ   =0   .          sides AEFD ( z=0
and z=0.01 )
              GROUP_NO = ' FACEAB'    DY   =0   .          face AB
FACE_IMPO=GROUP_MA      = ' FACEEF'   DNOR =0   .          face EF
             GROUP_MA = ' FACEAE'     DNOR =-6.10  -5     face AE
```

9.2 Characteristic of the mesh

Many nodes: 2064
Number of meshes: 1121 TETRA10

9.3 Quantities tested and results

displacements and the forced are evaluated at the points A and F. The components of field SIEQ_NOEU are tested at the point A formulates.

Standard		identification of reference	Reference	Tolerance
A	u	ANALYTIQUE	6. 10-5	2. 10-3
	v	ANALYTIQUE	0.	1. 10-3
	σ_{xx}	ANALYTIQUE	-59.9955	0.02
	σ_{yy}	ANALYTIQUE	99.9566	0.02
	σ_{zz}	ANALYTIQUE	19.9326	0.03
	σ_{xy}	ANALYTIQUE	0.	0.03
	VMIS	ANALYTIQUE	138.5226	0.002
	TRESCA	ANALYTIQUE	159.9521	0.002
	PRIN_1	ANALYTIQUE	-59.9955	0.02
	PRIN_2	ANALYTIQUE	19.9326	0.03
	PRIN_3	ANALYTIQUE	99.9566	0.015
	VMIS_SG	ANALYTIQUE	138.5226	0.002

Identification		Standard of reference	Reference	Tolerance
F	u	ANALYTIQUE	2.1217 10-5	1. 10-4
	v	ANALYTIQUE	2.1217 10-5	1. 10-4
	σ_{xx}	ANALYTIQUE	20.003	0.003
	σ_{yy}	ANALYTIQUE	20.003	0.005
	σ_{zz}	ANALYTIQUE	20.003	0.002
	σ_{xy}	ANALYTIQUE	-20.003	0.01

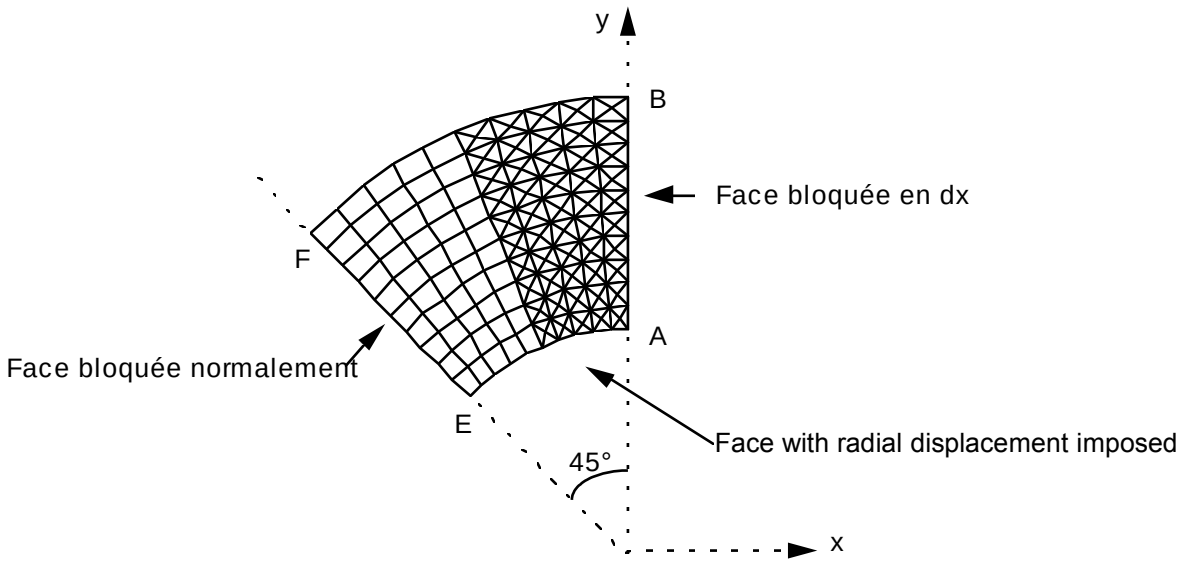
9.4 Remarks

the got results are completely correct since the stresses are obtained with an accuracy lower than $3\,\%$ even $1\,\%$ at the point F. the variation is a little more important here than for the HEXA20, but can be explained by the fact why the loading is imposed here in a way a little less precise since displacement U on the point A, is defined only with one accuracy of 0.158% against 0.077% (evening factor 2, that one finds on the stresses).

10 Modelization H

10.1 Characteristic of the modelization

Mesh with incompressible elements D_PLAN_INCO_LOG of type TRIA6 and QUAD8



limiting Conditions:

```
DDL_IMPO  =GROUP_NO  = ' GRNM11'      DX   =0.côté          AB
FACE_IMPO=GROUP_MA       = ' GRMA12'      DNOR =0.coté            EF
          =GROUP_MA  = ' GRMA13'      DNOR =-6 . 10-5face    AE
```

Name of the nodes:

$A=N2$, $B=N361$, $C=N121$, $D=N584$, $E=N155$, $F=N503$

10.2 Characteristics of the mesh

Many nodes: 591
Number of meshes: 200 TRIA6, 50 QUAD8.

10.3 Quantities tested and results

displacements and the forced are evaluated at the points A and F. The components of field SIEQ_NOEU are tested at the point A formulates.

Standard		identification of reference	Reference	Tolerance
A	u	ANALYTIQUE	0.	1. 10-5
	v	ANALYTIQUE	6. 10-5	1. 10-4
	σ_{xx}	ANALYTIQUE	99.9566	5. 10-3
	σ_{yy}	ANALYTIQUE	-59.9955	0.02
	σ_{zz}	ANALYTIQUE	19.9326	0.02
	σ_{xy}	ANALYTIQUE	0.	0.02
	VMIS	ANALYTIQUE	138.5226	0.006
	TRESCA	ANALYTIQUE	159.9521	0.006
	PRIN_1	ANALYTIQUE	-59.9955	0.02
	PRIN_2	ANALYTIQUE	19.9326	0.02
	PRIN_3	ANALYTIQUE	99.9566	0.003
	VMIS_SG	ANALYTIQUE	138.5226	0.006

Identification		Standard of reference	Reference	Tolerance
F	u	ANALYTIQUE	-2.1217 10-5	3. 10-4
	v	ANALYTIQUE	2.1217 10-5	3. 10-4
	σ_{xx}	ANALYTIQUE	20.003	5. 10-3
	σ_{yy}	ANALYTIQUE	20.003	2.5 10-3
	σ_{zz}	ANALYTIQUE	20.003	5. 10-4
	σ_{xy}	ANALYTIQUE	20.003	2. 10-3

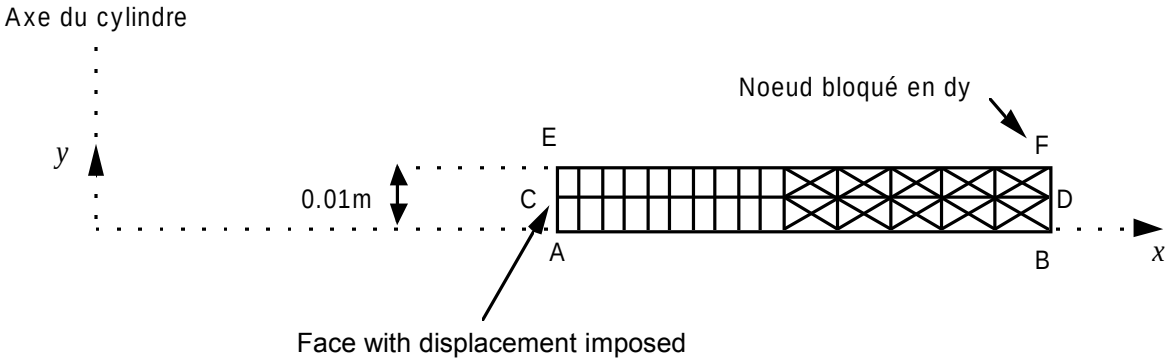
10.4 Remarks

As for the modelization 3D, the got results are completely satisfactory.

11 Modelization I

11.1 Characteristic of the modelization

Mesh with incompressible elements AXIS_INCO_LOG of type TRIA6 and QUAD8



limiting Conditions:

```
DDL_IMPO=GROUP_NO        = ' FACSUP'      DY = 0.          side EF
              GROUP_NO = ' FACINF'        DY = 0.          side AB
FACE_IMPO=       GROUP_MA = ' FACEAE'        DX =  6. 10-5face       AE
```

11.2 Characteristics of the mesh

Many nodes: 175.
Number of meshes and types: 20 QUAD8, 40 TRIA6.

11.3 Quantities tested and results

displacements and the forced are evaluated at the points A and F. The components of field SIEQ_NOEU are tested at the point A formulates.

Standard		identification of reference	Reference	Tolerance
A	u	ANALYTIQUE	6. 10-5	1. 10-3
	v	ANALYTIQUE	0.	1. 10-5
	σ_{xx}	ANALYTIQUE	-59.9955	5. 10-3
	σ_{yy}	ANALYTIQUE	19.9326	5. 10-3
	σ_{zz}	ANALYTIQUE	99.9566	5. 10-3
	σ_{xy}	ANALYTIQUE	0.	1. 10-5
	VMIS	ANALYTIQUE	138.5226	2. 10-3
	TRESCA	ANALYTIQUE	159.9521	2. 10-3
	PRIN_1	ANALYTIQUE	-59.9955	3. 10-3
	PRIN_2	ANALYTIQUE	19.9326	5. 10-3
	PRIN_3	ANALYTIQUE	99.9566	5. 10-4
	VMIS_SG	ANALYTIQUE	138.5226	2. 10-3

Standard		Identification of reference	Reference	Tolerance
F	u	ANALYTIQUE	3. 10-5	5. 10-4
	v	ANALYTIQUE	0.	1. 10-5
	σ_{xx}	ANALYTIQUE	0.	0.03
	σ_{yy}	ANALYTIQUE	20.0	3. 10-3
	σ_{zz}	ANALYTIQUE	40.006	3. 10-3
	σ_{xy}	ANALYTIQUE	0.5	. 10-3

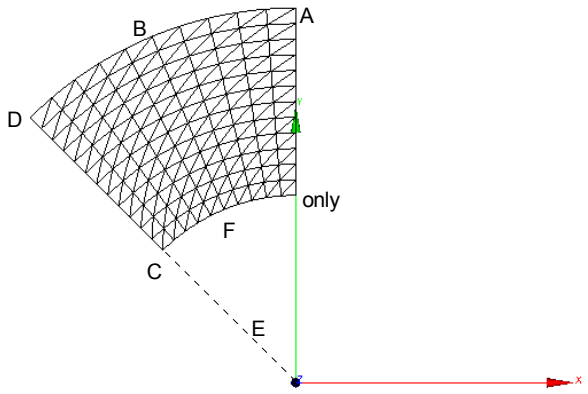
11.4 Remarks

the accuracy obtained is very good since all the stresses are obtained with an accuracy lower than 0.5%.

12 Modelization J

12.1 Characteristic of the modelization

Mesh with incompressible elements 3D_INCO_LOG of limiting type PENTA15



45° Conditions:

```
DDL_IMPO=GROUP_NO        = ' FACSUP'   DZ    =0   .
              GROUP_NO = ' FACINF'     DZ    =0   .
                GROUP_NO = ' FACEAB'    DX    =0   .
FACE_IMPO=GROUP_MA       = ' FACEEF'   DNOR  =0   .
              GROUP_MA = ' FACEAE'     DNOR =-6.10   -5
```

sides $AEFD$ ($z=0$ and $z=0.01$)

face AB

face EF

face AE

12.2 Characteristic of the mesh

Many nodes: 1861
Number of meshes: 480 PENTA15

12.3 Quantities tested and results

displacements and the forced are evaluated at the points A and F. The components of field SIEQ_NOEU are tested at the point A formulates.

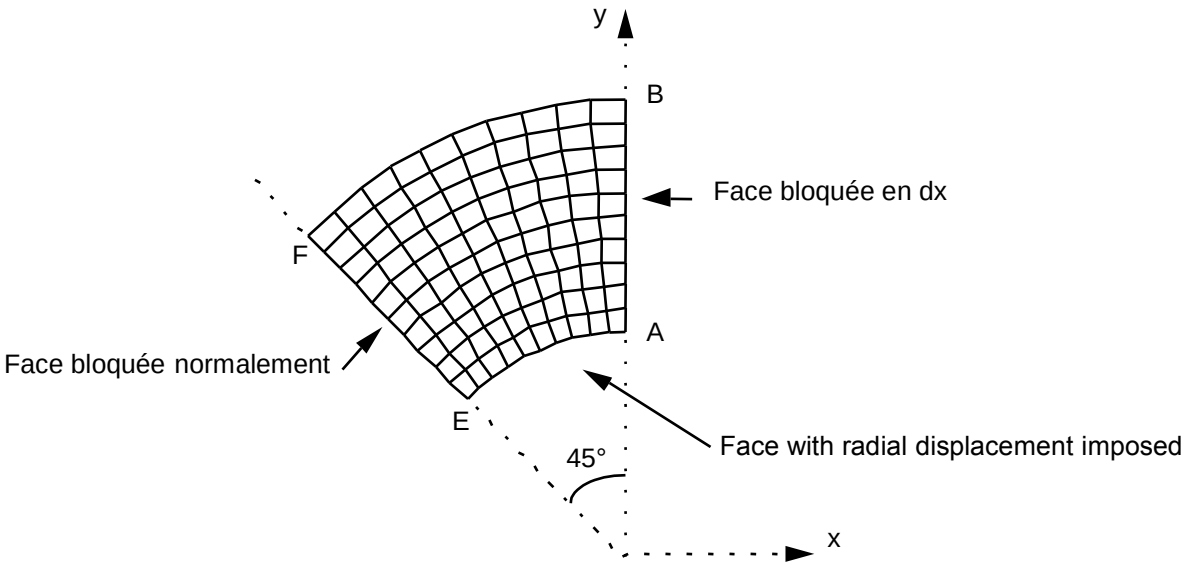
Standard		identification of reference	Reference	Tolerance
A	u	ANALYTIQUE	0.	1. 10-5
	v	ANALYTIQUE	6. 10-5	1. 10-3
	σ_{xx}	ANALYTIQUE	99.9566	0.0002
	σ_{yy}	ANALYTIQUE	-59.9955	0.003
	σ_{zz}	ANALYTIQUE	19.9326	0.007
	σ_{xy}	ANALYTIQUE	0.	0.0008
	VMIS	ANALYTIQUE	138.5226	0.05
	TRESCA	ANALYTIQUE	159.9521	0.05
	PRIN_1	ANALYTIQUE	-59.9955	0.09
	PRIN_2	ANALYTIQUE	19.9326	0.08
	PRIN_3	ANALYTIQUE	99.9566	0.03
	VMIS_SG	ANALYTIQUE	138.5226	0.05

Identification		Standard of reference	Reference	Tolerance
F	u	ANALYTIQUE	-2.1217 10-5	4. 10-3
	v	ANALYTIQUE	2.1217 10-5	4. 10-3
	σ_{xx}	ANALYTIQUE	20.003	0.007
	σ_{yy}	ANALYTIQUE	20.003	0.0006
	σ_{zz}	ANALYTIQUE	20.003	3. 10-5
	σ_{xy}	ANALYTIQUE	20.003	0.0004

13 Modelization K

13.1 Characteristic of the modelization

Mesh with incompressible elements 3D_INCO_LUP of type HEXA20 only



Along the axis z :

- total thickness $e=0.01$
- 2 layers of limiting

elements Conditions:

```
DDL_IMPO=GROUP_NO       = ' FACSUP'   DZ   =0   .
             GROUP_NO = ' FACINF'     DZ   =0   .          sides AEFD ( z=0
and z=0.01 )
              GROUP_NO = ' FACEAB'    DX   =0   .          face AB
FACE_IMPO=GROUP_MA      = ' FACEEF'   DNOR =0   .          face EF
             GROUP_MA = ' FACEAE'     DNOR =-6.10  -5      face AE
```

13.2 Characteristic of the mesh

Many nodes: 1501 nodes
Number of meshes: 240 HEXA20

13.3 Quantities tested and results

displacements and the forced are evaluated at the points A and F. The components of field SIEQ_NOEU are tested at the point A formulates.

Standard		identification of reference	Reference	Tolerance
A	u	ANALYTIQUE	0.	1. 10-3
	v	ANALYTIQUE	6. 10-5	1. 10-4
	σ_{xx}	ANALYTIQUE	99.9566	0.01
	σ_{yy}	ANALYTIQUE	-59.9955	0.03
	σ_{zz}	ANALYTIQUE	19.9326	0.05
	σ_{xy}	ANALYTIQUE	0.	0.03
	VMIS	ANALYTIQUE	138.5226	0.001
	TRESCA	ANALYTIQUE	159.9521	0.001
	PRIN_1	ANALYTIQUE	-59.9955	0.0025
	PRIN_2	ANALYTIQUE	19.9326	0.005
	PRIN_3	ANALYTIQUE	99.9566	0.0005
	VMIS_SG	ANALYTIQUE	138.5226	0.001

Identification		Standard of reference	Reference	Tolerance
F	u	ANALYTIQUE	-2.1217 10-5	0.005
	v	ANALYTIQUE	2.1217 10-5	0.005
	σ_{xx}	ANALYTIQUE	20.003	0.002
	σ_{yy}	ANALYTIQUE	20.003	0.002
	σ_{zz}	ANALYTIQUE	20.003	0.0025
	σ_{xy}	ANALYTIQUE	20.003	0.0015

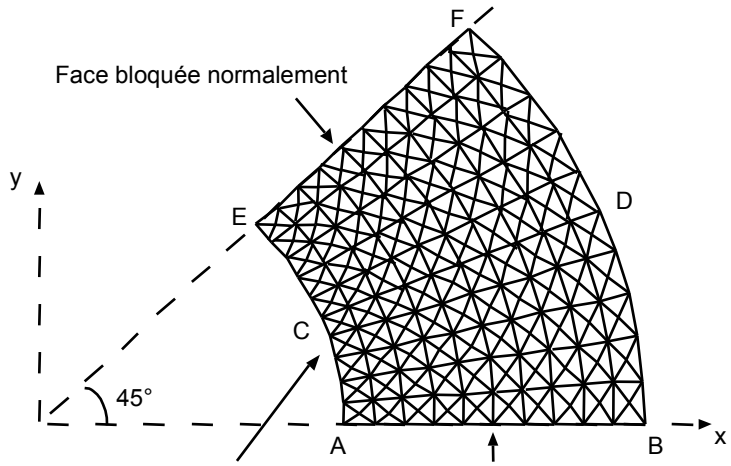
13.4 Remarks

One gets very good results since for all the examined quantities, the difference between the solution obtained with the code and the analytical solution is lower than 0.5% for displacements and lower than $5\ \%$ for the stresses.

14 Modelization L

14.1 Characteristic of the modelization

Mesh with incompressible elements 3D_INCO_LUP of type TETRA10 only



AB is on the axis OX (contrary to the modelization A).
The mesh was obtained with GMSH for one density of $0,01$.

Limiting conditions:

```
DDL_IMPO=GROUP_NO        = ' FACSUP'   DZ    =0   .
             GROUP_NO = ' FACINF'     DZ    =0   .          sides AEFD ( z=0
and z=0.01 )
              GROUP_NO = ' FACEAB'    DY    =0   .          face AB
FACE_IMPO=GROUP_MA       = ' FACEEF'   DNOR =0   .          face EF
             GROUP_MA = ' FACEAE'     DNOR =-6.10  -5      face AE
```

14.2 Characteristic of the mesh

Many nodes: 2064
Number of meshes: 1121 TETRA10

14.3 Quantities tested and results

displacements and the forced are evaluated at the points A and F. The components of field SIEQ_NOEU are tested at the point A formulates.

Standard		identification of reference	Reference	Tolerance
A	u	ANALYTIQUE	6. 10-5	2. 10-4
	v	ANALYTIQUE	0.	1. 10-3
	σ_{xx}	ANALYTIQUE	-59.9955	0.02
	σ_{yy}	ANALYTIQUE	99.9566	0.02
	σ_{zz}	ANALYTIQUE	19.9326	0.03
	σ_{xy}	ANALYTIQUE	0.	0.03
	VMIS	ANALYTIQUE	138.5226	0.002
	TRESCA	ANALYTIQUE	159.9521	0.002
	PRIN_1	ANALYTIQUE	-59.9955	0.02
	PRIN_2	ANALYTIQUE	19.9326	0.03
	PRIN_3	ANALYTIQUE	99.9566	0.015
	VMIS_SG	ANALYTIQUE	138.5226	0.002

Identification		Standard of reference	Reference	Tolerance
F	u	ANALYTIQUE	2.1217 10-5	1. 10-4
	v	ANALYTIQUE	2.1217 10-5	1. 10-4
	σ_{xx}	ANALYTIQUE	20.003	0.003
	σ_{yy}	ANALYTIQUE	20.003	0.005
	σ_{zz}	ANALYTIQUE	20.003	0.002
	σ_{xy}	ANALYTIQUE	-20.003	0.01

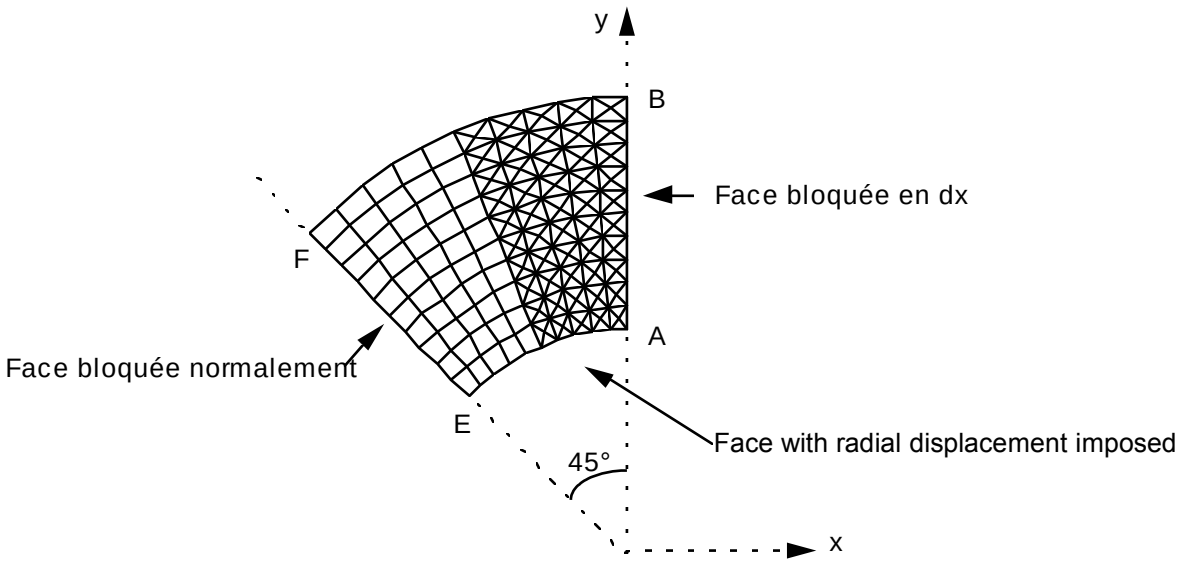
14.4 Remarks

the got results are completely correct since the stresses are obtained with an accuracy lower than 3% even 1% at the point F. the variation is a little more important here than for the HEXA20, but can be explained by the fact why the loading is imposed here in a way a little less precise since displacement U on the point A, is defined only with one accuracy of 0.158% against 0.077% (evening factor 2, that one finds on the stresses).

15 Modelization M

15.1 Characteristic of the modelization

Mesh with incompressible elements D_PLAN_INCO_LUP of type TRIA6 and QUAD8



limiting Conditions:

```
DDL_IMPO  =GROUP_NO  = ' GRNM11'      DX   =0.côté          AB
FACE_IMPO=GROUP_MA       = ' GRMA12'      DNOR =0.coté            EF
          =GROUP_MA  = ' GRMA13'      DNOR =-6 . 10-5face    AE
```

Name of the nodes:

$A=N2$, $B=N361$, $C=N121$, $D=N584$, $E=N155$, $F=N503$

15.2 Characteristics of the mesh

Many nodes: 591
Number of meshes: 200 TRIA6, 50 QUAD8.

15.3 Quantities tested and results

displacements and the forced are evaluated at the points A and F. The components of field SIEQ_NOEU are tested at the point A formulates.

Standard		identification of reference	Reference	Tolerance
A	u	ANALYTIQUE	0.	1. 10-5
	v	ANALYTIQUE	6. 10-5	1. 10-4
	σ_{xx}	ANALYTIQUE	99.9566	5. 10-3
	σ_{yy}	ANALYTIQUE	-59.9955	0.02
	σ_{zz}	ANALYTIQUE	19.9326	0.02
	σ_{xy}	ANALYTIQUE	0.	0.02
	VMIS	ANALYTIQUE	138.5226	0.006
	TRESCA	ANALYTIQUE	159.9521	0.006
	PRIN_1	ANALYTIQUE	-59.9955	0.02
	PRIN_2	ANALYTIQUE	19.9326	0.02
	PRIN_3	ANALYTIQUE	99.9566	0.003
	VMIS_SG	ANALYTIQUE	138.5226	0.006

Identification		Standard of reference	Reference	Tolerance
F	u	ANALYTIQUE	-2.1217 10-5	3. 10-4
	v	ANALYTIQUE	2.1217 10-5	3. 10-4
	σ_{xx}	ANALYTIQUE	20.003	5. 10-3
	σ_{yy}	ANALYTIQUE	20.003	2.5 10-3
	σ_{zz}	ANALYTIQUE	20.003	5. 10-4
	σ_{xy}	ANALYTIQUE	20.003	2. 10-3

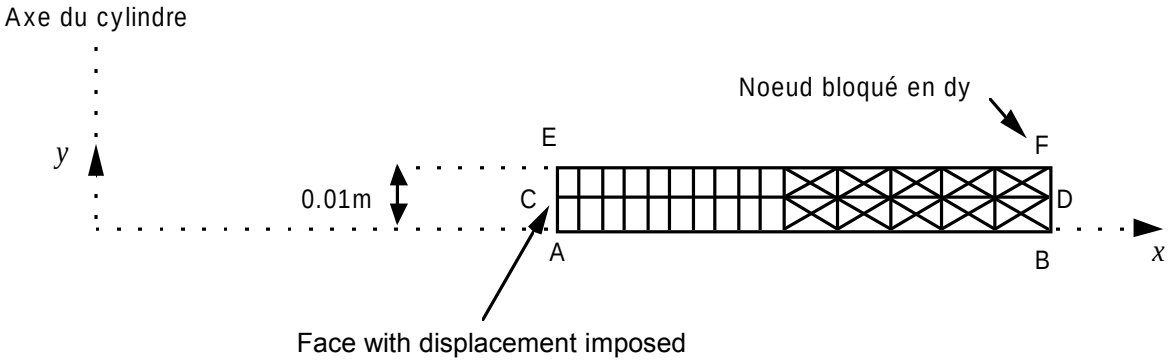
15.4 Remarks

As for the modelization 3D, the got results are completely satisfactory.

16 Modelization N

16.1 Characteristic of the modelization

Mesh with incompressible elements AXIS_INCO_LUP of type TRIA6 and QUAD8



limiting Conditions:

```
DDL_IMPO=GROUP_NO        = ' FACSUP'      DY = 0.          side EF
              GROUP_NO = ' FACINF'        DY = 0.          side AB
FACE_IMPO=       GROUP_MA = ' FACEAE'        DX =  6. 10-5face       AE
```

16.2 Characteristics of the mesh

Many nodes: 175.
Number of meshes and types: 20 QUAD8, 40 TRIA6.

16.3 Quantities tested and results

displacements and the forced are evaluated at the points A and F. The components of field SIEQ_NOEU are tested at the point A formulates.

Standard		identification of reference	Reference	Tolerance
A	u	ANALYTIQUE	6. 10-5	1. 10-3
	v	ANALYTIQUE	0.	1. 10-5
	σ_{xx}	ANALYTIQUE	-59.9955	5. 10-3
	σ_{yy}	ANALYTIQUE	19.9326	5. 10-3
	σ_{zz}	ANALYTIQUE	99.9566	5. 10-3
	σ_{xy}	ANALYTIQUE	0.	1. 10-5
	VMIS	ANALYTIQUE	138.5226	2. 10-3
	TRESCA	ANALYTIQUE	159.9521	2. 10-3
	PRIN_1	ANALYTIQUE	-59.9955	3. 10-3
	PRIN_2	ANALYTIQUE	19.9326	5. 10-3
	PRIN_3	ANALYTIQUE	99.9566	5. 10-4
	VMIS_SG	ANALYTIQUE	138.5226	2. 10-3

Standard		Identification of reference	Reference	Tolerance
F	u	ANALYTIQUE	3. 10-5	5. 10-4
	v	ANALYTIQUE	0.	1. 10-5
	σ_{xx}	ANALYTIQUE	0.	0.03
	σ_{yy}	ANALYTIQUE	20.0	3. 10-3
	σ_{zz}	ANALYTIQUE	40.006	3. 10-3
	σ_{xy}	ANALYTIQUE	0.5	. 10-3

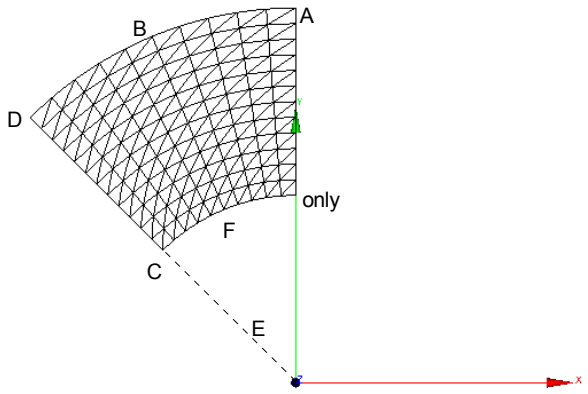
16.4 Remarks

the accuracy obtained is very good since all the stresses are obtained with an accuracy lower than 0.5%.

17 Modelization O

17.1 Characteristic of the modelization

Mesh with incompressible elements 3D_INCO_LUP of limiting type PENTA15



45° Conditions:

```
DDL_IMPO=GROUP_NO        = ' FACSUP'   DZ    =0   .
              GROUP_NO = ' FACINF'     DZ    =0   .
               GROUP_NO = ' FACEAB'    DX    =0   .
FACE_IMPO=GROUP_MA       = ' FACEEF'   DNOR  =0   .
              GROUP_MA = ' FACEAE'     DNOR =-6.10   -5
```

sides $AEFD$ ($z=0$ and $z=0.01$)

face AB

face EF

face AE

17.2 Characteristic of the mesh

Many nodes: 1861
Number of meshes: 480 PENTA15

17.3 Quantities tested and results

displacements and the forced are evaluated at the points A and F. The components of field SIEQ_NOEU are tested at the point A formulates.

Standard		identification of reference	Reference	Tolerance
A	u	ANALYTIQUE	0.	1. 10-5
	v	ANALYTIQUE	6. 10-5	1. 10-3
	σ_{xx}	ANALYTIQUE	99.9566	0.0002
	σ_{yy}	ANALYTIQUE	-59.9955	0.003
	σ_{zz}	ANALYTIQUE	19.9326	0.007
	σ_{xy}	ANALYTIQUE	0.	0.0008
	VMIS	ANALYTIQUE	138.5226	0.001
	TRESCA	ANALYTIQUE	159.9521	0.001
	PRIN_1	ANALYTIQUE	-59.9955	0.003
	PRIN_2	ANALYTIQUE	19.9326	0.008
	PRIN_3	ANALYTIQUE	99.9566	0.0002
	VMIS_SG	ANALYTIQUE	138.5226	0.001

Identification		Standard of reference	Reference	Tolerance
F	u	ANALYTIQUE	-2.1217 10-5	4. 10-3
	v	ANALYTIQUE	2.1217 10-5	4. 10-3
	σ_{xx}	ANALYTIQUE	20.003	0.007
	σ_{yy}	ANALYTIQUE	20.003	0.0006
	σ_{zz}	ANALYTIQUE	20.003	3. 10-5
	σ_{xy}	ANALYTIQUE	20.003	0.0004

18 Summary of the results

With a Poisson's ratio ν very close to 0.5, one finds the results of the incompressible analytical solution in large deformations, with a completely correct accuracy.