SSNV104 - Contact of two phères

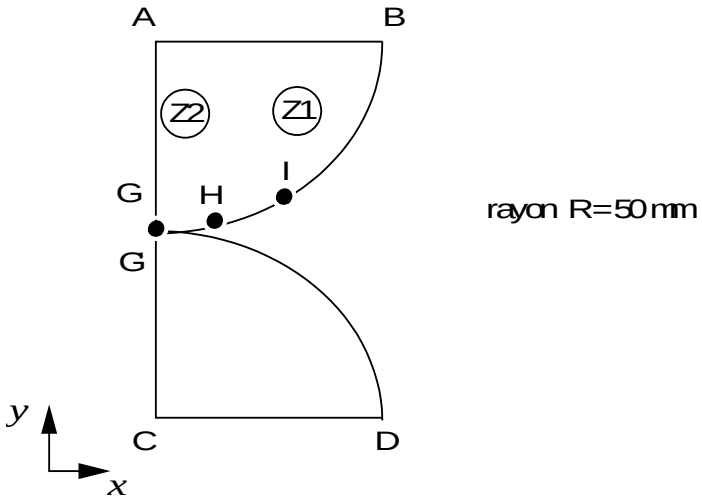
Summarized:

The computation two hemispheres one consists in crushing on the other to test the unilateral contact algorithm in static. The solution is compared with result analytical of Hertz.
The modelizations implement computations into axisymmetric as in $3D$ with linear/quadratic elements and various methods of resolutions of the contact (dualisation, penalization, continuous formulation).
This classical test for which one has an analytical solution is also used to validate several options of the algorithm of pairing.

1 Problem of reference

1.1 Geometry

the geometry consists of two half-spheres of identical radius which one represents a cut in the figure below.



Taking into account the differences in mesh, the positions of the nodes H and I indicated here in an approximate way are different in the 3 modelizations.

1.2 Properties of the material

$E = 20\,000.\,MPa$

$\nu = 0.3$

1.3 Boundary conditions and loadings

Crushing imposed on the spheres:

- DY imposed $-2\,mm$ for the nodes of AB
- DY imposed $+2\,mm$ for the nodes of CD

For axisymmetric computations:

- DX blocked on the axis AC

For computations $3\mathrm{D}$ (2 modelled quarters of sphere):

- conditions of symmetry on each quarter of sphere

2 Reference solution

2.1 Method of calculating used for the reference solution

In 1881, Hertz established under certain assumptions a solution with the problem which bears its name[1]. Thus by supposing a contact without friction and for which the half width of contact a is very small in front of the radius of the spheres R ($a \ll R$), the contact pressure at the points $C1$ and $C2$ is worth [2] :

$$P_0 = -\frac{E}{\pi(1-\nu^2)}\sqrt{\frac{2h}{R}} \qquad \text{éq 2.1-1}$$

where h corresponds to the imposed crushing, which is worth here $4\,mm$. That is to say $P_0 = -2798.3\,Mpa$.

The half-width of contact a is expressed according to imposed crushing and of the radius of the spheres:

$$a = \sqrt{\frac{Rh}{2}} \qquad \text{éq 2.1-2}$$

In this test, for a crushing of $4\,mm$ $a = 10\,mm$.
Contact surface is a disc of radius a, the distribution of pressure in this zone is the following one:

$$\text{So } x \leqslant a \text{ then } \quad P(x) = P_0\sqrt{1-\left(\frac{x}{a}\right)^2} \qquad \text{éq 2.1-3}$$

2.2 Quantities and results of reference

σ_{yy} to the point G (analytical solution).

Displacements in three points of the edge (NON-regression except for G analytical).

σ_{yy} in meshes leaning on G (NON-regression).

Statute and clearance in several points of contact surface (analytical).

2.3 Uncertainties on the solution

The computation analytical although valid for $a \ll R$ in general gives a very good approximation of the solution.

2.4 Bibliographical reference

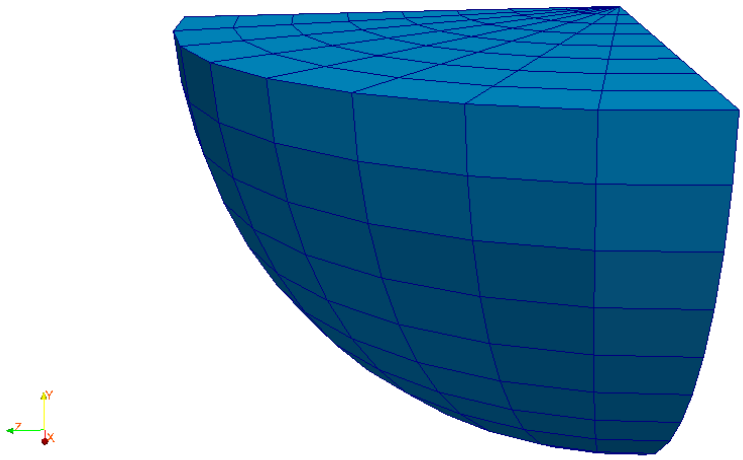
[1] HERTZ H. Über die Berührung fester elastischer Körper. *Newspaper für die queen und angewandte Mathematik*, 92:156-171, 1881.
[2] DUMONT G, *method of the active stresses applied to the unilateral contact*. Note technical, EDF-DER, 1993. HI-75/93/016.

3 Modelization A

3.1 Characteristic of the modelization

One uses a modelization 3D. Due to symmetry, only one quarter of hemisphere is represented, a condition of type unilateral connection makes it possible to supplement the modelization.
In this modelization, the points H and H_p (respectively I and I_p) are the same points (compared to those described in the geometry) but located on the two sides supporting the conditions of symmetry.
The results got with this modelization are used as reference (AUTRE_ASTER) for the three-dimensional modelizations I and J.

3.2 Characteristic of the mesh



Nodes: 1849.

Meshes : 96 TETRA4, 2936 PENTA6, 112 PYRAM5.

3.3 Quantities tested and results

the First computation (formulation "LIAISON_UNIL")

Standard	Identification of reference	Value of reference	Tolerance
σ_{yy} does not net $M2948$ G	"ANALYTIQUE"	-2798,3 N	14,0%
σ_{yy} mesh $M2948$ not G	"NON_REGRESSION"	-3176,3 N	1,0E-4%
σ_{yy} does not net $M2960$ G	"NON_REGRESSION"	-3176,3 N	1,0E-4%
σ_{yy} does not net $M2972$ G	"NON_REGRESSION"	-3176,3 N	1,0E-4%
σ_{yy} does not net $M2984$ G	"NON_REGRESSION"	-3176,3 N	1,0E-4%
σ_{yy} does not net $M2996$ G	"NON_REGRESSION"	-3176,3 N	1,0E-4%
σ_{yy} does not net $M3008$ G	"NON_REGRESSION"	-3176,3 N	1,0E-4%
σ_{yy} does not net $M3020$ G	"NON_REGRESSION"	-3176,3 N	1,0E-4%

σ_{yy} does not net $M3032$ G	"NON_REGRESSION"	-3176,3 N	1,0E-4%
DX point G	"ANALYTIQUE"	0 mm	1.0E-10
DX not H	"NON_REGRESSION"	-0,11343 mm	1,0E-4%
DZ point H_p	"NON_REGRESSION"	-0,11343 mm	1,0E-4%
DY point H	"NON_REGRESSION"	-0,16291 mm	1,0E-4%
DY point H_p	"NON_REGRESSION"	-0,16291 mm	1,0E-4%
DX point I	"NON_REGRESSION"	-0,17845 mm	1,0E-4%
DZ point I_p	"NON_REGRESSION"	-0,17845 mm	1,0E-4%
DY point I	"NON_REGRESSION"	-0,62966 mm	1,0E-4%
DY point I_p	"NON_REGRESSION"	-0,62966 mm	1,0E-4%

3.4 Remarks

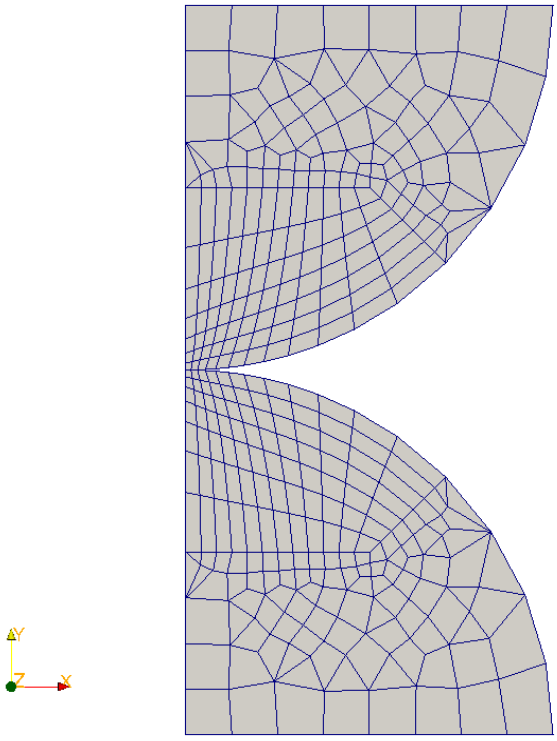
the results got in this three-dimensional modelization are in concord with the analytical solution. One notes moreover than the radiant mesh makes it possible to preserve the axisymetry of the problem, this is why the results got in each mesh which contributes to the point G are similar.
This modelization validates the use of a unilateral condition function of the space, i.e. whose "clearance" is not the same one for all the constrained nodes. The got results are identical to a modelization of contact.

4 Modelization B

4.1 Characteristic of the modelization

One uses a modelization AXIS. Three computations are carried out with options of pairing or contact algorithms different.

4.2 Characteristics of the mesh



Nodes: 376 nodes.

Meshes : 30 TRIA3 and 324 QUAD4.

4.3 Quantities tested and results

the First computation (nodal pairing, normal slave)

Standard	Identification of reference	Value of reference	Tolerance
σ_{yy} nets $M31$ node $N291(G)$	"ANALYTIQUE"	-2798,3 N	7.0%
σ_{yy} mesh $M31$ node $N291(G)$	"NON_REGRESSION"	-2971,27 N	0.1%
DX node $N291(G)$	"ANALYTIQUE"	0 mm	1.0E-10
DX node $N287(H)$	"NON_REGRESSION"	-0,111283 mm	0.1%
DY node $N287(H)$	"NON_REGRESSION"	-0,161842 mm	0.1%
DX node $N285(I)$	"NON_REGRESSION"	-0,168540 mm	0.1%
DY node $N285(I)$	"NON_REGRESSION"	-0,628587 mm	0,1%

the Second computation (" LAGRANGIAN" algorithm)

Standard	Identification of reference	Value of reference	Tolerance
σ_{yy} nets $M31$ node $N291(G)$	"ANALYTIQUE"	-2798,3 N	7.0%
σ_{yy} mesh $M31$ node $N291(G)$	"NON_REGRESSION"	-2973,0 N	0.1%
DX node $N291(G)$	"ANALYTIQUE"	0 mm	1.0E-10
DX node $N287(H)$	"NON_REGRESSION"	-0,108104 mm	0.1%
DY node $N287(H)$	"NON_REGRESSION"	-0,164375 mm	0.1%
DX node $N285(I)$	"NON_REGRESSION"	-0,160912 mm	0.1%
DY node $N285(I)$	"NON_REGRESSION"	-0,630986 mm	0,1%

the Third computation (algorithm "FORCED")

Standard	Identification of reference	Value of reference	Tolerance
σ_{yy} nets $M31$ node $N291(G)$	"ANALYTIQUE"	-2798,3 N	7.0%
σ_{yy} mesh $M31$ node $N291(G)$	"NON_REGRESSION"	-2973,0 N	0.1%
DX node $N291(G)$	"ANALYTIQUE"	0 mm	1.0E-10
DX node $N287(H)$	"NON_REGRESSION"	-0,108104 mm	0.1%
DY node $N287(H)$	"NON_REGRESSION"	-0,164375 mm	0.1%
DX node $N285(I)$	"NON_REGRESSION"	-0,160912 mm	0.1%
DY node $N285(I)$	"NON_REGRESSION"	-0,630986 mm	0,1%

4.4 Remarks

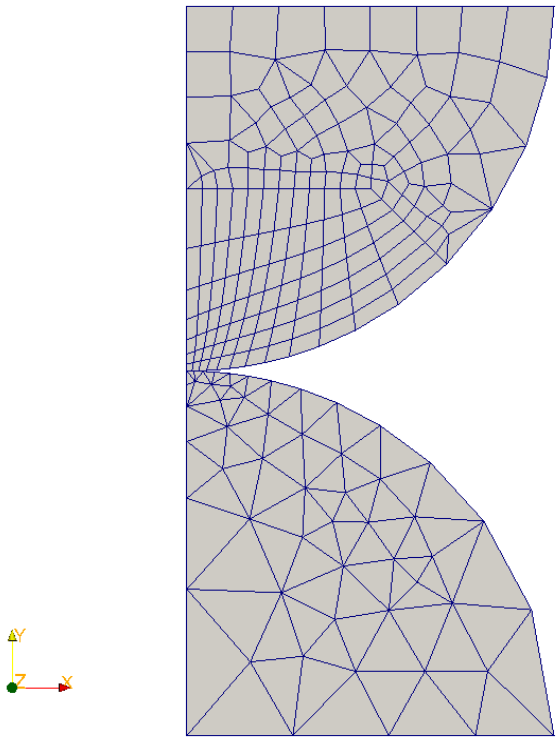
the difference between the contact pressure at the point G and the analytical solution is explained on the one hand by extrapolation (transition Gauss points towards nodes) and on the other hand by the weak refinement of the mesh.
It is noted that the dualized contact algorithms "LAGRANGIAN" and "STRESS" give the same results exactly and that nodal pairing makes it possible to preserve the symmetry of the problem while giving results very close to pairing node-facet.

5 Modelization D

5.1 Characteristic of the modelization

One uses a modelization AXIS. Two computations are carried out with different options of pairing. In this modelization, imposed crushing goes until $10\,mm$.

5.2 Characteristics of the mesh



Nodes: 243.

Meshes : 100 TRIA3, 162 QUAD4.

5.3 Quantities tested and results

the First computation (lissage)

Standard	Identification of reference	Value of reference	Tolerance
σ_{yy} nets $M31$ urgent $N291(G)$ node 1.0	"ANALYTIQUE"	-1978,7 N	7,0%
σ_{yy} mesh $M286$ urgent $N55(G')$ node 1.0	"ANALYTIQUE"	-1978,7 N	1,0%
σ_{yy} nets $M293$ urgent $N55(G')$ node 1.0	"ANALYTIQUE"	-1978,7 N	15,0%
σ_{yy} mesh $M31$ urgent $N291(G)$ node 2.0	"ANALYTIQUE"	-2798,3 N	5,0%
σ_{yy} mesh $M286$ urgent $N55(G')$ node 2.0	"ANALYTIQUE"	-2798,3 N	3,0%
σ_{yy} mesh $M293$ urgent $N55(G')$ node 2.0	"ANALYTIQUE"	-2798,3 N	15,0%

σ_{yy} mesh $M31$ urgent $N291(G)$ node 3.0	"ANALYTIQUE"	-3427,2 N	2,0%
σ_{yy} mesh $M286$ urgent $N55(G')$ node 3.0	"ANALYTIQUE"	-3427,2 N	3,0%
σ_{yy} mesh $M293$ urgent $N55(G')$ node 3.0	"ANALYTIQUE"	-3427,2 N	14,0%
σ_{yy} mesh $M31$ urgent $N291(G)$ node 4.0	"ANALYTIQUE"	-3957,4 N	3,0%
σ_{yy} mesh $M286$ urgent $N55(G')$ node 4.0	"ANALYTIQUE"	-3957,4 N	4,0%
σ_{yy} mesh $M293$ urgent $N55(G')$ node 4.0	"ANALYTIQUE"	-3957,4 N	15,0%
σ_{yy} mesh $M31$ urgent $N291(G)$ node 5.0	"ANALYTIQUE"	-4424,5 N	6,0%
σ_{yy} mesh $M286$ urgent $N55(G')$ node 5.0	"ANALYTIQUE"	-4424,5 N	4,0%
σ_{yy} mesh $M293$ urgent $N55(G')$ node 5.0	"ANALYTIQUE"	-4424,5 N	15,0%

the Second computation (nodal pairing and normal slave)

Standard	Identification of reference	Value of reference	Tolerance
σ_{yy} net $M31$ urgent $N291(G)$ node 1.0	"ANALYTIQUE"	-1978,7 N	2,0%
σ_{yy} mesh $M286$ urgent $N55(G')$ node 1.0	"ANALYTIQUE"	-1978,7 N	1,0%
σ_{yy} nets $M293$ urgent $N55(G')$ node 1.0	"ANALYTIQUE"	-1978,7 N	4,0%
σ_{yy} mesh $M31$ urgent $N291(G)$ node 2.0	"ANALYTIQUE"	-2798,3 N	7,0%
σ_{yy} mesh $M286$ urgent $N55(G')$ node 2.0	"ANALYTIQUE"	-2798,3 N	7,0%
σ_{yy} mesh $M293$ urgent $N55(G')$ node 2.0	"ANALYTIQUE"	-2798,3 N	4,0%
σ_{yy} mesh $M31$ urgent $N291(G)$ node 3.0	"ANALYTIQUE"	-3427,2 N	18,0%
σ_{yy} mesh $M286$ urgent $N55(G')$ node 3.0	"ANALYTIQUE"	-3427,2 N	13,0%
σ_{yy} mesh $M293$ urgent $N55(G')$ node 3.0	"ANALYTIQUE"	-3427,2 N	3,0%
σ_{yy} mesh $M31$ urgent $N291(G)$ node 4.0	"ANALYTIQUE"	-3957,4 N	28,0%
σ_{yy} mesh $M286$ urgent $N55(G')$ node 4.0	"ANALYTIQUE"	-3957,4 N	20,0%
σ_{yy} mesh $M293$ urgent $N55(G')$ node 4.0	"ANALYTIQUE"	-3957,4 N	2,0%
σ_{yy} mesh $M31$ urgent $N291(G)$ node 5.0	"ANALYTIQUE"	-4424,5 N	38,0%
σ_{yy} mesh $M286$ urgent $N55(G')$ node 5.0	"ANALYTIQUE"	-4424,5 N	27,0%
σ_{yy} mesh $M293$ urgent $N55(G')$ node 5.0	"ANALYTIQUE"	-4424,5 N	1,0%

5.4 Remarks

The mesh consist of 2 half-spheres with a grid differently (the higher sphere is in QUAD4, the lower sphere in TRIA3). The contact pressure raised on the upper surface is in concord with the analytical solution (the variation of about 5% is explained by extrapolation).
On lower surface, one notes contrary to the considerable variations because there exist 2 meshes which contribute to the point G' (mesh of TRIA3), however if one meshes makes the average of the results got on these 2, one finds a pressure in concord with the analytical solution.

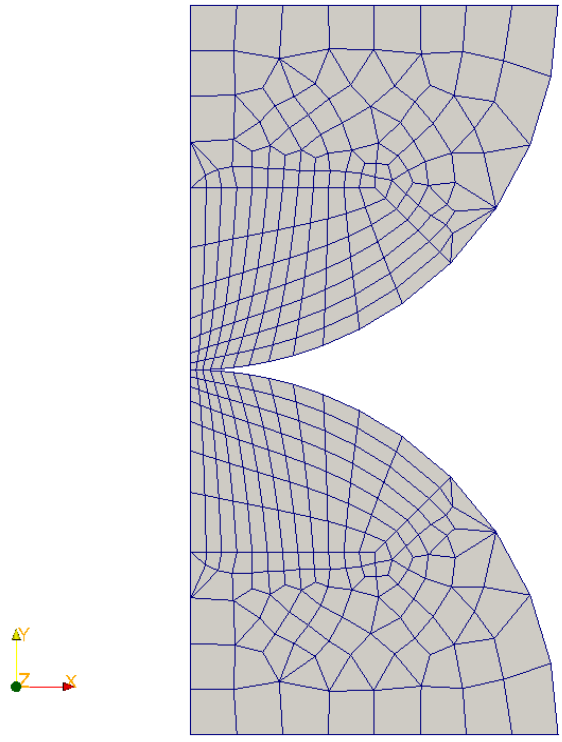
As regards the second computation the important errors are due to the use of a nodal pairing whereas the mesh is not compatible.

6 Modelization E

6.1 Characteristic of the modelization

One uses a modelization AXIS with the formulation CONTINUE of the contact. This modelization validates the functionality SLIDE by slackening imposed crushing.

6.2 Characteristics of the mesh



Nodes: 376 nodes.

Meshes : 30 TRIA3 and 324 QUAD4.

6.3 Quantities tested and Standard

Identification	results of reference	Value of reference	Tolerance
σ_{yy} nets $M31$ node $N291(G)$	"ANALYTIQUE"	-2798,3 N	7.0%
σ_{yy} mesh $M31$ node $N291(G)$	"NON_REGRESSION"	-2971,37 N	1,0%
DX node $N291(G)$	"ANALYTIQUE"	0 mm	1.0E-10
DX node $N287(H)$	"NON_REGRESSION"	-0,110211 mm	3,0%
DY node $N287(H)$	"NON_REGRESSION"	-0,162911 mm	1,0%
DX node $N285(I)$	"NON_REGRESSION"	-0,165946 mm	4,0%
DY node $N285(I)$	"NON_REGRESSION"	-0,629666 mm	0,5%

Standard	Identification of reference	Value of reference	Tolerance
Statute *CONT* urgent *N72* node 0.2	"ANALYTIQUE"	0.0,1	
Statute *CONT* urgent *N80* node 0.2	"ANALYTIQUE"	0.0,1	
Statute *CONT* urgent *N81* node 0.2	"ANALYTIQUE"	0.0,1	
Statute *CONT* urgent *N72* node 0.5	"ANALYTIQUE"	0.0,1	
Statute *CONT* urgent *N80* node 0.5	"ANALYTIQUE"	0.0,1	
Statute *CONT* urgent *N81* node 0.5	"ANALYTIQUE"	2.0,1	
Statute *CONT* urgent *N72* node 1.0	"ANALYTIQUE"	2.0,1	
Statute *CONT* urgent *N80* node 1.0	"ANALYTIQUE"	2.0,1	
Statute *CONT* urgent *N81* node 1.0	"ANALYTIQUE"	2.0,1	
Statute *CONT* urgent *N72* node 1.5	"ANALYTIQUE"	2.0,1	
Statute *CONT* urgent *N80* node 1.5	"ANALYTIQUE"	2.0,1	
Statute *CONT* urgent *N81* node 1.5	"ANALYTIQUE"	2.0,1	
Clearance *JEU* urgent *N72* node 1.5	"ANALYTIQUE"		0.1.0E-8
Clearance *JEU* urgent *N80* node 1.5	"ANALYTIQUE"		0.1.0E-8
Clearance *JEU* urgent *N81* node 1.5	"ANALYTIQUE"		0.1.0E-8
Statute *CONT* urgent *N72* node 2.0	"ANALYTIQUE"	2.0,1	
Statute *CONT* urgent *N80* node 2.0	"ANALYTIQUE"	2.0,1	
Statute *CONT* urgent *N81* node 2.0	"ANALYTIQUE"	2.0,1	
Clearance *JEU* urgent *N72* node 2.0	"ANALYTIQUE"		0.1.0E-8
Clearance *JEU* urgent *N80* node 2.0	"ANALYTIQUE"		0.1.0E-8
Clearance *JEU* urgent *N81* node 2.0	"ANALYTIQUE"		0.1.0E-8

6.4 Remarks

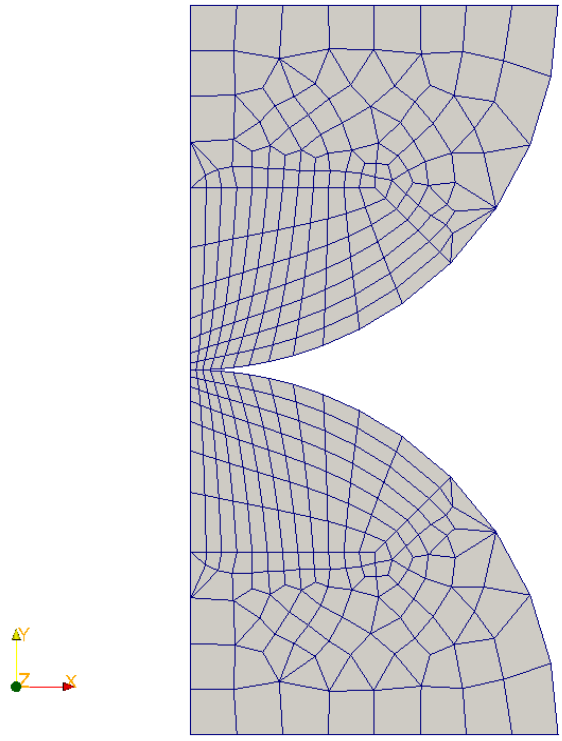
the results got with the formulation CONTINUE are very close to those obtained in formulation DISCRETE and in concord with the analytical solution.
The differences noted on displacements are explained by the diagram of integration used in formulation CONTINUE : it is about a diagram to the nodes which does not make it possible correctly to treat the nullity of the Jacobian on the axis of revolution.

7 Modelization F

7.1 Characteristic of the modelization

One uses a modelization AXIS. Four computations are carried out with options of pairing or contact algorithms different.

7.2 Characteristics of the mesh



Nodes: 376 nodes.

Meshes : 30 TRIA3 and 324 QUAD4.

7.3 Quantities tested and results

the First computation (nodal pairing, master-slave norm and " LAGRANGIAN" algorithm)

Standard	Identification of reference	Value of reference	Tolerance
σ_{yy} nets $M31$ node $N291(G)$	"ANALYTIQUE"	-2798,3 N	7.0%
σ_{yy} mesh $M31$ node $N291(G)$	"NON_REGRESSION"	-2971,37 N	0.1%
DX node $N291(G)$	"ANALYTIQUE"	0 mm	1.0E-10
DX node $N287(H)$	"NON_REGRESSION"	-0,110211 mm	0.1%
DY node $N287(H)$	"NON_REGRESSION"	-0,162911 mm	0.1%
DX node $N285(I)$	"NON_REGRESSION"	-0,165946 mm	0.1%
DY node $N285(I)$	"NON_REGRESSION"	-0,629666 mm	0,1%

the Second computation (master-slave norm, " LAGRANGIAN" algorithm)

Standard	Identification of reference	Value of reference	Tolerance
σ_{yy} nets $M31$ node $N291(G)$	"ANALYTIQUE"	-2798,3 N	7.0%
σ_{yy} mesh $M31$ node $N291(G)$	"NON_REGRESSION"	-2966,0 N	0.1%
DX node $N291(G)$	"ANALYTIQUE"	0 mm	1.0E-10
DX node $N287(H)$	"NON_REGRESSION"	-0,110678 mm	0.1%
DY node $N287(H)$	"NON_REGRESSION"	-0,162891 mm	0.1%
DX node $N285(I)$	"NON_REGRESSION"	-0,167194 mm	0.1%
DY node $N285(I)$	"NON_REGRESSION"	-0,628947 mm	0,1%

the Third computation (master-slave norm, algorithm "FORCED")

Standard	Identification of reference	Value of reference	Tolerance
σ_{yy} net $M31$ node $N291(G)$	"ANALYTIQUE"	-2798,3 N	7.0%
σ_{yy} mesh $M31$ node $N291(G)$	"NON_REGRESSION"	-2966,0 N	0.1%
DX node $N291(G)$	"ANALYTIQUE"	0 mm	1.0E-10
DX node $N287(H)$	"NON_REGRESSION"	-0,110678 mm	0.1%
DY node $N287(H)$	"NON_REGRESSION"	-0,162901 mm	0.1%
DX node $N285(I)$	"NON_REGRESSION"	-0,167194 mm	0.1%
DY node $N285(I)$	"NON_REGRESSION"	-0,628947 mm	0,1%

the Fourth computation (lissage, master-slave norm and algorithm "FORCED")

Standard	Identification of reference	Value of reference	Tolerance
σ_{yy} nets $M31$ node $N291(G)$	"ANALYTIQUE"	-2798,3 N	7.0%
σ_{yy} mesh $M31$ node $N291(G)$	"NON_REGRESSION"	-2971,37 N	0.1%
DX node $N291(G)$	"ANALYTIQUE"	0 mm	1.0E-10
DX node $N287(H)$	"NON_REGRESSION"	-0,110211 mm	0.1%
DY node $N287(H)$	"NON_REGRESSION"	-0,162911 mm	0.1%
DX node $N285(I)$	"NON_REGRESSION"	-0,165946 mm	0.1%
DY node $N285(I)$	"NON_REGRESSION"	-0,629666 mm	0,1%

7.4 Remarks

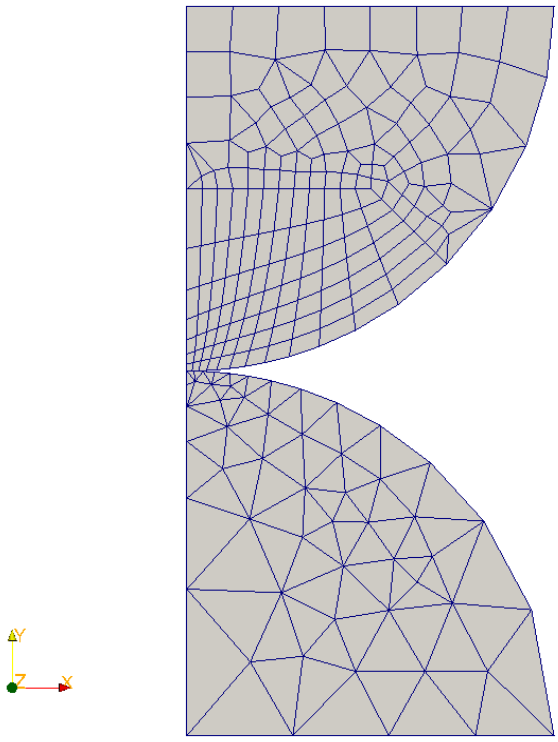
On this modelization, one note that the lissage makes it possible to find the symmetry of the problem (nodes perfectly in opposite once the established contact), last computation indeed which gets the same results as in nodal pairing.
The values obtained are in concord general with the analytical solutions.

8 Modelization G

8.1 Characteristic of the modelization

One uses a modelization AXIS. Two computations are carried out with different options of pairing. In this modelization, imposed crushing goes until $10\,mm$.

8.2 Characteristics of the mesh



Nodes: 243.

Meshes : 100 TRIA3, 162 QUAD4.

8.3 Quantities tested and results

the First computation (lissage, master-slave norm and " LAGRANGIAN" algorithm)

Standard	Identification of reference	Value of reference	Tolerance
σ_{yy} nets $M31$ urgent $N291(G)$ node 1.0	"ANALYTIQUE"	-1978,7 N	8,0%
σ_{yy} mesh $M286$ urgent $N55(G')$ node 1.0	"ANALYTIQUE"	-1978,7 N	1,0%
σ_{yy} nets $M293$ urgent $N55(G')$ node 1.0	"ANALYTIQUE"	-1978,7 N	15,0%
σ_{yy} mesh $M31$ urgent $N291(G)$ node 2.0	"ANALYTIQUE"	-2798,3 N	6,0%
σ_{yy} mesh $M286$ urgent $N55(G')$ node 2.0	"ANALYTIQUE"	-2798,3 N	4,0%
σ_{yy} mesh $M293$ urgent $N55(G')$ node 2.0	"ANALYTIQUE"	-2798,3 N	15,0%

σ_{yy} mesh $M31$ urgent $N291(G)$ node 3.0	"ANALYTIQUE"	-3427,2 N	1,0%
σ_{yy} nets $M286$ urgent $N55(G')$ node 3.0	"ANALYTIQUE"	-3427,2 N	3,0%
σ_{yy} mesh $M293$ urgent $N55(G')$ node 3.0	"ANALYTIQUE"	-3427,2 N	14,0%
σ_{yy} mesh $M31$ urgent $N291(G)$ node 4.0	"ANALYTIQUE"	-3957,4 N	4,0%
σ_{yy} mesh $M286$ urgent $N55(G')$ node 4.0	"ANALYTIQUE"	-3957,4 N	3,0%
σ_{yy} mesh $M293$ urgent $N55(G')$ node 4.0	"ANALYTIQUE"	-3957,4 N	14,5%
σ_{yy} mesh $M31$ urgent $N291(G)$ node 5.0	"ANALYTIQUE"	-4424,5 N	8,0%
σ_{yy} mesh $M286$ urgent $N55(G')$ node 5.0	"ANALYTIQUE"	-4424,5 N	3,0%
σ_{yy} mesh $M293$ urgent $N55(G')$ node 5.0	"ANALYTIQUE"	-4424,5 N	15,0%

the Second computation (lissage, master-slave norm and algorithm "FORCED")

Standard	Identification of reference	Value of reference	Tolerance
σ_{yy} net $M31$ urgent $N291(G)$ node 1.0	"ANALYTIQUE"	-1978,7 N	8,0%
σ_{yy} mesh $M286$ urgent $N55(G')$ node 1.0	"ANALYTIQUE"	-1978,7 N	1,0%
σ_{yy} nets $M293$ urgent $N55(G')$ node 1.0	"ANALYTIQUE"	-1978,7 N	15,0%
σ_{yy} mesh $M31$ urgent $N291(G)$ node 2.0	"ANALYTIQUE"	-2798,3 N	6,0%
σ_{yy} mesh $M286$ urgent $N55(G')$ node 2.0	"ANALYTIQUE"	-2798,3 N	4,0%
σ_{yy} mesh $M293$ urgent $N55(G')$ node 2.0	"ANALYTIQUE"	-2798,3 N	15,0%
σ_{yy} mesh $M31$ urgent $N291(G)$ node 3.0	"ANALYTIQUE"	-3427,2 N	1,0%
σ_{yy} nets $M286$ urgent $N55(G')$ node 3.0	"ANALYTIQUE"	-3427,2 N	3,0%
σ_{yy} mesh $M293$ urgent $N55(G')$ node 3.0	"ANALYTIQUE"	-3427,2 N	14,0%
σ_{yy} mesh $M31$ node $N291(G)$ time 4.0	"ANALYTIQUE"	-3957,4 N	4,0%
σ_{yy} mesh $M286$ urgent $N55(G')$ node 4.0	"ANALYTIQUE"	-3957,4 N	3,0%
σ_{yy} mesh $M293$ urgent $N55(G')$ node 4.0	"ANALYTIQUE"	-3957,4 N	14,0%
σ_{yy} mesh $M31$ urgent $N291(G)$ node 5.0	"ANALYTIQUE"	-4424,5 N	8,0%
σ_{yy} mesh $M286$ urgent $N55(G')$ node 5.0	"ANALYTIQUE"	-4424,5 N	3,0%
σ_{yy} mesh $M293$ urgent $N55(G')$ node 5.0	"ANALYTIQUE"	-4424,5 N	15,0%

8.4 Remarks

The mesh consist of 2 half-spheres with a grid differently (the higher sphere is in QUAD4, the lower sphere in TRIA3). The contact pressure raised on the upper surface is in concord with the analytical solution (the variation of about 5% is explained by extrapolation).
On lower surface, one notes contrary to the considerable variations because there exist 2 meshes which contribute to the point G' (mesh of TRIA3), however if one meshes makes the average of the results got on these 2, one finds a pressure in concord with the analytical solution.

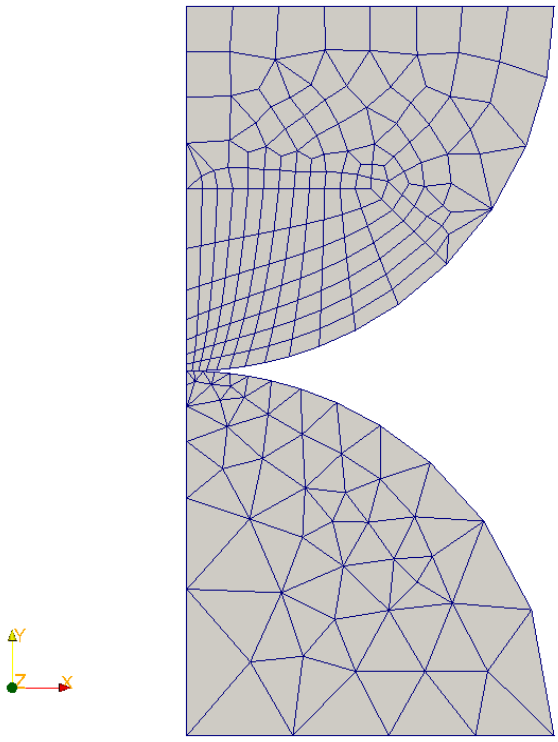
The two contact algorithms used give the same results appreciably.

9 Modelization H

9.1 Characteristic of the modelization

One uses a modelization AXIS. Only one computation with penalization of the contact is carried out. In this modelization, imposed crushing goes until $10\,mm$.

9.2 Characteristics of the mesh



Nodes: 243.

Meshes : 100 TRIA3, 162 QUAD4.

9.3 Quantities tested and results

the First computation (lissage, algorithm "PENALIZATION")

Standard	Identification of reference	Value of reference	Tolerance
σ_{yy} nets $M31$ urgent $N291(G)$ node 1.0	"ANALYTIQUE"	-1978,7 N	7,0%
σ_{yy} mesh $M286$ urgent $N55(G')$ node 1.0	"ANALYTIQUE"	-1978,7 N	1,0%
σ_{yy} nets $M293$ urgent $N55(G')$ node 1.0	"ANALYTIQUE"	-1978,7 N	15,0%
σ_{yy} mesh $M31$ urgent $N291(G)$ node 2.0	"ANALYTIQUE"	-2798,3 N	5,0%
σ_{yy} mesh $M286$ urgent $N55(G')$ node 2.0	"ANALYTIQUE"	-2798,3 N	3,0%
σ_{yy} mesh $M293$ urgent $N55(G')$ node 2.0	"ANALYTIQUE"	-2798,3 N	15,0%

σ_{yy} mesh $M31$ urgent $N291(G)$ node 3.0	"ANALYTIQUE"	-3427,2 N	2,0%
σ_{yy} mesh $M286$ urgent $N55(G')$ node 3.0	"ANALYTIQUE"	-3427,2 N	3,0%
σ_{yy} mesh $M293$ urgent $N55(G')$ node 3.0	"ANALYTIQUE"	-3427,2 N	14,0%
σ_{yy} mesh $M31$ urgent $N291(G)$ node 4.0	"ANALYTIQUE"	-3957,4 N	3,0%
σ_{yy} mesh $M286$ urgent $N55(G')$ node 4.0	"ANALYTIQUE"	-3957,4 N	4,0%
σ_{yy} mesh $M293$ urgent $N55(G')$ node 4.0	"ANALYTIQUE"	-3957,4 N	15,0%
σ_{yy} mesh $M31$ node $N291(G)$ time 5.0	"ANALYTIQUE"	-4424,5 N	6,0%
σ_{yy} mesh $M286$ urgent $N55(G')$ node 5.0	"ANALYTIQUE"	-4424,5 N	4,0%
σ_{yy} mesh $M293$ urgent $N55(G')$ node 5.0	"ANALYTIQUE"	-4424,5 N	15,0%

9.4 Remarks

The mesh consist of 2 half-spheres with a grid differently (the higher sphere is in QUAD4, the lower sphere in TRIA3). The contact pressure raised on the upper surface is in concord with the analytical solution (the variation of about 5% is explained by extrapolation).
On lower surface, one notes contrary to the considerable variations because there exist 2 meshes which contribute to the point G' (mesh of TRIA3), however if one meshes makes the average of the results got on these 2, one finds a pressure in concord with the analytical solution.

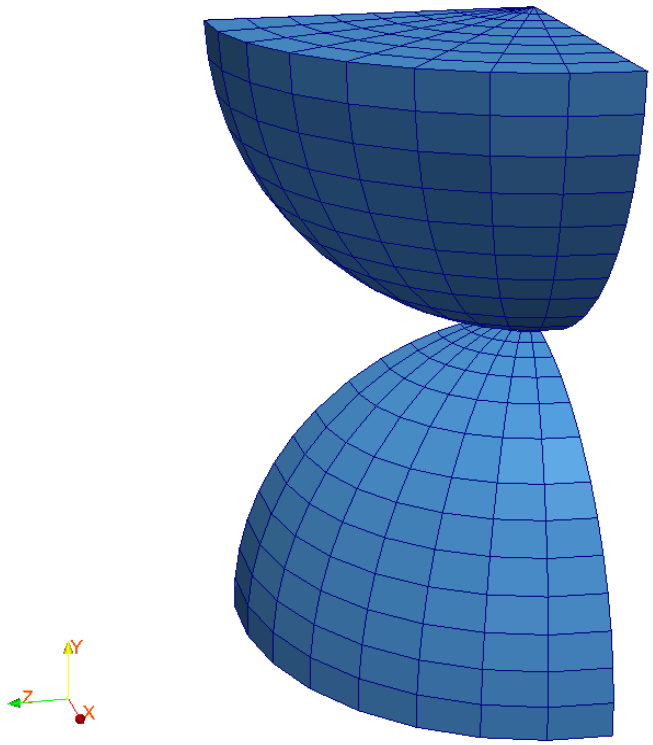
The coefficient of penalization used here (large in front $R_{sphere} * E$) makes it possible to get results comparable to those of the exact algorithms of dualisation (modelization G).

10 Modelization I

10.1 Characteristic of the modelization

One uses a modelization 3D. Only one computation with the parameters by default of the contact is carried out.
In this modelization, points H and H_p (resp. I and I_p) are the same points (compared to those described in the geometry) but located on the two sides supporting the conditions of symmetry.

10.2 Characteristics of the mesh



Nodes: 3698.
Meshes : 192 TETRA4, 5872 PENTA6, 224 PYRAM5.

10.3 Quantities tested and results

the First computation (algorithm "FORCED")

Standard	Identification of reference	Value of reference	Tolerance
σ_{yy} does not net $M5884$ G	"ANALYTIQUE"	-2798,3 N	14,0%
σ_{yy} mesh $M5884$ not G	"AUTRE_ASTER"	-3176,3 N	2,0%
σ_{yy} mesh $M5896$ not G	"AUTRE_ASTER"	-3176,3 N	2,0%
σ_{yy} mesh $M5908$ not G	"AUTRE_ASTER"	-3176,3 N	2,0%

σ_{yy} mesh $M5920$ not G	"AUTRE_ASTER"	-3176,3 N	2,0%
σ_{yy} mesh $M5932$ not G	"AUTRE_ASTER"	-3176,3 N	2,0%
σ_{yy} mesh $M5944$ not G	"AUTRE_ASTER"	-3176,3 N	2,0%
σ_{yy} mesh $M5956$ not G	"AUTRE_ASTER"	-3176,3 N	2,0%
σ_{yy} mesh $M5968$ not G	"AUTRE_ASTER"	-3176,3 N	2,0%
DX point G	"ANALYTIQUE"	0 mm	1.0E-10
DX not H	"AUTRE_ASTER"	-0,11343 mm	4,0%
DZ point H_p	"AUTRE_ASTER"	-0,11343 mm	4,0%
DY point H	"AUTRE_ASTER"	-0,16291 mm	2,0%
DY point H_p	"AUTRE_ASTER"	-0,16291 mm	2,0%
DX point I	"AUTRE_ASTER"	-0,17845 mm	3,0%
DZ point I_p	"AUTRE_ASTER"	-0,17845 mm	3,0%
DY point I	"AUTRE_ASTER"	-0,62966 mm	0,1%
DY point I_p	"AUTRE_ASTER"	-0,62966 mm	0,1%

10.4 Remarks

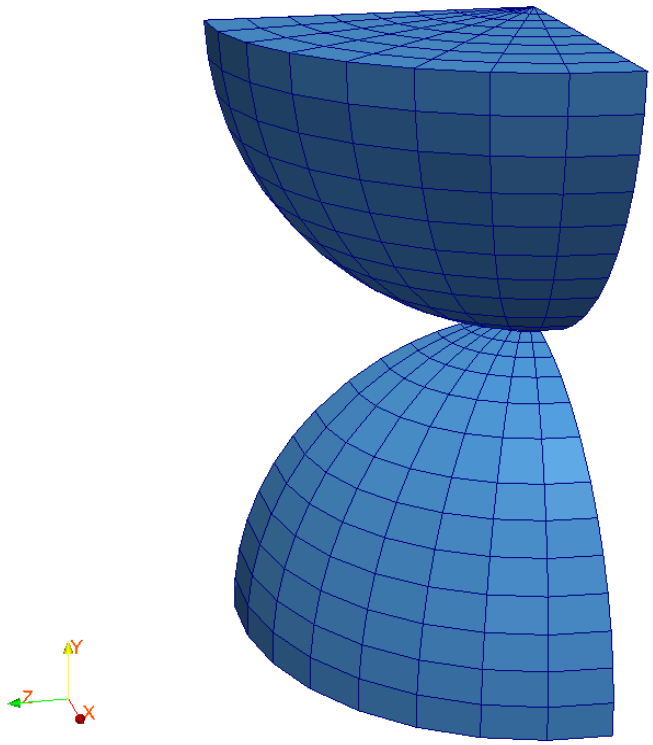
the results got in this modelization differ up to 4 % from the reference (modelization A): it is because geometrical non-linearity of the contact is voluntarily solved coarsely (criterion built-in to 5 %). One notes that the radiant mesh makes it possible to preserve the axisymetry of the problem, this is why the results got in each mesh which contributes to the point G are similar.

11 Modelization J

11.1 Characteristic of the modelization

One uses a modelization 3D. Only one computation with an alternative of the contact algorithm by default is carried out.
In this modelization, points H and H_p (resp. I and I_p) are the same points (compared to those described in the geometry) but located on the two sides supporting the conditions of symmetry.

11.2 Characteristics of the mesh



Nodes: 3698.
Meshes : 192 TETRA4, 5872 PENTA6, 224 PYRAM5.

11.3 Quantities tested and results

the First computation (algorithm "GCP")

Standard	Identification of reference	Value of reference	Tolerance
σ_{yy} does not net $M5884$ G	"ANALYTIQUE"	-2798,3 N	14,0%
σ_{yy} mesh $M5884$ not G	"AUTRE_ASTER"	-3176,3 N	0,1%
σ_{yy} mesh $M5896$ not G	"AUTRE_ASTER"	-3176,3 N	0,1%
σ_{yy} mesh $M5908$ not G	"AUTRE_ASTER"	-3176,3 N	0,1%

σ_{yy} mesh $M5920$ not G	"AUTRE_ASTER"	-3176,3 N	0,1%
σ_{yy} mesh $M5932$ not G	"AUTRE_ASTER"	-3176,3 N	0,1%
σ_{yy} mesh $M5944$ not G	"AUTRE_ASTER"	-3176,3 N	0,1%
σ_{yy} mesh $M5956$ not G	"AUTRE_ASTER"	-3176,3 N	0,1%
σ_{yy} mesh $M5968$ not G	"AUTRE_ASTER"	-3176,3 N	0,1%
DX point G	"ANALYTIQUE"	0 mm	1.0E-10
DX not H	"AUTRE_ASTER"	-0,11343 mm	0,1%
DZ point H_p	"AUTRE_ASTER"	-0,11343 mm	0,1%
DY point H	"AUTRE_ASTER"	-0,16291 mm	0,1%
DY point H_p	"AUTRE_ASTER"	-0,16291 mm	0,1%
DX point I	"AUTRE_ASTER"	-0,17845 mm	0,1%
DZ point I_p	"AUTRE_ASTER"	-0,17845 mm	0,1%
DY point I	"AUTRE_ASTER"	-0,62966 mm	0,1%
DY point I_p	"AUTRE_ASTER"	-0,62966 mm	0,1%

11.4 Remarks

the results got in this three-dimensional modelization are in very good agreement with the analytical solution as well as the reference (modelization A). One notes moreover than the radiant mesh makes it possible to preserve the axisymetry of the problem, this is why the results got in each mesh which contributes to the point G are similar.
The results differ from those obtained by the modelization I not because of the algorithm of resolution of the contact but because the geometrical convergence criterion is not the same one (5 % in modelization I, 1 % in modelization J).

12 Summary of the results

the comparison in each modelization to the analytical reference (restricted at the point G) is satisfactory.

The non-regression of the results is ensured by testing displacements in two nodes of edge, H and I. These nodes occupying of the slightly different positions according to the modelizations, one should not seek to compare them between modelizations.

This test in curved geometry highlights, the utility of certain parameters of pairing such as:

-lissage: it is about a process of modifications of the norms which are used to write the conditions of contact. Although the meshes of sphere are symmetric one notes always in practice a light shift on the last point of contact surface. The lissage of the norms makes it possible here to find symmetry. This parameter proves also useful to improve geometrical convergence of the modelizations using of the meshes coarse (it is the case of the modelizations $2D$ of this test).
-Nodal pairing or the average of norms (master-slave): little used in general, these two options of pairing, like the lissage, makes it possible to find the symmetry of the problem.

The modelization A where the contact is replaced by a unilateral connection the very important part played by the geometrical reactualization on the accuracy of the stresses puts also ahead. Indeed this modelization makes it possible to remove the geometrical non-linearity of this problem, it thus provides **a reference solution**. Thus when one compares the results got on the same problem $3D$ by the modelizations A, I and J, one notes that:
-the values of displacements obtained by the modelization I present variations going up to 4 % compared to the two other modelizations
-the modelizations A and J find a pressure of Hertz identical, while the modelization I finds I a pressure 10%
-lower in the modelization the geometrical criterion is satisfied to 5% close whereas it is it to 1% in the modelization J

It thus appears important to satisfy the geometrical criterion as well as possible to obtain the best possible accuracy on the values of stresses.