SSNV103 - Traction test shears models of Summarized

Rousselier:

It is about a nonlinear quasi-static problem in structural mechanics.

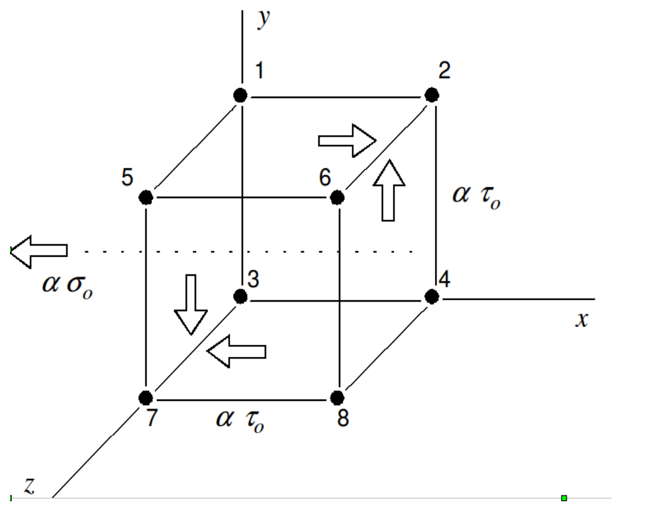
One analyzes the response of a volume element with a loading in tension-shears, carried out in such way that imposes a state of uniform stress-strain.

The case test understands 1 modelization: in $3D$.

It G validates the numerical integration of the elastoplastic model of behavior with damage of. Rousselier.

1 Problem of reference

1.1 Geometry



Face YZ : $(1,3,5,7)$
Face XZ : $(3,4,7,8)$
Face $1YZ$: $(2,4,6,8)$
Face $1XZ$: $(1,2,5,6)$

$\alpha \tau_0$ shears imposed $\begin{cases} \text{Face } 1XZ \\ \text{Face } 1YZ \end{cases}$

$\alpha \sigma_0$ imposed pressure Face YZ

$\alpha(t)$ function of force

1.2 Material properties

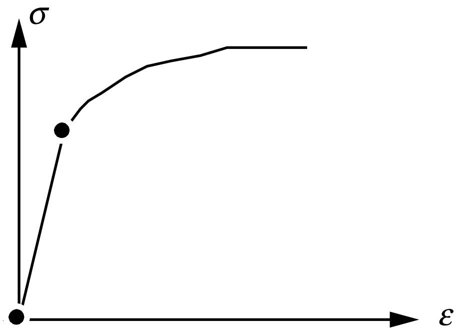
isotropic elasticity: $E=206\,400.\text{MPa}$ $\nu=0.3$

plasticity: (coefficients of the model of Rousselier)
$D=2.$
$f_0=5.10^{-4}$
$\sigma_1=490.\text{MPa}$

rational curve of tension entered point by point with:

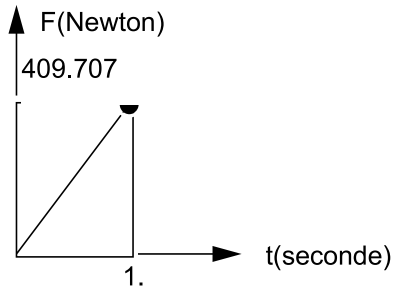
$$R(p)=r_i+(r_0-r_i)e^{-bp}$$

with p : cumulated plastic strain
and $r_i=1500\ MPa$
$r_0=520.\text{MPa}$
$b=2.4$



1.3 Boundary conditions and loadings

$N04$	$dx=dy=0$	Face YZ :	$FX=FY=-F(t)$
$N08$	$dx=dy=dz=0$	Face XZ :	$FX=-F(t)$
$N02, N06$	$dx=0$	Face $1YZ$:	$FY=F(t)$
		Face $1XZ$:	$FX=F(t)$



1.4 Forced

Initial conditions and null strains with $t=0$.

2 Reference solution

2.1 Method of calculating used for the reference solution

The model $3D$ of velocity is written:

$$\begin{cases} \dot{\sigma}-\dot{\rho}\,\Lambda\,\varepsilon_e-\rho\,\Lambda\,\dot{\varepsilon}_e &= 0 \quad (\Lambda \text{tenseur élasticité isotrope linéaire}) \\ \dot{\beta}-\dot{p}\,D\exp(\dfrac{\sigma_H}{\sigma_1\,\rho}) &= 0 \\ \dot{\varepsilon}-\dot{\varepsilon}_e-\rho\,\dot{p}\dfrac{\partial f}{\partial\sigma} &= 0 \\ \dot{f} &= 0 \end{cases}$$

what, in the case of a loading of imposed tension-shears $\left(\sigma(t)=\alpha(t)\begin{bmatrix}\sigma_0 & \tau_0 & 0\\ \tau_0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}\right)$ results in integrating a system of 6 ordinary differential equations in $y=(\varepsilon,\gamma,\varepsilon_e,\gamma_e,\beta,p)$ form $A(y,t)\dot{y}=G(y,t)$.

$$(S)\quad \begin{cases} \dot{\alpha}\,\sigma_o+\rho^2\,F_o\,e^{\beta}\,E\,\varepsilon_e\,\dot{\beta}-\rho\,E\,\dot{\varepsilon}_e \;=\; 0 \\ \dot{\alpha}\,\tau_o+\rho^2\,F_o\,e^{\beta}\,2\mu\,\gamma_e\,\dot{\beta}-2\rho\,\mu\,\dot{\gamma}_e \;=\; 0 \\ \dot{\varepsilon}-\dfrac{\sigma_o}{\sigma_{eq\ o}}\rho\,\dot{p}-\dot{\varepsilon}_e \;=\; 0 \\ \dot{\gamma}-\dfrac{3\,\tau_o}{2\,\sigma_{eq\ o}}\rho\,\dot{p}-\dot{\gamma}_e \;=\; 0 \\ \dot{\beta}-\dot{p}\,D\,\exp\left(\dfrac{\sigma_H}{\sigma_1\,\rho}\right) \;=\; 0 \\ \dot{\alpha}\,\sigma_o\left[\dfrac{\sigma_o}{\rho\,\sigma_{eq\ o}}+\dfrac{1}{3}D\,F_o\,e^{\beta}\exp\left(\dfrac{\sigma_H}{\sigma_1\,\rho}\right)\right]+3\,\dot{\alpha}\,\dfrac{\tau_o^2}{\rho\,\sigma_{eq\ o}}-\dfrac{\partial R}{\partial p}\,\dot{p} \\ \quad +\left[\sigma_{eq\ o}\,F_o\,e^{\beta}+D\,\sigma_1\,\rho\,F_o\,e^{\beta}\exp\left(\dfrac{\sigma_H}{\sigma_1\,\rho}\right)\left(1-\rho\,F_o\,e^{\beta}\left(1-\dfrac{\sigma_H}{\sigma_1\,\rho}\right)\right)\right]\dot{\beta} \;=\; 0 \end{cases}$$

with $t=0$:

$$f=0 \quad \rho(0)=1\,,\ \beta(0)=0$$

from where:

$$\alpha(0)\,\sigma_{eq0}-R(0)+D\sigma_1 F_0\exp(\frac{\alpha(0)\sigma_0}{3\,\sigma_1})$$

who is solved by a method of NEWTON for $\alpha(0)$:

$$\begin{cases} \varepsilon(0) & = & \frac{1}{E}\alpha(0)\sigma_0 & = & \varepsilon_e(0) \\ \gamma(0) & = & \frac{1}{2\mu}\alpha(0)\tau_0 & = & \gamma_e(0) \\ p(0) & = & 0 & & \end{cases}$$

2.2 Results of reference

One imposes $\alpha(t)=\alpha(0)+t$ with $\sigma_0=\tau_0=150\,MPa$.

One obtains $\alpha(0)=1.73138$ and $\alpha(1)=2.73138$.

The system (S) is then solved numerically by a “Backward difference formulated” using scientific library NAG on CRAY. Result of reference = $(\varepsilon,\gamma,\beta,\rho)$ to the nodes to $t=1$.

2.3 Uncertainty on the solution

Uncertainty related to library NAG.

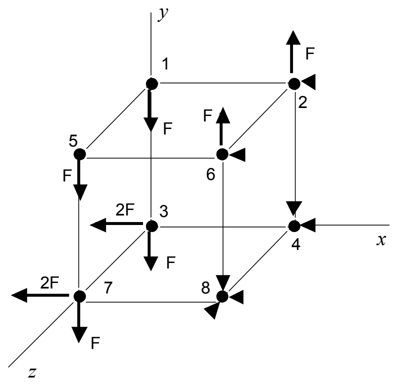
2.4 Bibliographical references

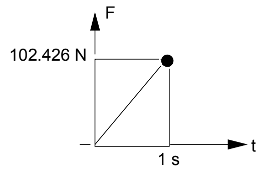
1) User's manual library NAG on CRAY.

3 Modelization A

3.1 Characteristic of the modelization

Modelization 3D





3.2 Characteristic of the mesh

The mesh contains 1 element of type `HEXA8`.

3.3 Quantities tested and results of the modelization A

Identification	Reference	Test	Tolerance
ε on $NO1$ to $t=1\mathrm{s}$	0,07830	`ANALYTIQUE`	0,11%
γ out of $NO1$ to $t=1\mathrm{s}$	0,11700	`ANALYTIQUE`	0,20%
p out of $NO1$ to $t=1\mathrm{s}$	0,15260	`ANALYTIQUE`	0,10%
σ_{11} out of $NO1$ to $t=1\mathrm{s}$	409.70700	`ANALYTIQUE`	0,05%

One also tests the parameters of the data structure results:

Identification	Reference	Test	Tolerance
`INST` for `NUME_ORDRE=8`	1	`ANALYTIQUE`	0%
`ITER_GLOB` for `NUME_ORDRE=8`	4	`NON_REGRESSION`	0%

3.4 Remarks

One could expect a better correlation, but it should be stressed that library NAG uses the function $R(p)$ in algebraic form, whereas *Code_Aster* uses it in the form of a point by point given curve.

Moreover, it seems that the integration of the rate of the function threshold poses problems with NAG, whatever the accuracy required in addition (the value of the threshold f being appreciably different from 0 at the end of the integration). However, one can note the constancy of this correlation throughout integration $(t\in[0,1])$.

4 Summary of the results

the values of *Code_Aster* are in concord with the values of reference.