SSNV102 - Traction test shears with the model of TAHERI

Summarized:

The problem is quasi-static nonlinear in structural mechanics.

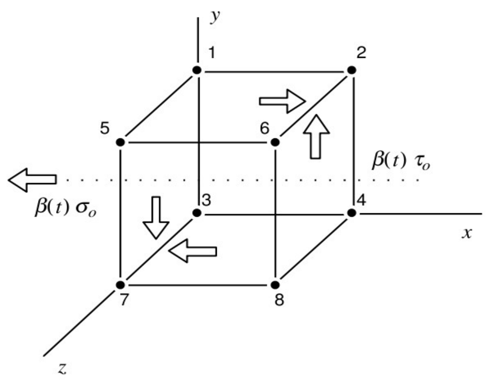
One analyzes the response of a volume element with a loading in tension-shears, carried out in such way that imposes a state of uniform stress-strain in the element.

There are 2 modelizations: one into $3D$ voluminal and another in plane stresses $2D$.

One validates by this test the numerical integration of the elastoplastic model of behavior of Saïd Taheri.

1 Problem of reference

1.1 Geometry



Face YZ : $(1,3,5,7)$
Face XZ : $(3,4,7,8)$
Face $1YZ$: $(2,4,6,8)$
Face $1XZ$: $(1,2,5,6)$

$\beta(t)\tau_0$ shears imposed $\begin{cases}\text{Face 1XZ}\\ \text{Face 1YZ}\end{cases}$

$\beta(t)\sigma_0$ imposed pressure

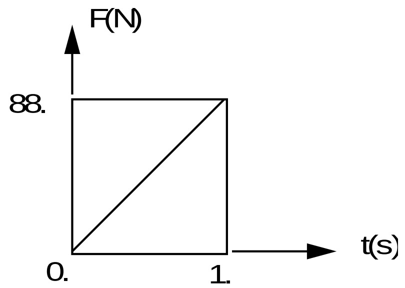
$\beta(t)$ function of force

1.2 Material properties

isotropic elasticity	$E=200\,000\,MPa$	$\nu=0,3$		
plasticity Taheri	$C_{inf}=0.065\,MPa$	$C_1=-0.012\,Mpa$	$s=450$	$b=30$
	$m=0.1$	$a=312$	$\alpha=0.3$	$R_o=72$

1.3 Boundary conditions and loadings

$N04$	$dx=dy=0$	Face YZ :	$FX=FY=-F(t)$
$N08$	$dx=dy=dz=0$	Face XZ :	$FX=-F(t)$
$N02$, $N06$	$dx=0$	Face $1YZ$:	$FY=F(t)$
		Face $1XZ$:	$FX=F(t)$



1.4 Forced

initial conditions and null strains with $t=0$.

2 Reference solution

2.1 Method of calculating used for the reference solution

One integrates the following system numerically enters $t=0$ and $t=1$.

(Nonlinear ordinary Differential connection of 6 equations to 6 unknowns solved using library NAG by a “Backward difference method”)

$$\dot{\varepsilon}-\dot{\varepsilon}\,p = \dot{\beta}\frac{\sigma_o}{E} \quad \text{éq 2.1-1}$$

$$\dot{\gamma}-\dot{\gamma}\,p = \dot{\beta}\frac{\tau_o}{2\mu} \quad \text{éq 2.1-2}$$

$$\dot{\varepsilon}\,p-\dot{p}\frac{\partial F}{\partial\sigma} = 0 \quad \text{éq 2.1-3}$$

$$\dot{\gamma}\,p-\dot{p}\frac{\partial F}{\partial\tau} = 0 \quad \text{éq 2.1-4}$$

$$\left[-\frac{3}{2}\frac{\partial F}{\partial\sigma}\left(Kx+Cs\frac{\partial F}{\partial\sigma}\right)-2\frac{\partial F}{\partial\tau}\left(Ky+Cs\frac{\partial F}{\partial\tau}\right)-HR-a\,D\,\alpha\,Z^{(\alpha-2)}\left(\left(\varepsilon_p-\varepsilon_p^n\right)\frac{\partial F}{\partial\sigma}+\frac{4}{3}\left(\gamma_p-\gamma_p^n\right)\frac{\partial F}{\partial\tau}\right)\right]\dot{p}+\left[\frac{3}{2}\frac{\partial F}{\partial\sigma}\left(Qx+C\varepsilon_p^n\right)+2\frac{\partial F}{\partial\tau}\left(Qy+C\gamma_p^n\right)+JR\right]\dot{\sigma}_p = -\frac{\partial F}{\partial\sigma}\dot{\beta}\,\sigma_o-2\frac{\partial F}{\partial\tau}\dot{\beta}\,\tau_o \quad \text{éq 2.1-5}$$

$$\left[1+JR+\frac{3C}{2U}\left(\frac{3}{2}C\left(\varepsilon_p\,s-\sigma_p\,\varepsilon_p^n\right)\varepsilon_p^n+2C\left(s\,\gamma_p-\sigma_p\,\gamma_p^n\right)\gamma_p^n\right)\right]\dot{\sigma}_p - \left[HR+a\,D\,\alpha\,Z^{(\alpha-2)}\left(\left(\varepsilon_p-\varepsilon_p^n\right)\frac{\partial F}{\partial\sigma}+\frac{4}{3}\left(\gamma_p-\gamma_p^n\right)\frac{\partial F}{\partial\tau}\right)+KX + 3\frac{CS}{2U}\left(\frac{3}{2}C\left(s\,\varepsilon_p-\sigma_p\,\varepsilon_p^n\right)\frac{\partial F}{\partial\sigma}+2C\left(s\,\gamma_p-\sigma_p\,\gamma_p^n\right)\frac{\partial F}{\partial\tau}\right)\right]\dot{p} = 0 \quad \text{éq 2.1-6}$$

avec

$$D=1-me^{-up}\begin{cases} u=b\left(1-\dfrac{\sigma_p}{S}\right) \\ v=\dfrac{C\infty-C}{C} \\ w=\dfrac{1-D}{D} \\ C=C\infty-C_1\,e^{-up} \\ K=v\,u \qquad Q=v\dfrac{bp}{s} \\ H=w\,u \qquad J=w\dfrac{bp}{s}\end{cases}$$

et

$$\begin{cases} X = C\left(s\,\varepsilon_p-\sigma_p\,\varepsilon_p^n\right) \\ Y = C\left(s\,\gamma_p-\sigma_p\,\gamma_p^n\right) \\ U = \left[\dfrac{9}{4}X^2+3\,Y^2\right]^{1/2} \\ R = D\left(a\,Z^{\alpha}+r0\right) \\ Z = \left[\left(\varepsilon_p-\varepsilon_p^n\right)^2+\dfrac{4}{3}\left(\gamma_p-\gamma_p^n\right)^2\right]^{1/2}\end{cases}$$

with the initial conditions:

$$\begin{cases} \beta(0) & = & \dfrac{R(0)}{\left(\sigma_o^2+3\,\tau_o^2\right)^{1/2}} \\ \varepsilon(0) & = & \beta(0)\,\dfrac{\sigma_o}{E} \\ \gamma(0) & = & \beta(0)\,\dfrac{\tau_o}{2\mu} \\ p(0) & = & \varepsilon_p(0) = \varepsilon_p^n = 0 \\ R(0) & = & (1-m)\,r_o = \sigma_p(o) \end{cases}$$

$$\text{d'où}\quad \sigma(t=1) = \begin{bmatrix} 88. & 88. & 0 \\ 88. & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

2.2 Results of reference

Values of $\varepsilon, \gamma, \varepsilon_p, \gamma_p, p$ and σ_p to the nodes to $t=1\ s$.

2.3 Uncertainty on the solution

Uncertainty of library NAG.

2.4 Bibliographical references

1) User's manual library NAG on CRAY.

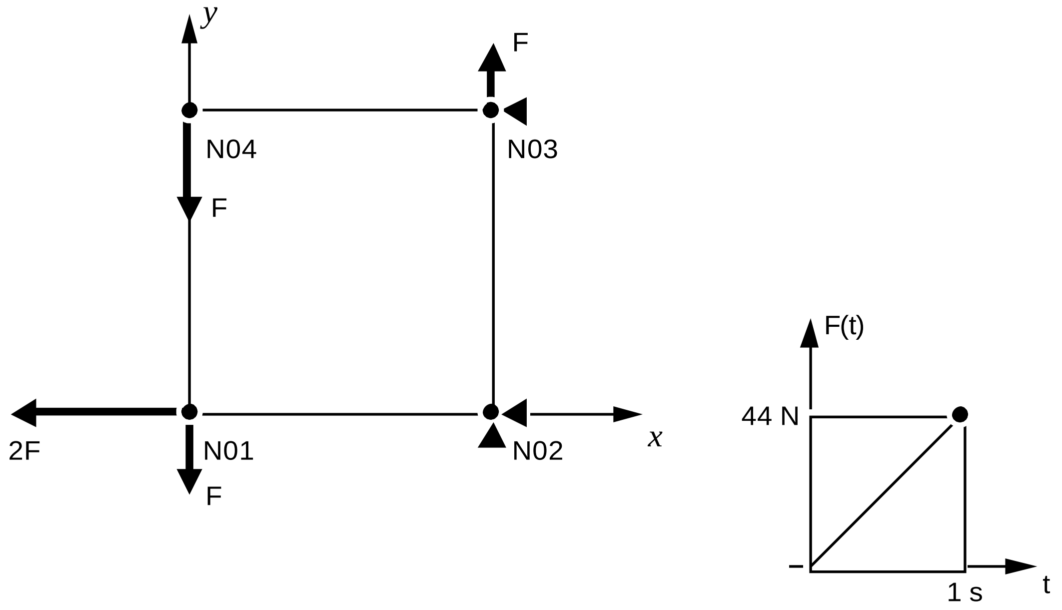
2) S. ANDRIEUX - P. SCHOENBERGER - S. TAHERI: A three dimensional cyclic constitutive law for metals with has variable semi-DISCRET memory - HI - 71/8147 (1992)

3) P. GEYER - J.M. PROIX - P. SCHOENBERGER - S. TAHERI: Modelization of the phenomena of progressive strain - Collection of the internal notes of DER 93NB00153

3 Modelization A

3.1 Characteristic of modelization

Modelization in plane stresses 2D **,** `C_PLAN`



3.2 Characteristic of the mesh

square Quadrangle to 4 nodes in plane stresses with:

$largeur=1\,mm$
$épaisseur=1\,mm$.

3.3 Quantities tested and results

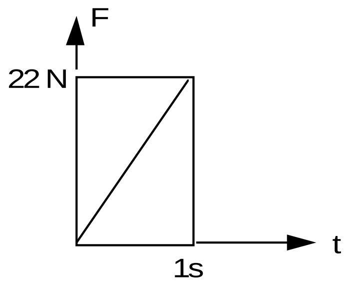
Identification	Reference	Test	Tolerance
ε on $NO4$ to $t=1s$	0,01721	ANALYTIQUE	2,5%
γ out of $NO4$ at $t=1s$	0,02573	ANALYTIQUE	2,5%
ε_p on $MA1$, Gauss point 4 with $t=1s$	0,01678	ANALYTIQUE	2,5%
γ_p on $MA1$, Gauss point 4 with $t=1s$	0,02515	ANALYTIQUE	2,5%
ε_p out of $NO4$ to $t=1s$	0,01678	ANALYTIQUE	2,5%
γ_p out of $NO4$ to $t=1s$	0,02515	ANALYTIQUE	2,5%
σ_p out of $NO1$ to $t=1s$	176,0	ANALYTIQUE	0,10%

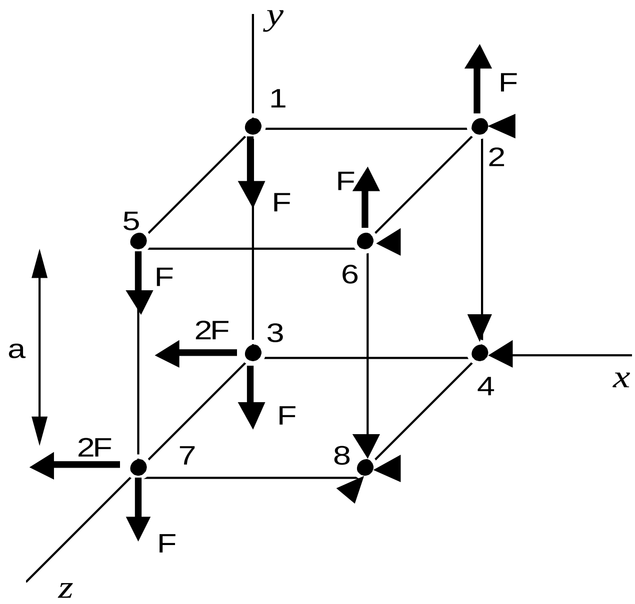
4 Modelization B

4.1 Characteristic of the modelization

Modelization 3D :

Cubic elementary with a grid using a hexahedron with 8 nodes.





4.2 Characteristics of mesh

1 nets HEXA8, width side $a=1\,mm$.

4.3 Quantities tested and Values

4.3.1 results tested

Identification	Reference	Test	Tolerance
ε on $NO4$ to $t=1\mathrm{s}$	0,01721	ANALYTIQUE	2,5%
γ out of $NO4$ to $t=1\mathrm{s}$	0,02573	ANALYTIQUE	2,5%
ε_p on $MA1$, Gauss point 1 with $t=1\mathrm{s}$	0,01678	ANALYTIQUE	2,5%
γ_p on $MA1$, Gauss point 1 with $t=1\mathrm{s}$	0,02515	ANALYTIQUE	2,5%
ε_p out of $NO4$ to $t=1\mathrm{s}$	0,01678	ANALYTIQUE	2,5%
γ_p out of $NO4$ to $t=1\mathrm{s}$	0,02515	ANALYTIQUE	2,5%
σ_p out of $NO1$ to $t=1\mathrm{s}$	176,0	ANALYTIQUE	0,10%
p out of $NO1$ to $t=1\mathrm{s}$	0.03	ANALYTIQUE	0,10%

One also tests the parameters of the data structure results:

Identification	Reference	Test	Tolerance
INST for NUME_ORDRE= 6	1	ANALYTIQUE	0%
ITER_GLOB for NUME_ORDRE= 6	12	NON_REGRESSION	8

4.3.2 Remarks

the reduction in the tolerance on total convergence in displacement does not bring significant gain in accuracy.

The number of increments of load (6) led to a satisfactory accuracy of result.

5 Summary of the results

Good accuracy at the time of the comparison with NAG in spite of some difficulties of convergence with this mathematical library.