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## SSNP505 - Plate multi-fissured in bitraction-shears with X-FEM

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### Summarized:

The purpose of this test is to validate the approach multi-cracking with X-FEM and junction. A plate in plane strain is requested in bi-tension and shears. Eight cracks are introduced in the center of the plate, including two with junction. One then uses `RAFF_XFEM` to refine around the crack tips and `CALC_G` to compute: the factors of intensity of the stresses.

A first test makes it possible to validate the approach without contact by comparison of the factors of intensity of the stresses with an semi-analytical solution. Then one shows in a second test that the addition of the contact, which corrects certain interpenetrations, influences the results.

## 1 Problem of reference

### 1.1 Geometry

One considers a square plate on side  $20\text{ m}$  in plane strains, centered in the reference  $(X, Y)$ . The cracks are defined by the points  $A$  with  $J$  and  $A'$  with  $J'$ , whose coordinates are given in table 1. The cracks are represented on figure 1.

Points	X	Y
A	-3,0851	0,75
B	0,50000	1,13327
C	0,20309	1,55730
D	-2,1454	1,09202
E	-1,3794	0,44923
F	-3,0851	-0,25
G	-2,3780	0,45711
H	-1,3794	1,09202
I	-0,5134	1,59202
J	-0,4397	0,79125

Points	X	Y
A'	3,08512	-0,75
B'	-0,5000	-0,3667
C'	-0,2031	0,05730
D'	2,14543	-0,4080
E'	1,37939	-1,0508
F'	3,08512	-1,75
G'	2,37802	-1,0429
H'	1,37939	-0,4080
I'	0,51336	0,09202
J'	0,43969	-0,7087

Table 1: coordinates of the points defining cracks.

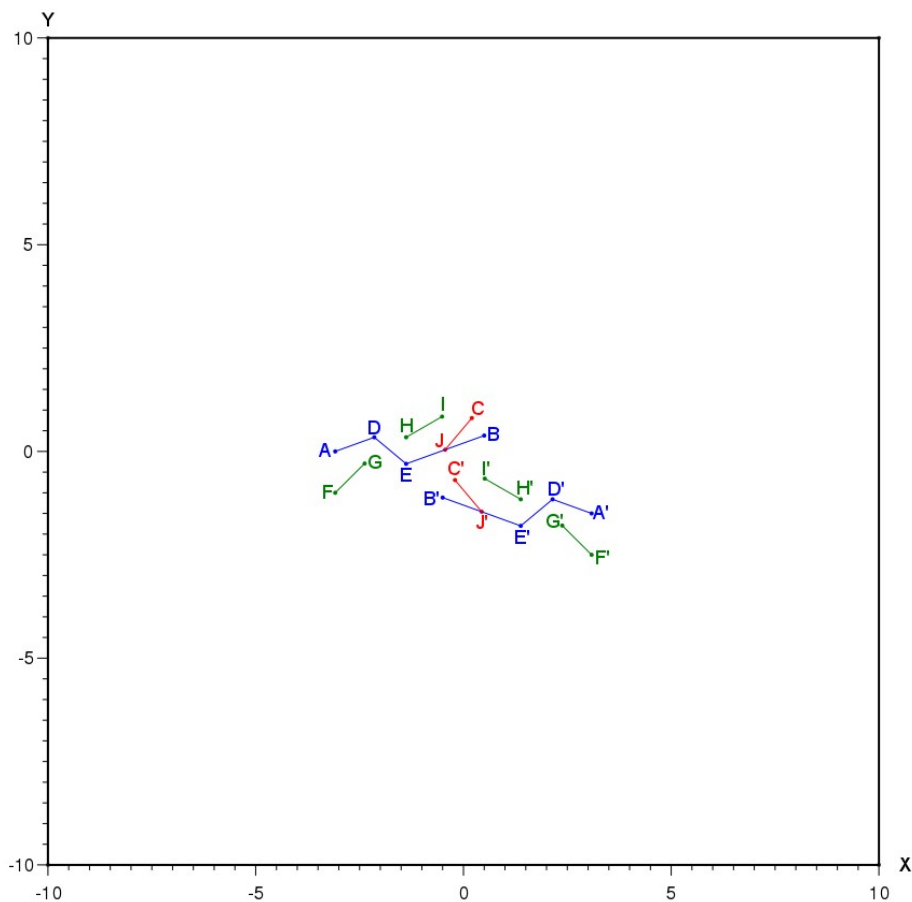


Figure 1: position of the points and cracks.

## 1.2 Properties of the material

the material is elastic isotropic with the following properties:

- $E=0,1 MPa$
- $\nu = 0,3$

## 1.3 Boundary conditions and loadings

One imposes the stress  $\sigma = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$  on all the contour of structure. That corresponds to a loading in bi-tension and unit shears. The rigid modes are blocked.

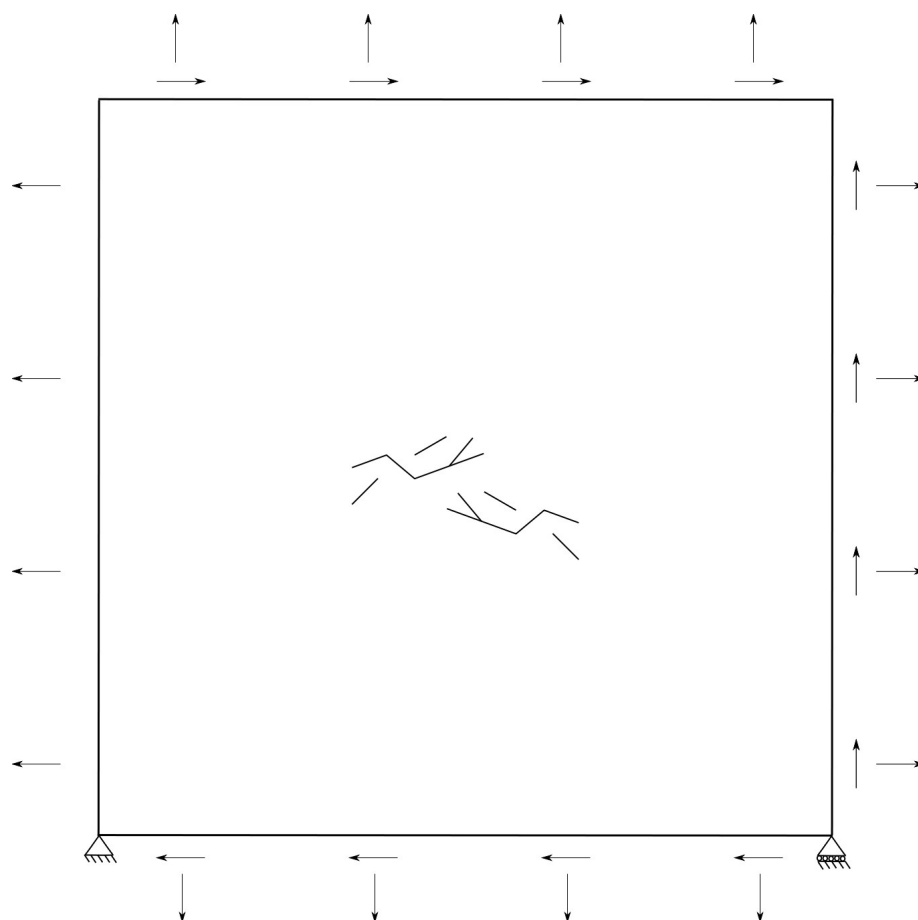


Figure 2: loading of structure.

## 2 Reference solution

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### 2.1 Method of calculating

The computation of the factors of intensity of the stresses is obtained in [1] by an semi-analytical procedure which consists in superimposing profiles of displacements checking of the free constraints on the lips of cracks.

### 2.2 Quantities and results of reference

One compares the values of  $K_1$  and  $K_2$  given in [1] to the crack tips.

Points	$K_1$	$K_2$
<i>A</i>	1,7943	2.8522
<i>B</i>	1,9932	2.4042
<i>C</i>	-1.6920	-0.1337
<i>F</i>	0.0510	0.2894
<i>G</i>	-0,5317	0.1885
<i>H</i>	-0,0517	-0.1979
<i>I</i>	-0,1933	0.0213

Items	$K_1$	$K_2$
<i>A'</i>	3,7215	2.3379
<i>B'</i>	2.6700	1.0248
<i>C'</i>	5.3966	-0.1143
<i>F'</i>	4.3255	-0.1661
<i>G'</i>	3.6812	0.9279
<i>H'</i>	0.4157	-0.3947
<i>I'</i>	1.0043	0.0648

Table 2: factors of intensity of the stresses to the crack tips obtained in [1].

### 2.3 Uncertainties on the solution

the number of significant figures given in table 2 reflects the quality of the semi-analytical solution. One can indeed check convergence towards these figures in the case of an increasingly fine mesh.

### 2.4 Bibliographical references

- [1] A.K. YAVUZ, S.L. PHOENIX, "Multiple Analysis Ace in Finite Punts", AIAA Newspaper, Flight 44, No 11, November 2006.
- [2] Mr. SIAVELIS, Mr. GUITON, P. MASSIN, N. Broad MOËS "sliding contact along branched discontinuities xith X-FEM", in relecture, 2012.

## 3 Modelization A

### 3.1 Characteristic of the modelization

One uses a modelization `D_PLAN`. The cracks are introduced with X-FEM using operands `GROUP_MA_FISS` and `GROUP_MA_FOND` of operator `DEFI_FISS_XFEM`. The points given in table 1 indeed make it possible to generate lines with a grid easily representing cracks of figure 1. The possible contact between the lips of cracks is not taken into account to respect the assumption of free stresses on the lips of cracks of the reference solution.

### 3.2 Characteristics of the initial

mesh The mesh regulated composed of 400 meshes of type `QUAD4` of figure 3 is refined in an adaptive way in 3 iterations using commands `RAFF_XFEM` and `MACR_ADAP_MAIL`. One meshes obtains the final mesh refined around the crack tips of figure 4, composed of 17105 of type `QUAD4` and 2076 meshes of type `TRIA3`.

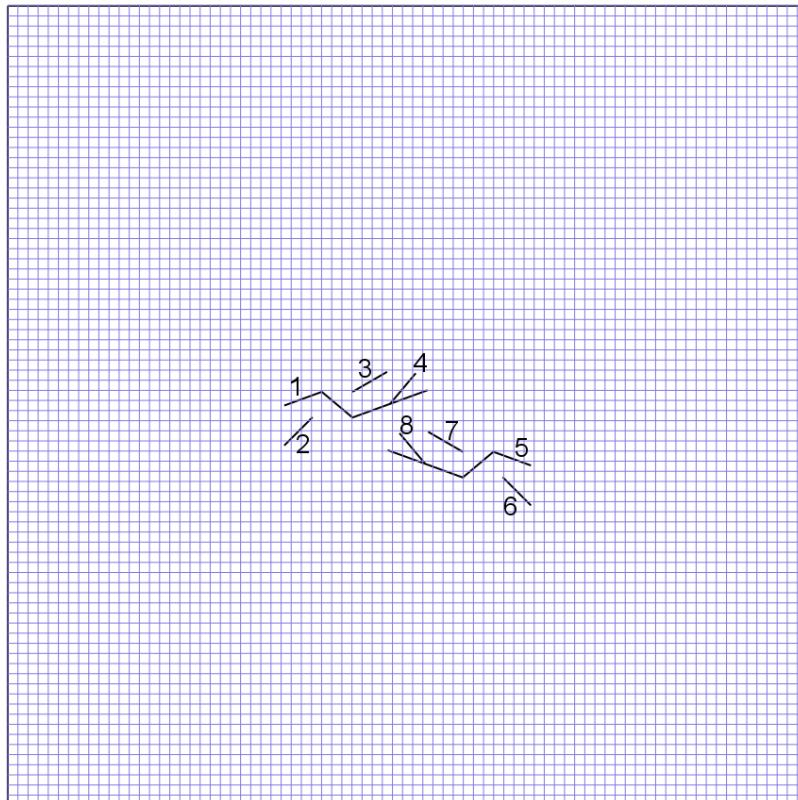


Figure 3: initial mesh and classification of cracks.

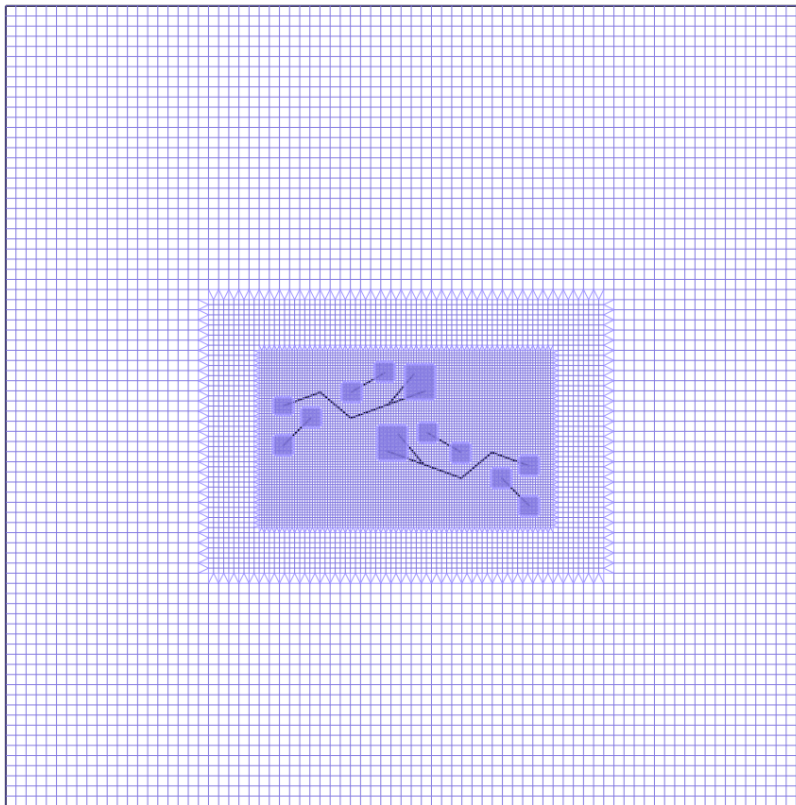


Figure 4: mesh refined around the crack tips used for computation.

## 3.3 Quantities tested and results

One tests the factors of intensity of the stresses on the points of the crack tips compared to the values given in table 2. The factors of intensity of the stresses are calculated using option `CALC_K_G` of operator `CALC_G`.

The values are validated with a tolerance of 1%. It is noted however that it is numerically possible to converge towards the semi-analytical solution by refining the mesh.

Standard	identification of reference	Value of reference	Tolerance
Not $A - K_1$	"ANALYTIQUE"	0.0000	1,1%
Point $A - K_2$	"ANALYTIQUE"	2.8522	1,1%
Point $B - K_1$	"ANALYTIQUE"	0.0000	1,1%
Point $B - K_2$	"ANALYTIQUE"	2.4042	1,1%
Point $C - K_1$	"ANALYTIQUE"	-1.6920	1,1%
Point $C - K_2$	"ANALYTIQUE"	-0.1337	1,1%
Point $F - K_1$	"ANALYTIQUE"	0.0510	1,1%
Point $F - K_2$	"ANALYTIQUE"	0.2894	1,1%
Point $G - K_1$	"ANALYTIQUE"	0.0000	1,1%

Point $G - K_2$	"ANALYTIQUE"	0.1885	1,1%
Point $H - K_1$	"ANALYTIQUE"	0.0000	1,1%
Point $H - K_2$	"ANALYTIQUE"	-0.1979	1,1%
Point $I - K_1$	"ANALYTIQUE"	0.0000	1,1%
Point $I - K_2$	"ANALYTIQUE"	0.0213	1,1%
Point $A' - K_1$	"ANALYTIQUE"	0.0000	1,1%
Point $A' - K_2$	"ANALYTIQUE"	2.3379	1,1%
Not $B' - K_1$	"ANALYTIQUE"	2.6700	1,1%
Point $B' - K_2$	"ANALYTIQUE"	1.0248	1,1%
Point $C' - K_1$	"ANALYTIQUE"	5.3966	1,1%
Point $C' - K_2$	"ANALYTIQUE"	-0.1143	1,1%
Point $F' - K_1$	"ANALYTIQUE"	4.3255	1,1%
Point $F' - K_2$	"ANALYTIQUE"	-0.1661	1,1%
Point $G' - K_1$	"ANALYTIQUE"	3.6812	1,1%
Point $G' - K_2$	"ANALYTIQUE"	0.9279	1,1%
Point $H' - K_1$	"ANALYTIQUE"	0.4157	1,1%
Point $H' - K_2$	"ANALYTIQUE"	-0.3947	1,1%
Point $I' - K_1$	"ANALYTIQUE"	1.0043	1,1%
Point $I' - K_2$	"ANALYTIQUE"	0.0648	1,1%

## 4 Modelization B

### 4.1 Characteristic of the modelization

the characteristics are the same ones as those of modelization A. One however takes into account the possible forces of contact between the lips of cracks via operator `DEFI_CONTACT` in order to correct interpenetrations. The assumption of free stresses on the lips of cracks made for the computation of the reference solution is not respected for cracks where the contact is active.

### 4.2 Characteristics of the mesh

The mesh is the same one as for modelization A.

### 4.3 Grandeurs testées et résultats

One tests the factors of intensity of the stresses on the points of the crack tips as for modelization A. the variations related to the taking into account of the contact on certain points do not make it possible to validate the approach with the semi-analytical solution of section 4. One thus makes a test of non regression. However the error obtained compared to an exact solution should be of the same order of magnitude as that obtained between the reference solution and the solution of modelization A. Indeed, the kinematical approximation allowing the computation of the factors of intensity of the stresses is exactly identical for the modelization A and B. Only the forces of contact on certain crack lips are added here compared to the Standard modelization

A.	Identification of reference	Value of reference	Tolerance
Not $A - K_1$	"NON_REGRESSION"	1.7479	1,1%
Point $A - K_2$	"NON_REGRESSION"	2.7678	1,1%
Point $B - K_1$	"NON_REGRESSION"	1.2384	1,1%
Point $B - K_2$	"NON_REGRESSION"	3.1770	1,1%
Point $C - K_1$	"NON_REGRESSION"	-0.5836	1,1%
Point $C - K_2$	"NON_REGRESSION"	0.4408	1,1%
Point $F - K_1$	"NON_REGRESSION"	0.1656	1,1%
Point $F - K_2$	"NON_REGRESSION"	0.2810	1,1%
Point $G - K_1$	"NON_REGRESSION"	-0.3466	1,1%
Point $G - K_2$	"NON_REGRESSION"	0.1555	1,1%
Point $H - K_1$	"NON_REGRESSION"	-0.0002	1,1%
Point $H - K_2$	"NON_REGRESSION"	-0.0410	1,1%
Point $I - K_1$	"NON_REGRESSION"	-0.3366	1,1%
Point $I - K_2$	"NON_REGRESSION"	0.2531	1,1%
Point $A' - K_1$	"NON_REGRESSION"	3.6999	1,1%
Point $A' - K_2$	"NON_REGRESSION"	2.3679	1,1%
Point $B' - K_1$	"NON_REGRESSION"	2.6639	1,1%



Point $B' - K_2$	"NON_REGRESSION"	1.0261	1,1%
Not $C' - K_1$	"NON_REGRESSION"	5.4041	1,1%
Point $C' - K_2$	"NON_REGRESSION"	-0.0961	1,1%
Point $F' - K_1$	"NON_REGRESSION"	4.3020	1,1%
Point $F' - K_2$	"NON_REGRESSION"	-0.1591	1,1%
Point $G' - K_1$	"NON_REGRESSION"	3.6790	1,1%
Point $G' - K_2$	"NON_REGRESSION"	0.9319	1,1%
Point $H' - K_1$	"NON_REGRESSION"	0.4022	1,1%
Point $H' - K_2$	"NON_REGRESSION"	-0.3641	1,1%
Point $I' - K_1$	"NON_REGRESSION"	0.9760	1,1%
Point $I' - K_2$	"NON_REGRESSION"	0.0989	1,1%

## 4.4 Remarks

figure 5 shows the deformed shape obtained. One notes a correction of the interpenetrations, which is obvious for crack 4 with the addition of the contact. This correction is at the origin of the variations obtained on the factors of intensity of the stresses.

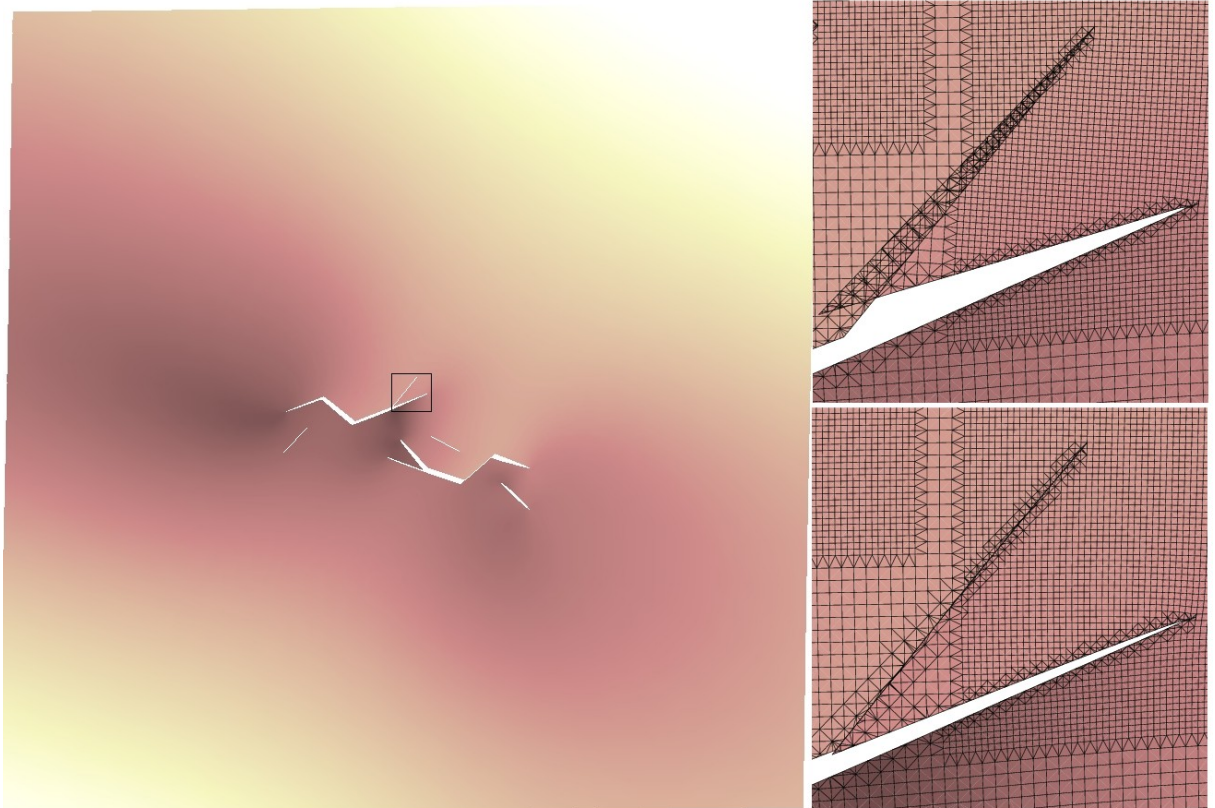


Figure 5: deformed shape obtained with amplification x1000 (on the left), zoom on crack 4, without and with contact (on the right).

## 5 Summary of the results

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the first computation without contact makes it possible to validate the approach multi-cracking with junctions in X-FEM by comparison of the factors of intensity of the stresses to an semi-analytical reference solution (which does not take into account the forces on the lips of cracks). Interpenetrations however are noted, although the structure is primarily requested in tension. Correction made by the second computation, with taking into account of the contact, watch that the new values obtained for the factors of intensity of the stresses can lead to light deviations of the directions of propagation [2].