SSNP503 – Contact in great slidings with X-FEM for horizontal cracks

Summarized:

The purpose of this test is testing and validating the sliding features of the approach large with X-FEM in cases presenting of horizontal cracks. One 2D tests structures in plane stresses and plane strains for elements quadrangles and triangles as well as structures 3D for elements hexahedrons, pentahedrons and tetrahedrons. This approach, described in [R5.03.53], of the contact allows in particular the taking into account, with friction, on the level of cracks introduced by the method X-FEM, in the presence of large displacements but under the assumption of the small strains. New algorithms, compared to the processing of contact with X-FEM under the assumption of the small disturbances [R7.02.12], which is tested and validated by this case test relate to the geometrical reactualization of the lips of cracks, master-slave pairing and the creation of the new hybrid elements of contact.

One considers a rectangular structure presenting two horizontal cracks crossing it completely, placed symmetrically compared to the median axis of structure. The two cracks thus cut the rectangle in three blocks. The blocking of horizontal displacements is imposed on the four corners of the rectangle, of vertical displacements are imposed on the edges inferior and superior of the rectangle in order to tighten the block medium which is seen forcing a horizontal displacement to make it slip along cracks. Following the request of compression thus created, contact pressures appear on the zones in contact, with an evolution of their values according to the advance of the block medium.

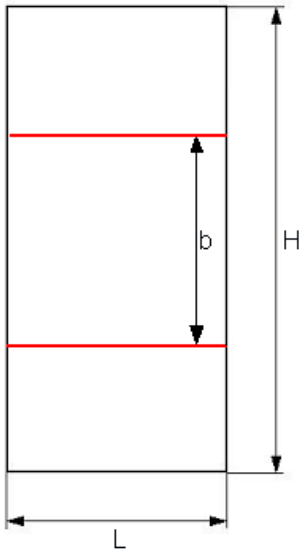
The validation is done by comparison of the values of contact pressure with the similar values obtained from a homologous test (even geometry, same boundary conditions etc), treated in the classical frame of the finite element method, with *Code_Aster*, where the cracks are in conformity with the mesh.

1 Problem of reference

1.1 Geometry

the structure is a healthy rectangle into which two horizontal cracks are introduced. The cracks are placed symmetrically compared to the median axis of structure, as shown on the Figure 1.1-a. Dimensions of structure as well as the distance between cracks are:

$H = 9\,m$;
$L = 4\,m$;
$b = 4\,m$;



Appear 1.1-a: Geometry of structure and positioning of cracks.

1.2 Properties of the material

Young's modulus: $E = 100\,MPa$

Poisson's ratio: $\nu = 0.3$

1.3 Boundary conditions and loadings

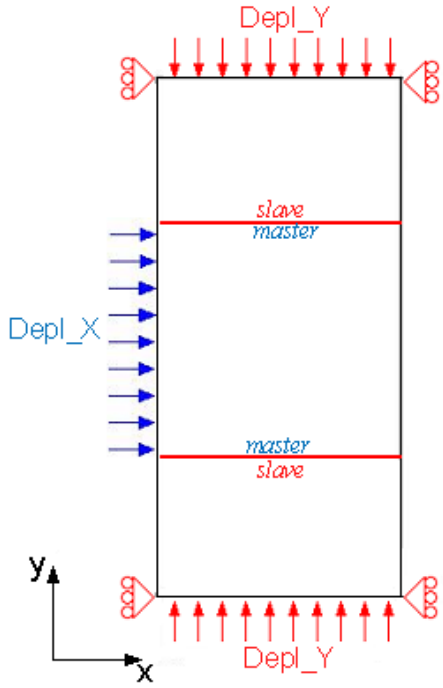
the blocking of horizontal displacements is imposed on the four corners of structure (Figure 1.3-a). On the edges inferior and superior of the rectangle, one imposes displacements along the axis Y which will close cracks in order to generate contact pressure. The block medium is subjected to an important sliding by applying a displacement controlled along the axis X to its left edge.

The numerical values of imposed displacements are:

$Depl_X = 2.00\,m$
$Depl_Y = 1.0\mathrm{E}-3\,m$

Their application is done according to a function crawls classical, in 4 steps of load.

The coefficient of kinetic friction of Coulomb is taken equal to $0,5$.



Appear 1.3-a: Illustration of the boundary conditions and the loadings.

2 Reference solution

the reference solutions for this benchmark are provided by the results resulting from Aster computations, for same structure but with cracks respecting the mesh, carried out with the continuous method of contact already existing for the classical frame of the finite element method [R5.03.52].

The geometry (except for the introduction cracks), the boundary conditions, the loadings as well as the parameters of contact are the same ones as those considered for this benchmark modelled with X-FEM.

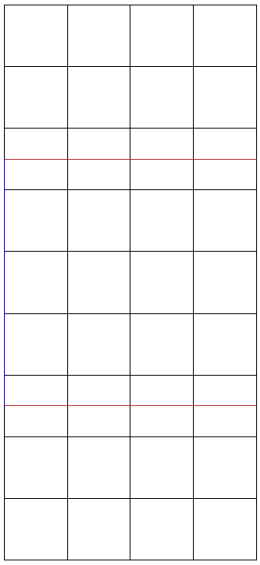
3 Modelization A

3.1 Characteristic of the modelization

It acts of a modelization FEM, in plane strains. The three blocks are with a grid conformément and the conditions of contact are imposed on the edges of these blocks. One and the declares the edges Masters on the block medium edges slaves on the blocks inferior and superior in order to conform to the figure 1.3-a.

3.2 Characteristics of the mesh

The mesh is regulated (Figure 3.2-a) and comprises 3 blocks made up of meshes of type `QUAD4`. The blocks superior and inferior have each one meshes 12, while the block medium has 20 of them.



Appear 3.2-a: The mesh of modelization A.

3.3 Grandeurs tested and results

One tests contact pressures on the upper lip of the lower block, this one being declared slave, at the end of each step of load considered. The site of the nodes which store the degrees of freedom of contact which one tests the values, is illustrated on the Figure 3.4-a.

Not	Identification	*Reference*
1	LAGS_C for N21	0.0000000E+00
	LAGS_C for N20	-8.1059677E+03
	LAGS_C for N19	-6.4501080E+03
	LAGS_C for N18	-6.4542489E+03
	LAGS_C for N17	-4.1383235E+03
2	LAGS_C for N21	0.0000000E+00
	LAGS_C for N20	-1.0392388E+04
	LAGS_C for N19	-1.3120234E+04
	LAGS_C for N18	-1.1868000E+04
	LAGS_C for N17	-8.7124511E+03
3	LAGS_C for N21	0.0000000E+00
	LAGS_C for N20	0.0000000E+00
	LAGS_C for N19	-2.1971033E+04
	LAGS_C for N18	-1.8731887E+04
	LAGS_C for N17	-1.1800265E+04
4	LAGS_C for N21	0.0000000E+00
	LAGS_C for N20	0.0000000E+00
	LAGS_C for N19	-1.9159216E+04
	LAGS_C for N18	-2.3596467E+04
	LAGS_C for N17	-1.7951778E+04

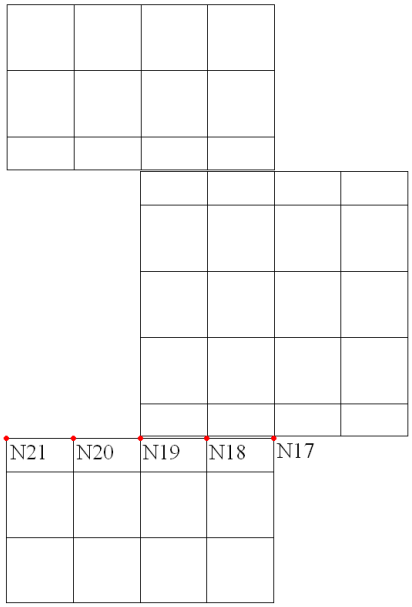


Figure 3.4-a: The site of the nodes which store the degrees of freedom of contact tested.

3.4 Remarks

This test is used as reference to the modelization B, it is thus normal to have residues close to zero ($10^{-10}\%$), which correspond to the accuracy required in operator IMPR_TABLE of the command file. It makes it possible to make sure that there is no evolution in the calculation algorithms for method FEM of contact in great slidings, in which case it would then be necessary also to re-examine the method X-FEM, based on this one.

4 Modelization B

4.1 Characteristic of the modelization

It acts of a modelization X-FEM, in plane strains, with definition of contact on the interfaces generated by cracks, themselves defined by functions of level (level set noted norm LN) directly in the command file using operator DEFI_FISS_XFEM [U4.82.08].

The statute main slave/for a contact surface X-FEM is given by the sign of the normal function of level LN : surface slave is negative side while surface Master is positive side.

The equations of the functions of levels for two horizontal cracks are the following ones:

$$\mathrm{LN}1 = Y - 2.5 \qquad \text{éq 4.1-1}$$
$$\mathrm{LN}2 = -Y + 6.5 \qquad \text{éq 4.1-2}$$

One does not define the levels set tangent because one does not model basic of crack. It is then necessary to inform key word TYPE_DISCONTINUITE with the option "INTERFACES" in operator DEFI_FISS_XFEM.

So that the approach great slidings with X-FEM is activated, it is necessary to inform key word REAC_GEOM in operator DEFI_CONTACT with option "AUTOMATIQUE". If not, for REAC_GEOM='SANS' (option by default), the processing HP is active.

For the activation of friction, it is necessary to inform key word FROTTEMENT in operator DEFI_CONTACT with option "COULOMB" then to indicate the value of the coefficient of kinetic friction for each zone by informing key word COULOMB under operand ZONE.

4.2 Characteristics of the mesh

The mesh is regulated (Figure 4.2-a) and meshes comprises 36 type QUAD4. Following the definition of cracks, the 8 meshes cut are transformed into QUAD8 in order to store the degrees of freedom of contact rubbing with the nodes tops and mediums.

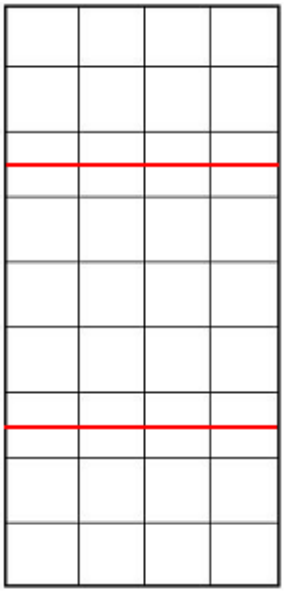


Figure 4.2-a : The mesh of the modelization B.

the choice amongst elements in the direction Y (on the vertical) was made by taking account of the restriction imposed by the modelization of multi-cracking with the method X-FEM: the cracks must be separate of at least two meshes healthy.

4.3 Quantities tested and results

One tests the values of the contact pressure on the lip slave of first crack, at the end of each step of load considered. On the cut elements, one can choose to arbitrarily record the values on the nodes above or below crack (one chose with the top), because the degrees of freedom of contact are bound by relation of equality (it is not necessary to make an interpolation). The site of the nodes which store the degrees of freedom of contact which one tests the values, is illustrated on the Figure 4.4-a.

Not	Identification	Reference
1	LAGS_C for N24	0.0000000E+00
	LAGS_C for N33	-8.1059677E+03
	LAGS_C for N34	-6.4501080E+03
	LAGS_C for N35	-6.4542489E+03
	LAGS_C for N10	-4.1383235E+03
2	LAGS_C for N24	0.0000000E+00
	LAGS_C for N33	-1.0392388E+04
	LAGS_C for N34	-1.3120234E+04
	LAGS_C for N35	-1.1868000E+04
	LAGS_C for N10	-8.7124511E+03
3	LAGS_C for N24	0.0000000E+00
	LAGS_C for N33	0.0000000E+00
	LAGS_C for N34	-2.1971033E+04
	LAGS_C for N35	-1.8731887E+04
	LAGS_C for N10	-1.1800265E+04
4	LAGS_C for N24	0.0000000E+00
	LAGS_C for N33	0.0000000E+00
	LAGS_C for N34	-1.9159216E+04
	LAGS_C for N35	-2.3596467E+04
	LAGS_C for N10	-1.7951778E+04

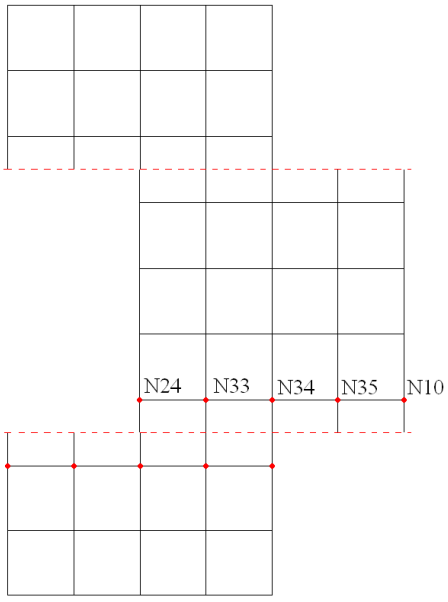


Figure 4.4-a: The site of the nodes which store the degrees of freedom of contact tested.

4.4 Remarks

In both cases (methods of contact X-FEM and classical FEM) there is the same number of degrees of freedom of displacements, the same number of degrees of freedom of contact, and the position of the nodes of the mesh (except the interface which is not with a grid for X-FEM) are the same ones, one integrates the contact on the same elements and the stiffness on the quasi-same elements (except for the cut elements). One thus solves the same numerical problem in the 2 cases. It is thus normal to observe differences close to zero ($10^{-7}\%$).

5 Modelization C

5.1 Characteristic of the modelization

It is acted of the same characteristics of modelization as the modelization A but as plane stresses.

5.2 Characteristics of the mesh

They is the same characteristics of mesh as modelization A.

5.3 Grandeurs tested and results

One tests contact pressures as for modelization A. the site of the nodes which store the degrees of freedom of contact, which one tests the values, is illustrated on the Figure 3.4-a.

Not	Identification	Reference
1	LAGS_C for N21	0.0000000E+00
	LAGS_C for N20	-7.1041388E+03
	LAGS_C for N19	-5.5751916E+03
	LAGS_C for N18	-5.6055420E+03
	LAGS_C for N17	-3.6920556E+03
2	LAGS_C for N21	0.0000000E+00
	LAGS_C for N20	-9.1709839E+03
	LAGS_C for N19	-1.1435732E+04
	LAGS_C for N18	-1.0391562E+04
	LAGS_C for N17	-7.7671507E+03
3	LAGS_C for N21	0.0000000E+00
	LAGS_C for N20	0.0000000E+00
	LAGS_C for N19	-1.9485289E+04
	LAGS_C for N18	-1.6399583E+04
	LAGS_C for N17	-1.0605893E+04
4	LAGS_C for N21	0.0000000E+00
	LAGS_C for N20	0.0000000E+00
	LAGS_C for N19	-1.7065781E+04
	LAGS_C for N18	-2.0872174E+04
	LAGS_C for N17	-1.6114706E+04

5.4 Remarks

This test is used as reference to modelization D.

6 Modélisation D

6.1 Characteristic of the modelization

It is acted of the same characteristics of modelization as the modelization B but as plane stresses.

6.2 Characteristics of the mesh

They is the same characteristics of mesh as the modelization B.

6.3 Quantities tested and results

One tests the values of contact pressure as for the modelization B.

Not	Identification	Reference
1	LAGS_C for N24	0.0000000E+00
	LAGS_C for N33	-7.1041388E+03
	LAGS_C for N34	-5.5751916E+03
	LAGS_C for N35	-5.6055420E+03
	LAGS_C for N10	-3.6920556E+03
2	LAGS_C for N24	0.0000000E+00
	LAGS_C for N33	-9.1709839E+03
	LAGS_C for N34	-1.1435732E+04
	LAGS_C for N35	-1.0391562E+04
	LAGS_C for N10	-7.7671507E+03
3	LAGS_C for N24	0.0000000E+00
	LAGS_C for N33	0.0000000E+00
	LAGS_C for N34	-1.9485289E+04
	LAGS_C for N35	-1.6399583E+04
	LAGS_C for N10	-1.0605893E+04
4	LAGS_C for N24	0.0000000E+00
	LAGS_C for N33	0.0000000E+00
	LAGS_C for N34	-1.7065781E+04
	LAGS_C for N35	-2.0872174E+04
	LAGS_C for N10	-1.6114706E+04

6.4 Remarks

Idem 4.4

7 Modelization E

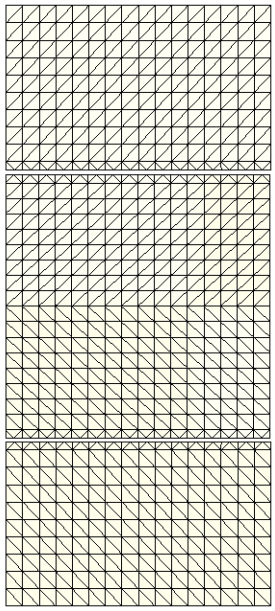
7.1 Characteristic of the modelization

It acts of the same characteristics of modelization as modelization A.

7.2 Caractéristiques of the mesh

The mesh is regulated (Figure 7.2-a) and comprises 3 blocks made up of meshes of type TRI3. The blocks superior and inferior have each one meshes 336, while the block medium has 608 of them.

The mesh is finer than in the preceding tests, because one wants to avoid a loss of accuracy between the methods of contact FEM and X-FEM caused by the differences in mesh. The mesh is in addition built so as to respect the median symmetry of the model.



Appear 7.2-a: The mesh of the modelization E.

7.3 Quantities tested and results

One tests the values of the contact pressure as for modelization A. the site of the nodes which store the degrees of freedom of contact, which one tests the values, is illustrated on the Figure 7.4-a.

Not	Identification	Reference
1	LAGS_C for N1254	0.0000000E+00
	LAGS_C for N1262	-7.1942608E+03
	LAGS_C for N1270	-6.5408344E+03
	LAGS_C for N1278	-6.1360067E+03
	LAGS_C for N1286	-2.4914329E+03
2	LAGS_C for N1254	0.0000000E+00
	LAGS_C for N1262	-2.4822528E+04
	LAGS_C for N1270	-1.2993429E+04
	LAGS_C for N1278	-1.2178296E+04
	LAGS_C for N1286	-3.4642908E+03
	LAGS_C for N1254	0.0000000E+00

3	LAGS_C for N1262	0.0000000E+00
	LAGS_C for N1270	-2.0219781E+04
	LAGS_C for N1278	-1.8168929E+04
	LAGS_C for N1286	-4.8271686E+03
4	LAGS_C for N1254	0.0000000E+00
	LAGS_C for N1262	0.0000000E+00
	LAGS_C for N1270	-4.3387251E+04
	LAGS_C for N1278	-2.4197889E+04
	LAGS_C for N1286	-7.2647040E+03

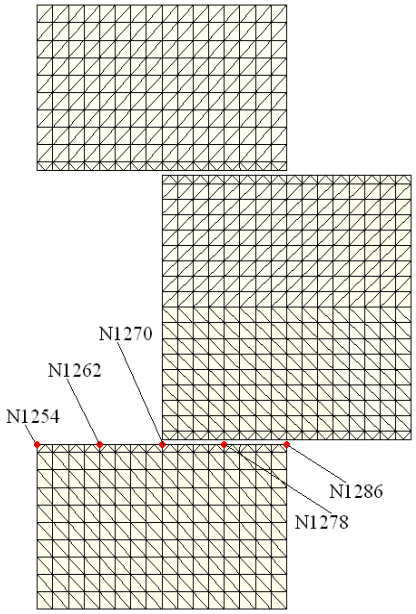


Figure 7.4-a: The site of the nodes which store the degrees of freedom of contact tested.

7.4 Remarks

This test is used as reference to the modelization F.

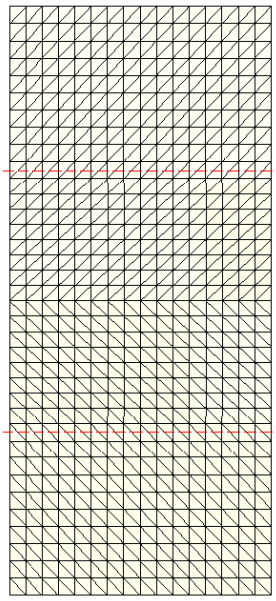
8 Modelization F

8.1 Characteristic of the modelization

It acts of the same characteristics of modelization as the modelization B.

8.2 Characteristic of the mesh

The mesh is regulated (Figure 8.2-a) and comprises 1152 meshes type TRI3. Following the definition of cracks, the 64 meshes cut store with their nodes the degrees of freedom of contact friction. The mesh is built as for the modelization E, so as to respect the median symmetry of the model.



Appear 8.2-a: The mesh of the modelization F.

8.3 Quantities tested and results

One tests the values of contact pressure as for the modelization B. the site of the nodes which store the degrees of freedom of contact, which one tests the values, is illustrated on the Figure 8.4-a.

One graphically compares on the figure 8.4-b the results of approaches FEM and X-FEMs.

Not	Identification	Reference
1	LAGS_C for N33	0.0000000E+00
	LAGS_C for N154	-7.1942608E+03
	LAGS_C for N230	-6.5408344E+03
	LAGS_C for N306	-6.1360067E+03
	LAGS_C for N677	-2.4914329E+03
2	LAGS_C for N33	0.0000000E+00
	LAGS_C for N154	-2.4822528E+04
	LAGS_C for N230	-1.2993429E+04
	LAGS_C for N306	-1.2178296E+04
	LAGS_C for N677	-3.4642908E+03
3	LAGS_C for N33	0.0000000E+00
	LAGS_C for N154	0.0000000E+00
	LAGS_C for N230	-2.0219781E+04

	LAGS_C for N306	-1.8168929E+04
	LAGS_C for N677	-4.8271686E+03
4	LAGS_C for N33	0.0000000E+00
	LAGS_C for N154	0.0000000E+00
	LAGS_C for N230	-4.3387251E+04
	LAGS_C for N306	-2.4197889E+04
	LAGS_C for N677	-7.2647040E+03

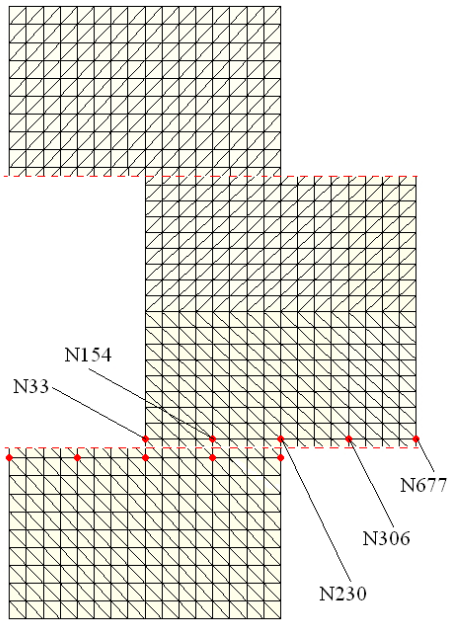
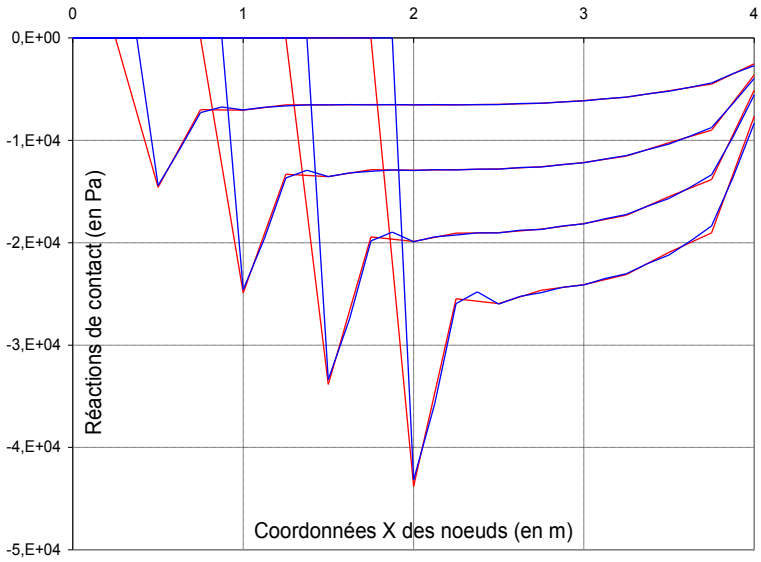


Figure 8.4-a: The site of the nodes which store the degrees of freedom of contact tested.



Appear 8.4-b: SSNP503-e and F: Comparison of the reactions of contact (4 steps of loads), for methods FEM (in blue) and XFEM (in red).

8.4 Remarks

These results validate in 2D the management of integration of the contact with slipping friction when a linear relation between the degrees of freedom of contact friction is imposed in the contacting passing/ not contacting by the algorithm of stabilization of the LBB (see [R5 03 53] and [D9 05 06]).

So that the results of two approaches (methods of contact X-FEM and classical FEM) correspond, one imposes that the positions of the nodes of the mesh as well as meshes the triangles (except the interface which is not with a grid for X-FEM) are the same ones, that integration of the contact is made on the same segments, and that the integration of the stiffness is made on the same triangles (except for the cut elements).

However by thus making, the interfaces of method of contact FEM have almost twice more degrees of freedom (contact and displacement) that those of method of contact X-FEM. One thus does not solve exactly the same numerical problem in the 2 cases.

Taking account of that, one can conclude that one observes very good performances (relative differences lower than 0.5%) if account of the edge effects is not taken (relative differences of about 5% on the last node).

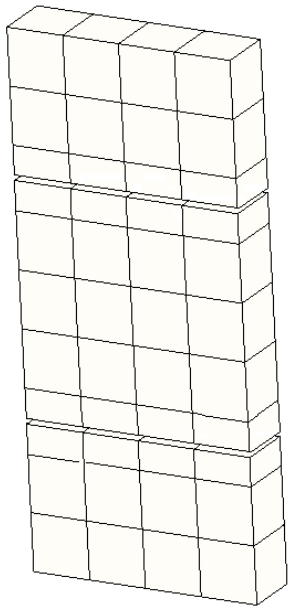
9 Modelization G

9.1 Characteristic of the modelization

It acts of the same characteristics of modelization as the modelization A, but in 3D. One thus extrudes the rectangle considered in the depth of the plane a unit length. One blocks rigid body motions in the new direction considered.

9.2 Characteristics of the mesh

The mesh is regulated (Figure9.2-a) and comprises 3 blocks made up of meshes of type HEXA8. The blocks superior and inferior have each one meshes 12, while the block medium has 20 of them.



Appear 9.2-a: The mesh of the modelization G.

9.3 Quantities tested and results

One tests contact pressures as for modelization A. the site of the nodes which store the degrees of freedom of contact, which one tests the values, is illustrated on the Figure 9.4-a.

Not	Identification	Reference
1	LAGS_C for N110	0.0000000E+00
	LAGS_C for N119	-7.4394584E+03
	LAGS_C for N121	-5.5601371E+03
	LAGS_C for N123	-5.7894274E+03
	LAGS_C for N108	-3.8723295E+03
2	LAGS_C for N110	0.0000000E+00
	LAGS_C for N119	-9.9465544E+03
	LAGS_C for N121	-1.1457450E+04
	LAGS_C for N123	-1.0644747E+04
	LAGS_C for N108	-8.1152553E+03
3	LAGS_C for N110	0.0000000E+00
	LAGS_C for N119	0.0000000E+00
	LAGS_C for N121	-2.0437614E+04

	LAGS_C for N123	-1.6583379E+04
	LAGS_C for N108	-1.1024368E+04
4	LAGS_C for N110	0.0000000E+00
	LAGS_C for N119	0.0000000E+00
	LAGS_C for N121	-1.8510255E+04
	LAGS_C for N123	-2.1051214E+04
	LAGS_C for N108	-1.6537311E+04

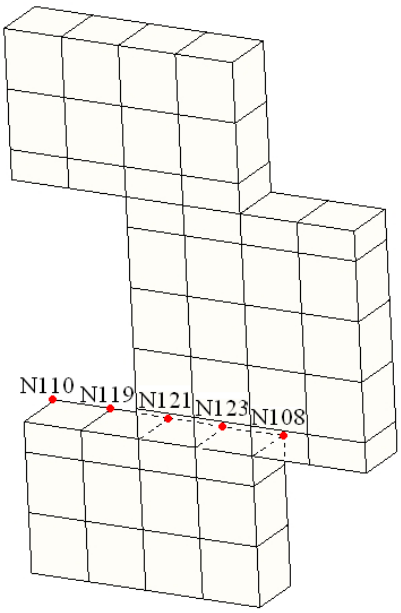


Figure 9.4-a: The site of the nodes which store the degrees of freedom of contact tested.

9.4 Remarks

This test is used as reference to the modelization H.

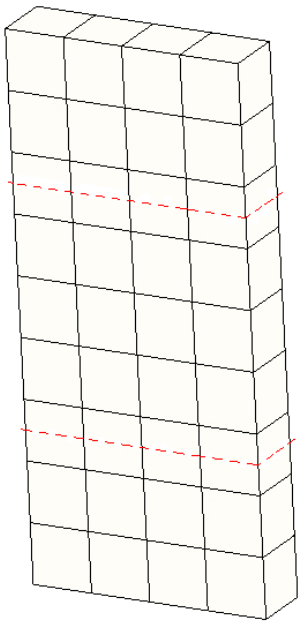
10 Modélisation H

10.1 Characteristic of the modelization

It acts of the same characteristics of modelization as the modelization B but in 3D. One thus extrudes the rectangle considered in the depth of the plane a unit length. One blocks rigid body motions in the new direction considered.

10.2 Characteristics of the mesh

The mesh is regulated (Figure 10.2-a) and meshes comprises 36 type HEXA8. Following the definition of cracks, the 8 meshes cut store with their nodes the degrees of freedom of contact friction.



Appear 10.2-a: The mesh of the modelization H.

10.3 Grandeurs tested and results

One tests the values of contact pressure as for the modelization B. the site of the nodes which store the degrees of freedom of contact, which one tests the values, is illustrated on the Figure 10.4-a.

Not	Identification	Reference
1	LAGS_C for N13	0.0000000E+00
	LAGS_C for N14	-7.4394584E+03
	LAGS_C for N15	-5.5601371E+03
	LAGS_C for N16	-5.7894274E+03
	LAGS_C for N85	-5.8084941D+03
2	LAGS_C for N13	0.0000000E+00
	LAGS_C for N14	-9.9465544E+03
	LAGS_C for N15	-1.1457450E+04
	LAGS_C for N16	-1.0644747E+04
	LAGS_C for N85	-1.2172882D+04
3	LAGS_C for N13	0.0000000E+00
	LAGS_C for N14	0.0000000E+00
	LAGS_C for N15	-2.0437614E+04

	LAGS_C for N16	-1.6583379E+04
	LAGS_C for N85	-1.6536552D+04
4	LAGS_C for N13	0.0000000E+00
	LAGS_C for N14	0.0000000E+00
	LAGS_C for N15	-1.8510255E+04
	LAGS_C for N16	-2.1051214E+04
	LAGS_C for N85	-2.4805965D+04

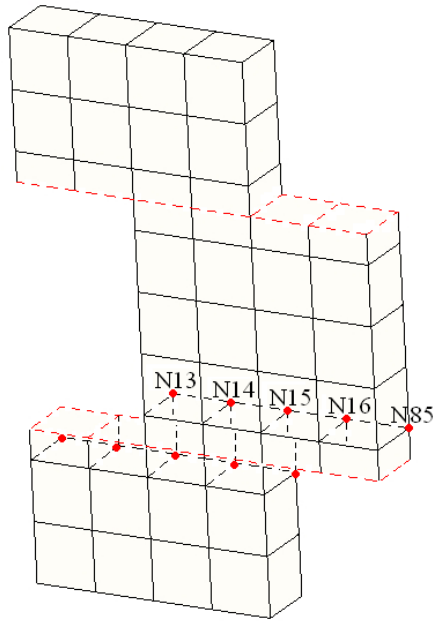
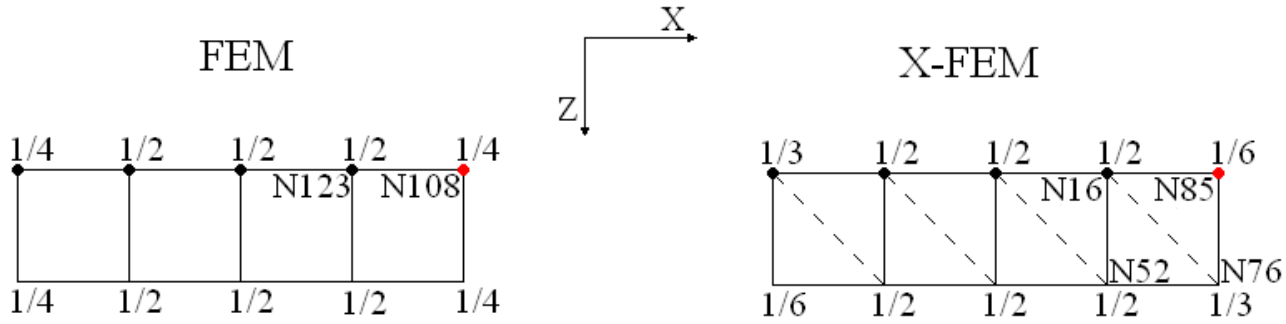


Figure 10.4-a: The site of the nodes which store the degrees of freedom of contact tested.



Appear 10.4-b: Surfaces (in m^2) on which the multipliers of contact of each node are divided into considering nodal integration (on the quadrangles for FEM, of the triangles for X-FEM).

10.4 Remarks

Idem 4.5 except that one does not integrate the contributions of contact on the same elements (see figure 10.4.b): in FEM meshes of contact are quadrangles, in X-FEM the facets are triangles. That causes a dissymmetry for the approach X-FEM between the nodes $N85$ (which contributes on a surface of $1/6\,m^2$) and $N76$ (which contributes on a surface of $1/3\,m^2$). The surfaces clarified on the figure 10.4-b act numerically like weights for the multipliers, one thus has logically $\frac{1}{6}\lambda_{N85}=\frac{1}{3}\lambda_{N76}$ is $\lambda_{N85}=2\,\lambda_{N76}$.

So now one compares the N108 nodes of approach FEM and N85 of the approach X-FEM, one can write $\frac{1}{6}\lambda_{N85}^{XFEM}=\frac{1}{4}\lambda_{N108}^{FEM}$ is $\lambda_{N85}^{XFEM}=\frac{3}{2}\lambda_{N108}^{FEM}$.

Value LAGS_C of $N108$ coming from the modelization G must thus be multiplied by 1,5 to be compared with LAGS_C of $N85$ modelization H. When one makes thus, one observes many differences close to zero ($10^{-3}\%$) on all the nodes tested.

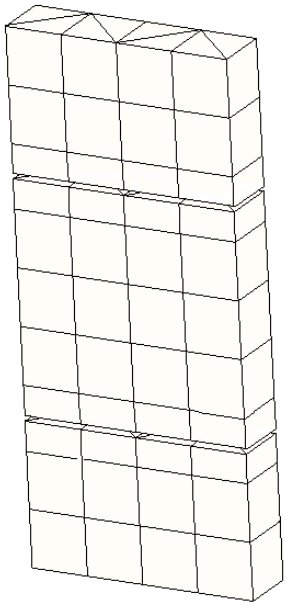
11 Modelization I

11.1 Characteristic of the modelization

It acts of the same characteristics of modelization as the modelization G.

11.2 Characteristic of the mesh

The mesh is regulated (Figure 11.2-a) and comprises 3 blocks made up of meshes of type PENTA6. The blocks superior and inferior have each one meshes 24, while the block medium has 40 of them.



Appear 11.2-a: The mesh of the modelization I.

11.3 Quantities tested and results

One tests contact pressures as for modelization A. the site of the nodes which store the degrees of freedom of contact, which one tests the values, is illustrated on the Figure 11.4-a.

Not	Identification	Reference
1	LAGS_C for N52	0.0000000E+00
	LAGS_C for N69	-9.2184408E+03
	LAGS_C for N70	-5.3136225E+03
	LAGS_C for N73	-6.3318943E+03
	LAGS_C for N50	-3.8408176E+03
2	LAGS_C for N52	0.0000000E+00
	LAGS_C for N69	-1.5003217E+04
	LAGS_C for N70	-8.0031951E+03
	LAGS_C for N73	-1.4765678E+04
	LAGS_C for N50	-6.7122964E+03
3	LAGS_C for N52	0.0000000E+00
	LAGS_C for N69	0.0000000E+00
	LAGS_C for N70	-1.7575098E+04
	LAGS_C for N73	-1.6339667E+04
	LAGS_C for N50	-1.2605826E+04
	LAGS_C for N52	0.0000000E+00

4	LAGS_C for N69	0.0000000E+00
	LAGS_C for N70	-1.7152487E+04
	LAGS_C for N73	-2.1471849E+04
	LAGS_C for N50	-1.7685794E+04

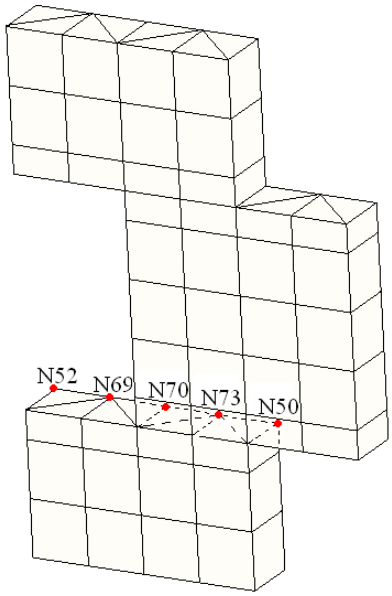


Figure 11.4-a: The site of the nodes which store the degrees of freedom of contact tested.

11.4 Remarks

This test is used as reference to the modelization J.

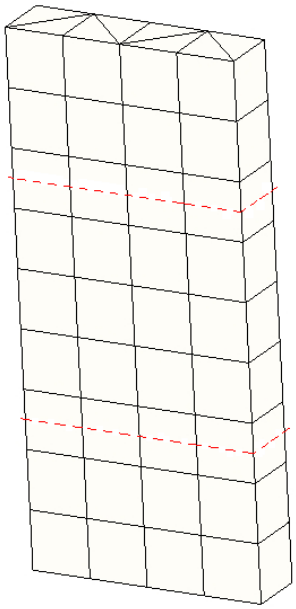
12 Modelization J

12.1 Characteristic of the modelization

It acts of the same characteristics of modelization as the modelization H.

12.2 Caractéristiques of the mesh

The mesh is regulated (Figure 12.2-a) and meshes comprises 72 type PENTA6. Following the definition of cracks, the 16 meshes cut store with their nodes the degrees of freedom of contact friction.



Appear 12.2-a: The mesh of the modelization J.

12.3 Quantities tested and results

One tests the values of contact pressure as for the modelization B. the site of the nodes which store the degrees of freedom of contact, which one tests the values, is illustrated on the Figure 12.4-a.

Not	Identification	Reference
1	LAGS_C for N19	0.0000000E+00
	LAGS_C for N20	-9.2184408E+03
	LAGS_C for N22	-5.3136225E+03
	LAGS_C for N23	-6.3318943E+03
	LAGS_C for N85	-3.8408176E+03
2	LAGS_C for N19	0.0000000E+00
	LAGS_C for N20	-1.5003217E+04
	LAGS_C for N22	-8.0031951E+03
	LAGS_C for N23	-1.4765678E+04
	LAGS_C for N85	-6.7122964E+03
3	LAGS_C for N19	0.0000000E+00
	LAGS_C for N20	0.0000000E+00
	LAGS_C for N22	-1.7575098E+04
	LAGS_C for N23	-1.6339667E+04

	LAGS_C for N85	-1.2605826E+04
4	LAGS_C for N19	0.0000000E+00
	LAGS_C for N20	0.0000000E+00
	LAGS_C for N22	-1.7152487E+04
	LAGS_C for N23	-2.1471849E+04
	LAGS_C for N85	-1.7685794E+04

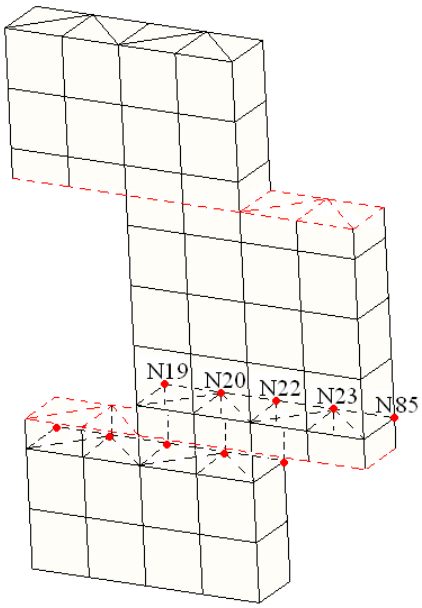


Figure 12.4-a: The site of the nodes which store the degrees of freedom of contact tested.

12.4 Remarks

Idem 4.5. One observes differences close to zero ($10^{-3}\,\%$).

13 Modelization K

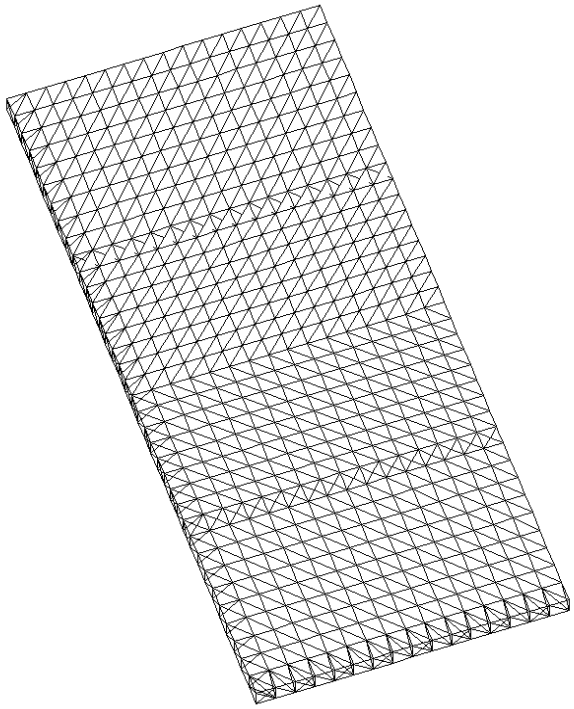
13.1 Characteristic of the modelization

It acts of the same characteristics of modelization as the modelization G.

13.2 Characteristic of the mesh

The mesh is regulated (Figure 13.2-a) and comprises 3 blocks made up of meshes of type TETRA4. The blocks superior and inferior have each one meshes 1098, while the block medium has 1824 of them.

The mesh is finer than in the preceding tests, because one wants to limit a loss of accuracy between the methods of contact FEM and X-FEM caused by the differences in mesh. The mesh is in addition built so as to respect the median symmetry of the model.



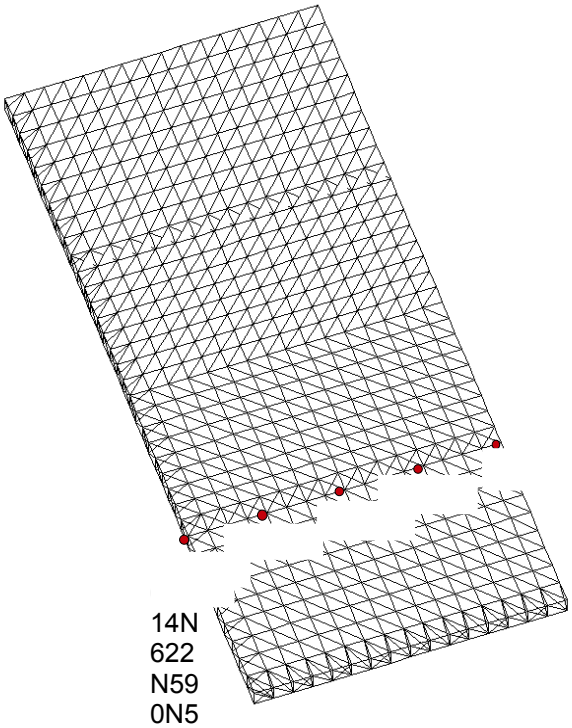
Appear 13.2-a: The mesh of the modelization K.

13.3 Quantities tested and results

One tests contact pressures as for modelization A. the site of the nodes which store the degrees of freedom of contact, which one tests the values, is illustrated on the Figure 13.4-a.

Not	Identification	Reference
1	LAGS_C for N3779	0.00E+000
	LAGS_C for N3802	-7.24E+003
	LAGS_C for N3826	-6.43E+003
	LAGS_C for N3850	-5.50E+003
	LAGS_C for N3874	-9.82E+002
2	LAGS_C for N3779	0.00E+000
	LAGS_C for N3802	-2.64E+004
	LAGS_C for N3826	-1.28E+004

	LAGS_C for N3850	-1.11E+004
	LAGS_C for N3874	-1.01E+003
3	LAGS_C for N3779	0.00E+000
	LAGS_C for N3802	0.00E+000
	LAGS_C for N3826	-1.93E+004
	LAGS_C for N3850	-1.68E+004
	LAGS_C for N3874	-1.57E+003
4	LAGS_C for N590	0.00E+000
	LAGS_C for N598	0.00E+000
	LAGS_C for N606	-4.43E+004
	LAGS_C for N614	-2.25E+004
	LAGS_C for N622	-3.17E+003



Appears 13.4-a: The site of the nodes which store the degrees of freedom of contact tested.

13.4 Remarks

This test is used as reference to the modelization L.

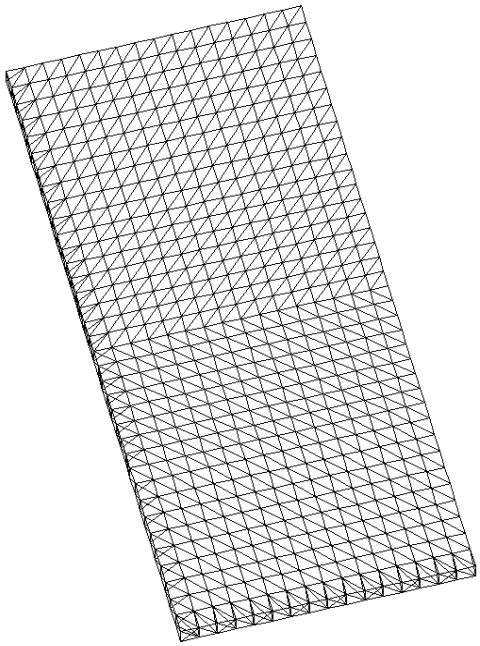
14 Modelization L

14.1 Characteristic of the modelization

It acts of the same characteristics of modelization as the modelization H.

14.2 Caractéristiques of the mesh

The mesh is regulated (Figure 14.2-a) and meshes comprises 3456 type TETRA4. Following the definition of cracks, the 192 meshes cut store with their nodes the degrees of freedom of contact friction. The mesh is built as for the modelization K, so as to respect the median symmetry of the model.



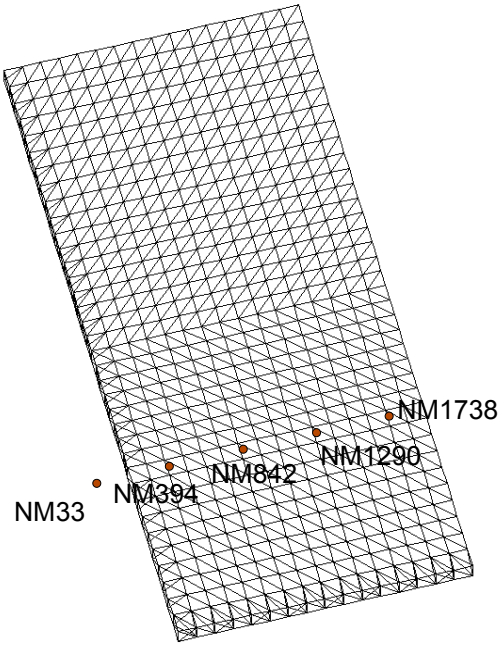
Appear 14.2-a: The mesh of the modelization L.

14.3 Quantities tested and results

One tests the values of contact pressure as for the modelization B. the site of the nodes which store the degrees of freedom of contact, which one tests the values, is illustrated on the Figure 14.4-a.
One graphically compares on the figure 14.4-b the results of approaches FEM and X-FEMs.

Not	Identification	Reference
1	LAGS_C for NM33	0.00E+000
	LAGS_C for NM394	-7.24E+003
	LAGS_C for NM842	-6.43E+003
	LAGS_C for NM1290	-5.50E+003
2	LAGS_C for NM33	0.00E+000
	LAGS_C for NM394	-2.64E+004
	LAGS_C for NM842	-1.28E+004
	LAGS_C for NM1290	-1.11E+004
3	LAGS_C for NM33	0.00E+000
	LAGS_C for NM394	0.00E+000
	LAGS_C for NM842	-1.93E+004

	LAGS_C for NM1290	-1.68E+004
4	LAGS_C for NM33	0.00E+000
	LAGS_C for NM394	0.00E+000
	LAGS_C for NM842	-4.43E+004
	LAGS_C for NM1290	-2.25E+004



Appears 14.4-a: The site of the nodes which store the degrees of freedom of contact tested.

Appear 14.4-b: SSNP503-k and L: Comparison of the reactions of contact (4 steps of loads), for methods FEM (in blue) and XFEM (in red).

14.4 Remarks

These results validate in 3D the management of integration of the contact with slipping friction when a linear relation between the degrees of freedom of contact friction is imposed in the contacting passing/ not contacting by the algorithm of stabilization of the LBB (see [R5 03 53] and [D9 05 06]).

So that the results of two approaches (methods of contact X-FEM and classical FEM) correspond, one imposes that the positions of the nodes of the mesh as well as the meshes tetrahedral ones (except the interface which is not with a grid for X-FEM) are the same ones, that integration of the contact is made on the same triangles, and that the integration of the stiffness is made on the same tetrahedrons (except for the cut elements).
However by thus making, the interfaces of method of contact FEM have almost twice more degrees of freedom (contact and displacement) that those of method of contact X-FEM. One thus does not solve exactly the same numerical problem in the 2 cases.

Taking account of that, one observes correct results which make it possible to validate the method (relative differences lower than 5%) if account of the edge effects is not taken (differences relative of about 20% on the points at the ends).

Nevertheless, these results are not as good as in 2D (comparison of the modelizations E and F). One can justify it by the fact that the reduction of the algorithm of the LBB is more severe on tetrahedrons than on triangles.

15 Modelization M

15.1 Characteristic of the modelization

It acts of a modelization X-FEM in small slidings (one does not use key word ITER_GEOM_MAXI in operator CONTACT).

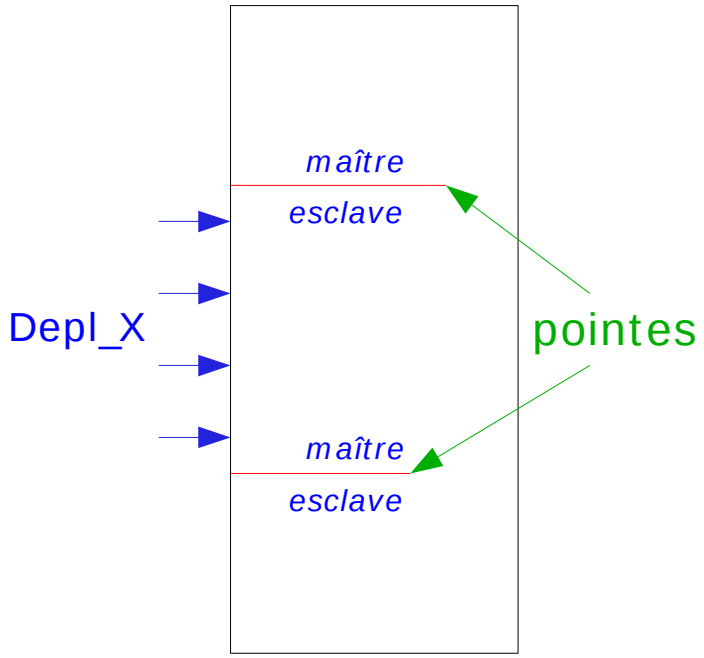
It has the same characteristics of modelization as the modelization B, but the two cracks have a point (see figure 15.1-a). One thus introduces into the modelization of cracks the level sets tangents (noted LT). The points are positioned in $\mathrm{LT}=0$.

In order to test all the possible types of pairing is the late elements $H-H$ $HCT-H$ $H-HCT$, $HCT-HCT$ and CT (see part 3 of [R5.03.53]), one inverts for the second fissures the statutes Master/slave by changing the sign of $LN2$.

The first point (defined by $LT1$) is located inside the element, whereas the second (defined by $LT2$) is on edge of the element.

Equations 4.1-1 and 4.1-2 defining the level set initially are replaced by the following ones:

$$LN1=Y-2{,}5 \qquad \text{éq 15.1-1}$$
$$LT1=X-2{,}5 \qquad \text{éq 15.1-2}$$
$$LN2=Y-6{,}5 \qquad \text{éq 15.1-3}$$
$$LT2=X-3 \qquad \text{éq 15.1-4}$$



Appears 15.1-a: The site of cracks for the modelization Mr.

Enfin, the introduction of the crack tips does not make it possible any more to impose a too large displacement according to X ; one imposes now:
$Depl_X=0.20\,m$

15.2 Characteristics of the mesh

Idem modelization B

15.3 Quantities tested and results

One tests contact pressures as for the modelization B. the site of the nodes which store the degrees of freedom of contact, which one tests the values, is illustrated on the figure 4.4-a.

Not	Identification	*Reference*
1	LAGS_C for N24	-2.1285803E+05
	LAGS_C for N33	-6.6645436E+04
	LAGS_C for N34	-2.8138419E+04
	LAGS_C for N35	-2.8138419E+04
2	LAGS_C for N24	-4.2571786E+05
	LAGS_C for N33	-1.3329046E+05
	LAGS_C for N34	-5.6276612E+04
	LAGS_C for N35	-5.6276612E+04
3	LAGS_C for N24	-6.3857768E+05
	LAGS_C for N33	-1.9993548E+05
	LAGS_C for N34	-8.4414806E+04
	LAGS_C for N35	-8.4414806E+04
4	LAGS_C for N24	-8.5143751E+05
	LAGS_C for N33	-2.6658050E+05
	LAGS_C for N34	-1.1255300E+05
	LAGS_C for N35	-1.1255300E+05

15.4 Remarks

This test is used as reference for the modelization N, and is also used as test of non regression for the contact with point in 2D small sliding.

16 Modelization N

16.1 Characteristic of the modelization

It is acted of the same characteristics of modelization as the modelization M, but as great slidings.

16.2 Characteristics of the mesh

Idem modelization B

16.3 Quantities tested and results

One tests contact pressures as for the modelization B. the site of the nodes which store the degrees of freedom of contact, which one tests the values, is illustrated on the figure 4.4-a.

Not	Identification	*Reference*
1	LAGS_C for N24	-2.1285803E+05
	LAGS_C for N33	-6.6645436E+04
	LAGS_C for N34	-2.8138419E+04
	LAGS_C for N35	-2.8138419E+04
2	LAGS_C for N24	-4.2571786E+05
	LAGS_C for N33	-1.3329046E+05
	LAGS_C for N34	-5.6276612E+04
	LAGS_C for N35	-5.6276612E+04
3	LAGS_C for N24	-6.3857768E+05
	LAGS_C for N33	-1.9993548E+05
	LAGS_C for N34	-8.4414806E+04
	LAGS_C for N35	-8.4414806E+04
4	LAGS_C for N24	-8.5143751E+05
	LAGS_C for N33	-2.6658050E+05
	LAGS_C for N34	-1.1255300E+05
	LAGS_C for N35	-1.1255300E+05

16.4 Remarks

the only difference between the two modelizations is that for the great slidings a phase of réappariement is introduced. This of results explains the differences between the two methods, since into our case one introduces a considerable sliding compared to the size of structure. It is also noticed that the difference increases linearly compared to the imposed sliding. Moreover by not imposing any sliding on these two benchmarks, the difference becomes null.

These differences of results would thus be explained by the made mistake when one neglects réappariement. That shows the utility to take into account the great slidings, even on zones close to the points of cracks.

17 Modelization O

17.1 Characteristic of the modelization

It acts of the same characteristics of modelization as the modelization M, but in 3D. One thus extrudes the rectangle considered in the depth of the plane a unit length. One blocks rigid body motions in the new direction considered.

17.2 Characteristics of the mesh

Idem modelization H.

17.3 Grandeurs tested and results

One test contact pressures as for the modelization B. the site of the nodes which store the degrees of freedom of contact, which one tests the values, is illustrated on the figure 10.4-a.

Not	Identification	*Reference*
1	LAGS_C for N13	-1.4922085E+05
	LAGS_C for N14	-4.4964660E+04
	LAGS_C for N15	-1.1560082E+04
	LAGS_C for N16	-1.1560082E+04
2	LAGS_C for N13	-2.9844255E+05
	LAGS_C for N14	-8.9929003E+04
	LAGS_C for N15	-2.3119982E+04
	LAGS_C for N16	-2.3119982E+04
3	LAGS_C for N13	-4.4766424E+05
	LAGS_C for N14	-1.3489335E+05
	LAGS_C for N15	-3.4679883E+04
	LAGS_C for N16	-3.4679883E+04
4	LAGS_C for N13	-5.9687934E+05
	LAGS_C for N14	-1.7986014E+05
	LAGS_C for N15	-4.6241201E+04
	LAGS_C for N16	-4.6241201E+04

17.4 Remarks

This test is used as reference for the modelization P, and is also used as test of non regression for the contact with crack tip in 3D small sliding.

18 Modelization P

18.1 Characteristic of the modelization

It is acted of the same characteristics of modelization as the modelization O, but as great slidings.

18.2 Characteristics of the mesh

Idem modelization H.

18.3 Grandeurs tested and results

One test contact pressures as for the modelization B. the site of the nodes which store the degrees of freedom of contact, which one tests the values, is illustrated on the figure 10.4-a.

Not	Identification	*Reference*
1	LAGS_C for N13	-1.4922085E+05
	LAGS_C for N14	-4.4964660E+04
	LAGS_C for N15	-1.1560082E+04
	LAGS_C for N16	-1.1560082E+04
2	LAGS_C for N13	-2.9844255E+05
	LAGS_C for N14	-8.9929003E+04
	LAGS_C for N15	-2.3119982E+04
	LAGS_C for N16	-2.3119982E+04
3	LAGS_C for N13	-4.4766424E+05
	LAGS_C for N14	-1.3489335E+05
	LAGS_C for N15	-3.4679883E+04
	LAGS_C for N16	-3.4679883E+04
4	LAGS_C for N13	-5.9687934E+05
	LAGS_C for N14	-1.7986014E+05
	LAGS_C for N15	-4.6241201E+04
	LAGS_C for N16	-4.6241201E+04

18.4 Remarks

Idem modelization N.

19 Modelization Q

19.1 Characteristic of the modelization

It acts of the same characteristics of modelization as the modelization F but by means of the penalized formulation.

19.2 Characteristics of the mesh

It is the same mesh as that of the modelization F.

19.3 Quantities tested and results

One tests the values of contact pressure with the same values of reference and tolerances that the modelization F. the site of the nodes which store the degrees of freedom of contact, which one tests the values, is illustrated on the Figure 8.4-a. It is checked that one gets results identical to the Lagrangian method.

Not	Identification	Reference	tolerance
1	LAGS_C for N33	0.0000000E+00	1.00E-010
	LAGS_C for N154	-7.1942608E+03	5.00%
	LAGS_C for N230	-6.5408344E+03	5.00%
	LAGS_C for N306	-6.1360067E+03	5.00%
	LAGS_C for N677	-2.4914329E+03	5.00%
2	LAGS_C for N33	0.0000000E+00	1.00E-010
	LAGS_C for N154	-2.4822528E+04	5.00%
	LAGS_C for N230	-1.2993429E+04	5.00%
	LAGS_C for N306	-1.2178296E+04	5.00%
	LAGS_C for N677	-3.4642908E+03	5.00%
3	LAGS_C for N33	0.0000000E+00	1.00E-10
	LAGS_C for N154	0.0000000E+00	1.00E-10
	LAGS_C for N230	-2.0219781E+04	5.00%
	LAGS_C for N306	-1.8168929E+04	5.00%
	LAGS_C for N677	-4.8271686E+03	6.50%
4	LAGS_C for N33	0.0000000E+00	0.00E+00
	LAGS_C for N154	0.0000000E+00	0.00E+00
	LAGS_C for N230	-4.3387251E+04	5.00%
	LAGS_C for N306	-2.4197889E+04	5.00%
	LAGS_C for N677	-7.2647040E+03	6.50%

20 Summary of the results

the purposes of this test are reached:

- They are of the contact to show the feasibility of the taking into account and friction on the lips horizontal cracks with the approach great slidings X-FEM.
- One also shows in 2D and 3D the effectiveness of the algorithms implemented to improve the results when the linear relations introduced on the degrees of freedom of contact by algorithm LBB enter in conflict with a change of contacting statute/not contacting (modelization F in 2D and modelization L in 3D).
- The approach was validated with the taking into account of frictions in 2D (plane strains and plane stresses, elements QUAD4 and TRI3) and in 3D (elements HEXA8, PENTA6 and TETRA4)

- It is possible to define interfaces (modelizations B, D, F, H, J and L) but also of cracks not cutting entirely the structure (modelizations N and P).