# SSNP306 - Validation of the criterion of buckling per selective search of the eigenvalues

**Summarized:**

This test allows the validation of the generalization of the criterion of buckling (loss of unicity of the solution, presence of bifurcation) for the models in mixed formulations. The new criterion which is written as a problem with the under-forced eigenvalues is validated for the modelization of damage NON-room GRAD_VARI. One compares the smallest modes of formulation GRAD_VARI and those of the local modelization in the case of a homogeneous study.

# 1 Problem of reference

## 1.1 Tallies theoretical

the criterion of usual buckling results in a checking of sign of the smallest eigenvalue of the tangent matrix $K\,\delta u = \mu\,\delta u$ . If $\mu \leq 0$ the solution of the mechanical problem is not single any more.

For the mixed formulations, the eigen modes associated with the presence with coefficients with Lagrange must be eliminated. This brings back for us towards a problem to the eigen modes under stress.

$K\begin{pmatrix}\delta u\\ \delta\lambda\end{pmatrix}=\mu\begin{pmatrix}\delta u\\ 0\end{pmatrix}$ is the new criterion of buckling for a mixed modelization.

In case GRAD_VARI the criterion is even more restrictive, because it is also necessary to take into account the constitutive law to obtain physical modes. This must be able to spread with other types of modelizations. The criterion is written then like a problem with the eigenvalues generalized:

$K\begin{pmatrix}\delta u\\ \delta\alpha\\ \delta\lambda\end{pmatrix}=\mu\begin{pmatrix}\delta u\\ 0\\ 0\end{pmatrix}$ new criterion of buckling for a modelization GRAD_VARI.

That provides us two stresses of equalities in the search of the mode in displacement. The first enables us to check the constitutive law of the material and the second the equality of the disturbances of the damage to the nodes with that calculated locally with Gauss points.

We validate the new criterion of buckling by making a comparison between the smallest modes of formulation GRAD_VARI and those of the local modelization in the case of a homogeneous study.

Note:
Technically in order to arrive there new commands RIGI_GEOM and DDL_EXCLUS were added in operator CRIT_STAB . One uses them in this precise case in the following way:

```
CRIT_STAB = _F (RIGI_GEOM = "NON", # Necessary to call DDL_EXCLUS then
                DDL_EXCLUS = ("VARI", "LAG_GV"))
```

## 1.2 Geometry

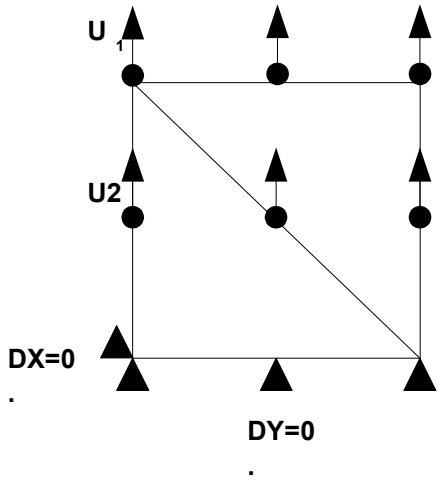


**Figure 1** : Representation of the problem in two meshes quadratic

## 1.3 Properties of the material

Damage model: elastic `material`

ENDO_FRAGILE Characteristics:

- $E=3\cdot 10^4\,\mathrm{Pa}$
- $\nu=0.25$

Characteristics related to the damage model:

- Elastic limit: $\sigma^Y=3.0\,\mathrm{Pa}$
- Slope of softening: $E^T=-1.95\cdot 10^3\,\mathrm{Pa}$
- NON-local characteristics: $L_c=1.0$ ; $r=100.$

## 1.4 Boundary conditions and loadings

**Fixed support** : Null displacements imposed $DY=0.$ on stops horizontal bottom ( $y=0.$ ) and $DX=0.$ on the node far left ( $x=y=0.$ ). See figure1.

**Loading 1** : Displacement imposed $U_1$ on stops horizontal top ( $y=1.$ ):
$DY=2.10^{-6}\mathrm{t}$

**Loading 2** : Displacement imposed $U_2$ on all the nodes of coordinates $y=0.5$ :
$DY=1.10^{-6}\mathrm{t}$

# 2 Reference solution

the same study is made in room and nonroom. One compares the smallest eigenvalues of the tangent operator calculated with the criterion of standard buckling in room (which one will take as reference) and with the new criterion in nonroom. The modelization `D_PLAN` is used locally, modelization `D_PLAN_GRAD_VARI` is used in nonroom. The loading is carried out over a period of 100 time step, of kind to visualize the transition by a state of damage NON-no one of structure. The successful test is considered, if the relative variation does not exceed 0.01%.

# 3 Modelization A

## 3.1 Characteristic of the mesh

The mesh consists of 2 TRIA6 as presented of Figure 1.

## 3.2 Quantities tested and results

| NUME_ORDRE | Reference | *Aster* | % difference | Tolerance |
|---|---|---|---|---|
| 100 | 980.12 | 981.05 | 0.095% | 0.02% |
| 90 | 1142.93 | 1056.03 | 0.093% | 0.02% |
| 80 | 1142.93 | 1143.95 | 0.089% | 0.02% |
| 70 | 1249.79 | 1249.79 | 0.084% | 0.02% |
| 60 | 1381.74 | 1381.74 | 0.076% | 0.02% |
| 50 | 1553.5 | 1554.52 | 0.066% | 0.02% |
| 40 | 1641.67786584879 | 1641.67786584879 | 0.0E+00% | 0.02% |
| 1 | 1641.67786584879 | 1641.67786584879 | 1.4E-14% | 0.02% |

**Table 1: Comparison of eigenvalues in room and NON-room**

# 4 Summaries of the results

One finds identical results in modelization local and nonlocal, which validates the introduction of the new criterion of buckling for the mixed problems.