SSNP159 – Elastic strain energy in plastic large deformations of a tensile test bar

Summarized:

This quasi-static mechanical test consists to subject to a simple tension a bar of section rectangular ($3D$) or cylindrical ($2D$ axisymmetric). The object is to validate the computation of elastic strain energies in three formalisms of strain: “PETIT”, “GDEF_LOG” and “SIMO_MIEHE”, with command POST_ELEM.

The bar is modelled by an element voluminal (HEXA20, modelization A) or quadrangular (QUAD4, for an axisymmetric modelization, modelization B).

The solution is analytical.

1 Problem of reference

1.1 Geometry

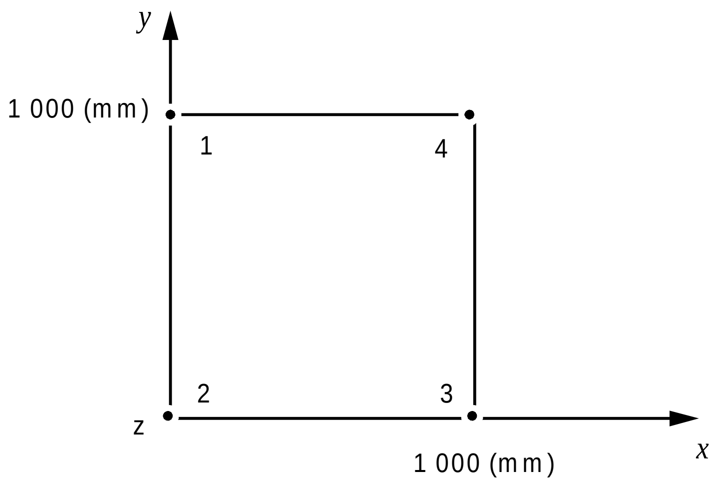

Figure 1.1-1: Geometry of reference

1.2 Properties of the material

the material obeys a plastic constitutive law in large deformations with linear isotropic hardening. Curve of tension is given in the logarithmic strain plane - rational stress.

$$\sigma=\frac{F}{S}=\frac{F}{S_0}.\frac{l}{l_0}$$

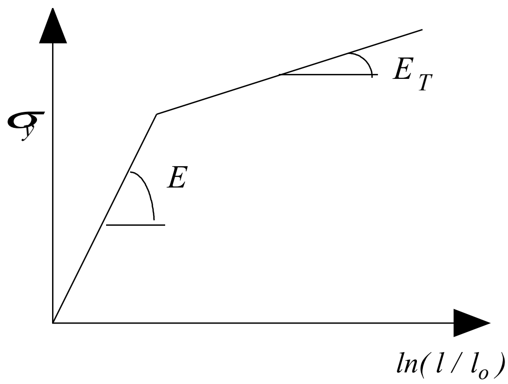

Figure 1.2-1: Curve of tension

$$\nu=0{,}3$$
$$E=200000\text{MPa}$$
$$E_T=2000\text{MPa}$$
$$\sigma_y=1000\text{MPa}$$

l_0 and l are, respectively, the initial length and the current length of the useful part of the test-tube.

S_0 and S are, respectively, initial and current surface.

1.3 Boundary conditions and loadings

the bar, initial length l_0 , blocked in the direction Ox on the face [1,2] with a mechanical displacement of tension u^{meca} varying linearly in time on the face $[3,4]$:

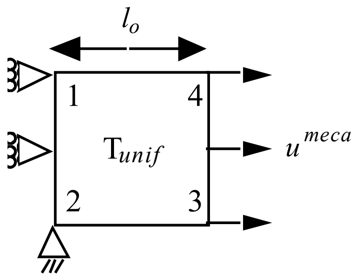

Figure 1.3-1: Limiting conditions and loading

2 Reference solution

the analytical solution of SIMO_MIEHE makes it possible to define the loading to be applied to obtain a stress of Kirchhoff of $1500\mathrm{MPa}$. This loading is then applied to the other models.

2.1 Result generic with the formalisms

For a uniaxial traction test according to the direction $\boldsymbol{x}$, the stress tensors of Kirchhoff $\boldsymbol{\tau}$ and Cauchy $\boldsymbol{\sigma}$ are form:

$$\boldsymbol{\sigma}=\begin{bmatrix}\sigma & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix},\boldsymbol{\tau}=J\,\boldsymbol{\sigma}=\begin{bmatrix}\tau & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix},$$

The tensor gradients of the transformation $\mathbf{F}$ and $\overline{\mathbf{F}}$ are expressed:

$$\boldsymbol{F}=\begin{bmatrix}F & 0 & 0\\ 0 & F_y & 0\\ 0 & 0 & F_y\end{bmatrix},\overline{\boldsymbol{F}}=J^{\frac{-1}{3}}=\begin{bmatrix}\bar{F} & 0 & 0\\ 0 & \bar{F}_y & 0\\ 0 & 0 & \bar{F}_y\end{bmatrix}$$

Displacement checks:

$$F=1+\frac{u^{meca}}{l_0}$$

The function of isotropic hardening linear is written:

$$R(p)=\frac{E\,E_T}{E-E_T}$$

2.2 Results for Simo_Miehe

the isochoric tensor of plastic strains G^P is form:

$$\boldsymbol{G^P}=\begin{bmatrix}G^P & 0 & 0\\ 0 & G_y^P & 0\\ 0 & 0 & G_y^P\end{bmatrix}$$, with $G_y^P=\frac{1}{\sqrt{G^P}}$ because $det(\boldsymbol{G^P})=1$

the constitutive law (left hydrostatic) is written:

$$tr(\boldsymbol{\tau})=\frac{3\mathrm{K}}{2}(J^2-1)\Rightarrow J^2=\frac{2\,\tau}{3\mathrm{K}}+1$$

The function threshold of plasticity is written:

$$f=0=\tau-R(p)-\sigma_y\Rightarrow p=\frac{E-E_T}{E\,E_T}(\tau-\sigma_y)$$

The model of yielding is written:

$$\overline{\boldsymbol{F}}.\dot{\boldsymbol{G}}^{\boldsymbol{P}}.\overline{\boldsymbol{F}}^{\boldsymbol{T}}=-3\,\dot{p}\frac{1}{\tau_{eq}}dev(\boldsymbol{\tau}).(\overline{\boldsymbol{F}}.\boldsymbol{G^P}.\overline{\boldsymbol{F}}^{\boldsymbol{T}})$$

By taking the first component, one obtains:

$\frac{\dot{G}^P}{G^P}=-2\,\dot{p}\Rightarrow G^p=e^{-2p}$ because $G^P(0)=1$

to conclude the problem, one uses the first component of the deviatoric part of the stress:

$$dev(\boldsymbol{\tau})=\mu\, dev(\bar{\boldsymbol{b}}^e)\Rightarrow\tau=\mu\left(\bar{F}^2 G^P-\bar{F}_y^2 G_y^P\right)\Rightarrow\bar{F}^3-\bar{F}\frac{\tau}{\mu G^P}-\frac{1}{\left(G^P\right)^{(3/2)}}=0\ ,$$

because $\bar{\boldsymbol{b}}^e=\bar{F}\,.\,\boldsymbol{G}^P\,.\,\bar{\boldsymbol{F}}^T$

elastic strain energy is written then:

$$\psi_{SM}^{elas}=\frac{K}{2}\left[\frac{J^2-1}{2}-\ln J\right]+\frac{\mu}{2}\left[tr\,\bar{\boldsymbol{b}}^e-3\right]=\frac{K}{2}\left[\frac{J^2-1}{2}-\ln J\right]+\frac{\mu}{2}\left[\bar{F}^2 G^P+\frac{2}{\bar{F}\sqrt{G^P}}-3\right]$$

For a stress of Kirchhoff of 1500MPa, can then successively determine:

$J=1{,}003$
$\sigma=1495\,MPa$
$p=0{,}2475$
$G^p=0{,}6096$
$\bar{F}=1{,}289$
$F=1{,}290$
$u^{meca}=290\,mm$
$\psi_{SM}^{elas}=5{,}63\,MPa$ at the material point.

The displacement applied for the two modelizations and the 3 formalisms will be thus $u^{meca}=290\,mm$.

2.3 Results for GDEF_LOG

the logarithmic strain is written:

$$\boldsymbol{E}_{\log}=\frac{1}{2}\ln[\boldsymbol{C}]=\frac{1}{2}\ln[\boldsymbol{F}^T.\boldsymbol{F}]=\begin{bmatrix}\ln F & 0 & 0\\ 0 & \ln F_y & 0\\ 0 & 0 & \ln F_y\end{bmatrix}$$

The left higher quarter of the projector lagragien from of deduced, in notation of Voigt:

$$\boldsymbol{P}=2\frac{\partial\boldsymbol{E}_{\log}}{\partial\boldsymbol{C}}=\frac{1}{2}\begin{bmatrix}\frac{2}{F^2} & 0 & 0\\ 0 & \frac{2}{F_y^2} & 0\\ 0 & 0 & \frac{2}{F_y^2}\end{bmatrix}$$

And because of statement of the second Piola-Kirchhoff stress:

$$\boldsymbol{S}=\boldsymbol{T}:\boldsymbol{P}=\boldsymbol{F}^{-1}.\boldsymbol{\tau}\,.\,\boldsymbol{F}^{-T}\Rightarrow\boldsymbol{T}=\boldsymbol{\tau}$$

The constitutive law is written:

$T=E(\ln F-p)$

Because of the threshold of plasticity:

$$f=0=T-R(p)-\sigma_y \Rightarrow p=\frac{E-E_T}{E\,E_T}(T-\sigma_y)=\frac{E\ln F-\sigma_y}{E+\frac{E\,E_T}{E-E_T}}\ ,$$

The elastic strain energy of this formalism is thus written:

$$\psi_{\log}^{elas}=\frac{1}{2}\frac{T^2}{E}$$

Imposed displacement $u^{meca}=290\,mm$, one deduces:

$$F=1{,}290$$
$$\ln F=0{,}255$$
$$p=0{,}2475$$
$$T=1500\text{MPa}$$
$$\sigma=1495\text{MPa}$$
$\psi_{\log}^{elas}=5{,}625\,MPa$ at the material point.

2.4 Results in small strains

In small strains, result is classical.
Axial strain:

$$\varepsilon_x=\frac{u^{meca}}{l_0}$$

Behavior:

$$\sigma=E(\varepsilon_x-p)$$

Function threshold:

$$\sigma-R(p)-\sigma_y=0$$

From where:

$$p=\frac{E\,\varepsilon_x-\sigma_y}{E+\frac{E\,E_T}{E-E_T}}$$

$$\sigma=E(\varepsilon_x-p)$$

Elastic strain energy:

$$\psi_{HPP}^{elas}=\frac{\sigma^2}{2E}$$

Imposed displacement $u^{meca}=290\,mm$, one deduces:

$$\varepsilon_x=0{,}029$$
$$p=0{,}281$$
$$\sigma=1570\text{MPa}$$
$\psi_{\log}^{elas}=6{,}16\,MPa$ at the material point.

2.5 Tests carried out

For each of the formalisms and each modelization, one tests the values of imposed displacement, the stress of Cauchy σ , the cumulated plastic strain p and elastic strain energy.

Attention, elastic strain energy tested is the value for the bar (not of the material point). For the axisymmetric modelization, it is thus necessary multiplied the value of the elastic strain energy of the material point par. $\frac{\pi R^2}{2\pi}$

3 Modelization A

3.1 Characteristic of the voluminal

modelization Modelization: 1 mesh HEXA20
1 nets QUAD8

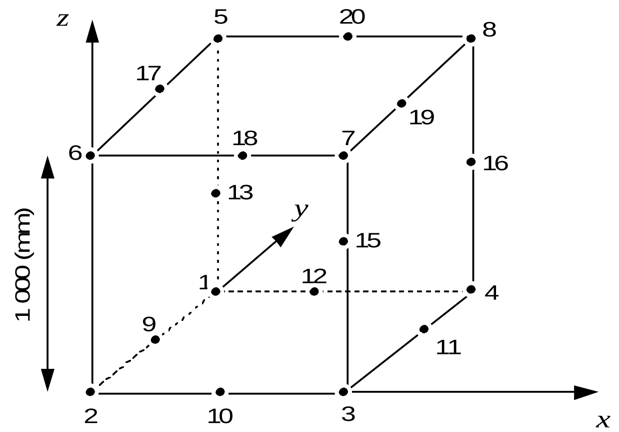

Figure 3.1-1: Mesh of the modelization A

Boundary conditions:

$N2$:	$U_x=U_y=U_z=0$	$N9\quad N13\quad N14\quad N5\quad N17$: $U_x=0$
$N1$:	$U_x=U_z=0$	
$N6$:	$U_x=U_y=0$	

Table 3.1-1: Limiting conditions modelization A

Load: Tension on the face $[3,4,8,7,11,16,19,15]$

the nombre total of increments is of 20 (20 increments enters $t=0\text{s}$ and 2s)
convergence is carried out if residue RESI_GLOB_RELA is lower or equal to 10-6.

3.2 Characteristics of the mesh

Many nodes: 20

Number of meshes: 2

1 HEXA20
1 QUAD8

3.3 Quantities tested and results

Identification	Reference			Tolerance
	SIMO_MIEHE	**GDEF_LOG**	**HP**	
$t=2$ Displacement DX ($N8$)	290.290.290	290	290	1.00%
$t=2$ Stresses $SIGXX$ ($PG1$)	1495	1495	1570	1.00%
$t=2$ Variable p $VARI$ ($PG1$)	0,2475	0,2475	0,282	1.50%
$t=2$ ENER ELAS, TOTAL	5.63E+009	5,625E9	6,16E9	5.00%

Table 3.3-1: Results of the modelization A

4 Modelization B

4.1 Characteristic of the axisymmetric

modelization $2D$ Modelization: 1 mesh QUAD4
1 nets SEG2

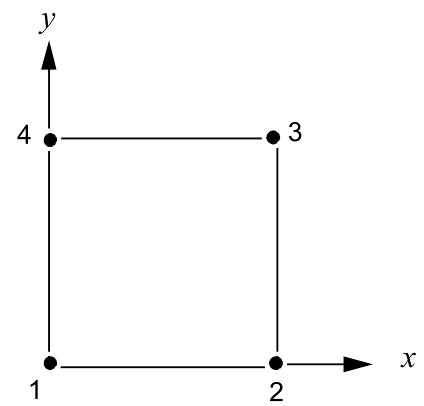

Figure 4.1-1: Mesh of the modelization B

Boundary conditions:

$N1$:	$U_Y=0$
$N2$:	$U_Y=0$

Table 4.1-1: Limiting conditions of the modelization B

Loading:

Tension on the face $[3,4]$ (mesh SEG2)
the nombre total of increments is of 20 (20 increments enters $t=0\text{s}$ and 2s)
convergence is carried out if residue RESI_GLOB_RELA is lower or equal to 10-6.

4.2 Characteristics of the mesh

Many nodes: 4
Number of meshes: 2

1 QUAD4
1 SEG2

4.3 Quantities tested and results

Identification	Reference			Tolerance
	SIMO_MIEHE	**GDEF_LOG**	**HP**	
$t=2$ Displacement DX ($N8$)	290.290.290	290	290	1.00%
$t=2$ Stresses $SIGXX$ ($PG1$)	1495	1495	1570	1.00%
$t=2$ Variable p $VARI$ ($PG1$)	0,2475	0,2475	0,282	1.50%
$t=2$ ENER ELAS, TOTAL	2.82E+009	2,81 E9	3,08 E9	5.00%

Table 4.3-1: Results of the modelization B

5 Summary of the results

the results found in term of elastic strain energy with *Code_Aster* are very satisfactory in small strains and logarithmic strains, with variations with analytical the lower than 0,1%, and correct in strain of SIMO_MIEHE, with a variation with analytical of approximately 3%, due at the end $tr(\bar{\boldsymbol{b}}^e)$, on which the fifth decimal plays a significant part.