SSNP150 – Method of the solutions manufactured in contact 2D and large deformations

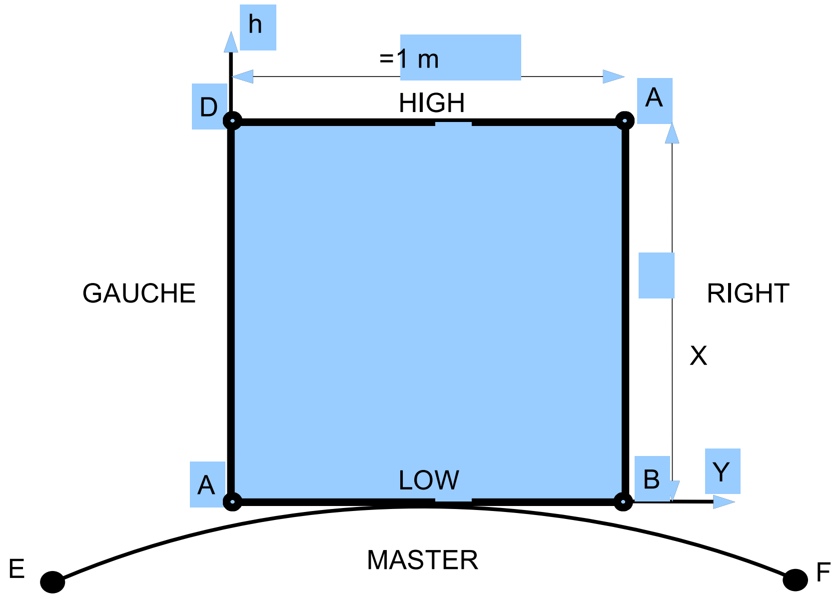
Summarized:

The purpose of this test is to 2D check the modelization of contact in large deformations thanks to the method of the manufactured solutions [bib1].

1 Problem of reference

1.1 Geometry

One considers a square of with dimensions $1\,m$.



1.2 Properties of the material

$E=1\mathrm{MPa}$	Modulus Young
$\nu=0.3$	Poisson's ratio

1.3 Boundary conditions and loadings

On edge HAUT , one forces a displacement (see paragraph 3.
On edges GAUCHE , BAS and DROITE , one forces a pressure (see paragraph 3.
In all the field, one forces a body force (see paragraph 3.
The surface MAITRE of natural paraboloid is described by the equation:

$$Y=-0.05\times(X-0.5)^2 \quad (1)$$

1.4 Initial conditions

Nothing

2 Reference solution

2.1 Method of calculating

the analytical reference solution is given by:

$$\begin{aligned} Ux &= -0.2\times Y\times Y\times Y\times(X-0.5) \\ Uy &= -0.05\times(X-0.5)\times(X-0.5)\times(1+Y)-0.01\times Y \end{aligned} \quad (2$$

the conditions of Dirichlet, Neumann and the source term are obtained by the method of the manufactured solutions [bib1].
One starts by determining the gradient of the transformation $\underline{\underline{F}}$:

$$\underline{\underline{F}}=\nabla \underline{U}+\underline{\underline{Id}} \quad (3)$$

Knowing the norm $\underline{N}=[0,-1]^T$ on the surface slave in the NON-deformed configuration, one obtains his statement in the configuration deformed by the formula of Nanson:

$$\underline{n}=\frac{\underline{\underline{F}}^{-T}\underline{N}}{\|\underline{\underline{F}}^{-T}\underline{N}\|} \quad (4)$$

Knowing the tensor of Hooke $\underline{\underline{\underline{\underline{A}}}}$ and the tensor of Green-Lagrange $\underline{\underline{E}}$, one calculates the second tensor of Piola-Kirchhoff $\underline{\underline{S}}$:

$$\underline{\underline{E}}=\frac{1}{2}(\underline{\underline{F}}^T\cdot\underline{\underline{F}}-\underline{\underline{Id}}) \quad (5)$$

$$\underline{\underline{S}}=\underline{\underline{\underline{\underline{A}}}}:\underline{\underline{E}} \quad (6)$$

It is pointed out that the second tensor of Piola-Kirchhoff $\underline{\underline{S}}$ makes it possible to obtain forces in undistorted configuration per not deformed unit of area:

$$\frac{d\,f_0}{dA}=\underline{\underline{S}}\cdot\underline{N} \quad (7)$$

As we seek to determine forces in deformed configuration, we will determine the first tensor of Piola-Kirchhoff $\underline{\underline{\Pi}}$

$$\underline{\underline{\Pi}}=\underline{\underline{F}}\cdot\underline{\underline{S}} \quad (8)$$

One can thus determine the body forces $\underline{f}_{vol}$:

$$\underline{f}_{vol}=-div\,\underline{\underline{\Pi}} \quad (9)$$

Knowing the norm in initial configuration on the various sides and the first tensor of Piola-Kirchhoff $\underline{\underline{\Pi}}$, one can calculate the forces of surface in deformed configuration:

$$\underline{f}_{surf}=\underline{\underline{\Pi}}\cdot\underline{N} \tag{10}$$

On the surface BAS which is in contact, one needs a particular processing. Indeed, the normal force is taken there into account by the contact:

$$\begin{aligned}\underline{f}_{surf}^{BAS} &= \underline{f}_{surf_n}^{BAS} + \underline{f}_{surf_t}^{BAS}\\ &= \underline{f}_{contact} + \underline{f}_{surf_t}^{BAS}\\ &= p*\underline{n} + \underline{f}_{surf_t}^{BAS}\end{aligned} \tag{11}$$

Where p the contact pressure indicates. It can be given by the statement:

$$p=(\underline{\underline{\Pi}}\cdot\underline{N})\cdot\underline{n} \tag{12}$$

One thus should apply only the tangential stresses to it. One calculates them by the statement:

$$\begin{aligned}\underline{f}_{surf_t}^{BAS} &= \underline{f}_{surf}^{BAS}-\underline{f}_{surf_n}^{BAS}\\ &= \underline{f}_{surf}^{BAS}-(\underline{f}_{surf_n}^{BAS}\cdot\underline{n})\underline{n}\end{aligned} \tag{13}$$

Concerning the forces of contact, *it is absolutely essential to build the solution manufactured so that they check the equations of the contact* [bib2], namely:

$$\begin{aligned} gap(\underline{U}) &\geq 0\\ p &\leq 0\\ p\cdot gap(\underline{U}) &= 0\end{aligned} \tag{14}$$

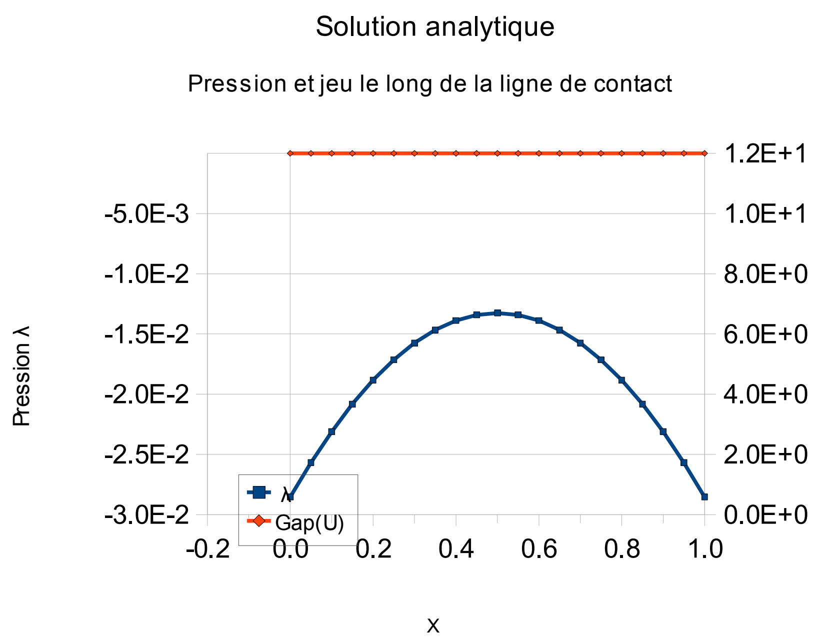


Figure 2.1-1: checking has postérori validity of the solution

This checking is done after having calculated in an analytical way the pressure and the jump of displacement associated with the manufactured solution, in general with a formal computational tool (in fact, it is the modulus Python *sympy*). One must then visualize them, in order to check *retrospectively* that the solution which one has contruite checks well (14). In the case of this test, we represented pressure and analytical jump of displacement in fig.2.1-12.1-1. One notices that it check $p<0$ and $\mathrm{gap}(\underline{U})=0$, which is characteristic of a surface entirely contacting, and conforms to (14).

2.2 Quantities and results of reference

the value of the difference between solutions analytical and calculated on mesh: $\sum^{\text{noeuds } n} |\underline{U}_{\mathrm{n}}^{\mathrm{calc}}-\underline{U}_{\mathrm{n}}^{\mathrm{ref}}|$
.
In the case of the modelizations which carry out an analysis of convergence with the smoothness of the mesh, velocity of convergence with the smoothness of the mesh of the solution calculated towards the analytical solution in norm L_2 :

–the greatest reality $\alpha_U>0$ such as $\|\underline{U}^{\mathrm{calc}}-\underline{U}^{\mathrm{ref}}\|_{0,\Omega}<C_U\times h^{\alpha_U}$ where C_U is independent of h for displacement;

–the greatest reality $\alpha_p>0$ such as $\|p^{\mathrm{calc}}-p^{\mathrm{ref}}\|_{0,\Gamma_c}<C_p\times h^{\alpha_p}$ where C_p is independent of h for the contact pressure.

2.3 Uncertainties on the solution

No

2.4 bibliographical References

1 Document U2.08.08, Use of the Method of the Solutions Manufactured for the software validation, Documentation U2 of Code_Aster

2 R5.03.50 Document, discrete Formulation of contact-friction, Documentation R of Code_Aster

3 Modelization A

3.1 Characteristic of the modelization

One uses a modelization D_PLAN.

3.2 Characteristics of the mesh

The mesh contains 65 elements of the type SEG3 and 256 elements of the type QUAD8.
Curved surface Master is represented by a single SEG3 .

3.3 Quantities tested and results

One tests the sum of the absolute values of the difference between the calculated solution and the analytical solution.

Standard	identification of reference	Value of reference
$\sum^{\text{noeuds } n} \lvert \underline{U}_{\text{n}}^{\text{calc}} - \underline{U}_{\text{n}}^{\text{ref}} \rvert$	"NON_REGRESSION"	4.03888411513E-05

4 Modelization B

4.1 Characteristic of the modelization

One uses a modelization D_PLAN.

4.2 Characteristics of the mesh

One carries out a study of convergence with the smoothness of the mesh of the solution calculated towards the analytical solution. A succession of meshes obtained by uniform refinement using command MACR_ADAP_MAIL is used:

- mesh 0: 5 SEG3, 1 QUAD8
- mesh 1: 9 SEG3, 4 QUAD8
- mesh 2: 17 SEG3, 16 QUAD8
- mesh 3: 32 SEG3, 64 QUAD8
- mesh 4: 64 SEG3, 256 QUAD8

surface main curves is represented by a single SEG3 .

4.3 Quantities tested and results

One tests the velocity of convergence with the smoothness of the mesh of the solution calculated towards the analytical solution in norm L_2 :

-the greatest reality $\alpha_U > 0$ such as $\|\underline{U}^{\text{calc}} - \underline{U}^{\text{ref}}\|_{0,\Omega} < C_U \times h^{\alpha_U}$ where C_U is independent of h for displacement;

-the greatest reality $\alpha_p > 0$ such as $\|p^{\text{calc}} - p^{\text{ref}}\|_{0,\Gamma_c} < C_p \times h^{\alpha_p}$ where C_p is independent of h for the contact pressure.

One tests also the sum of the absolute values of the difference between the calculated solution and the analytical solution for displacement.

Standard	identification of reference	Value of reference
$\sum^{\text{noeuds } n} \lvert \underline{U}_n^{\text{calc}} - \underline{U}_n^{\text{ref}} \rvert$	"NON_REGRESSION"	6.62799356621E-07
α_U	"ANALYTIQUE"	3.0
α_p	2.5 "formulates	"
α_p	ANALYTIQUE NON_REGRESSION"	2.8035

5 Summary of the results

the results are in very good agreement with the theory.