SSNP130 - Detection of the singularities in a fissured plate

Summarized:

The goal of this test is to test the detection of the singularities in a plate fissured as well as the computation of card of size (for an error targets given).

1 Problem of reference

1.1 Geometry

the geometry is a fissured plate of which one models only the upper part.

1.2 Properties of the material

the solid mass consists of a linear isotropic elastic material:

- Isotropic elasticity Young Modulus: $E=200000\,Pa$
- Poisson's ratio: $\nu=0,3$

1.3 Boundary conditions and loadings

One imposes a unit pressure on the top of the plate.

For reasons of symmetry, the ligament of crack is blocked according to the axis Y and the node at the end of ligament is blocked according to the axis X.

2 Reference solution

2.1 Method of calculating

When the exact solution of the studied problem present of the singularities, the order of convergence of the solution finite elements is modified. Let us consider, for example, a problem of plane elasticity discretized with triangular elements of degree p .
If the exact solution U_{ex} is regular, it is known that ([bib1]):

$$\|u-u_h\|_\Omega=\|e\|_\Omega\leq C\,h^p \qquad \text{éq 2.1-1}$$

With C a constant, h size of the elements.

Where $\|e\|_\Omega\leq C\,h^p$ is the contribution to the error in energy, that is to say:

$$\|e\|_\Omega\leq\frac{1}{2}\int_\Omega \varepsilon\left(e_h\right)\ K\ \varepsilon\left(e_h\right)d\,\Omega \qquad \text{éq 2.1-2}$$

On the other hand, if the exact solution presents a singularity, for example if, locally in the vicinity of a point M_0 , the field of displacement is form (with r and θ polar coordinates in the vicinity of the point M_0):

$$U_{ex}=r^\alpha V\left(\theta\right)+W \text{ with } 0<\alpha<1 \qquad \text{éq 2.1-3}$$

With V a function of θ and W a constant.

Then, it is shown that [bib1]:

$$\|e_h\|_\Omega\leq C\,h^\alpha \qquad \text{éq 2.1-4}$$

It results from it that the rate of convergence of the total error in energy becomes independent of the degree p of the finite elements used and it is the same of that of the measurement of the error (for example, if $p=1$ or $p=2$ then $\alpha=1/2$ for a crack).

Thus, at a peak of crack the order of the singularity will be worth 0.5, and far from the singularity (where the solution finite element is regular the order of the singularity is worth p (1 for the linear elements, 2 for the quadratic elements)

2.2 Quantities and results of reference

One will test the value of the singularity at a peak of crack (analytical solution), in its vicinity (NON-regression) and far from the crack (analytical solution).

One also tests the ratio of size to be applied to the mesh for a target error (NON-regression) and the new size of the elements (NON-regression).

It is noted that for the modelization A, the target error is the error (in quantity of interest) on average displacement on the structure. For the other modelizations, the error used is the error in norm of energy.

2.3 Uncertainty on the solution

analytical Solution and NON-regression

2.4 bibliographical References

[1] STRANG & FIX: Year analysis of the finite element method, Prentice hall, 1976.

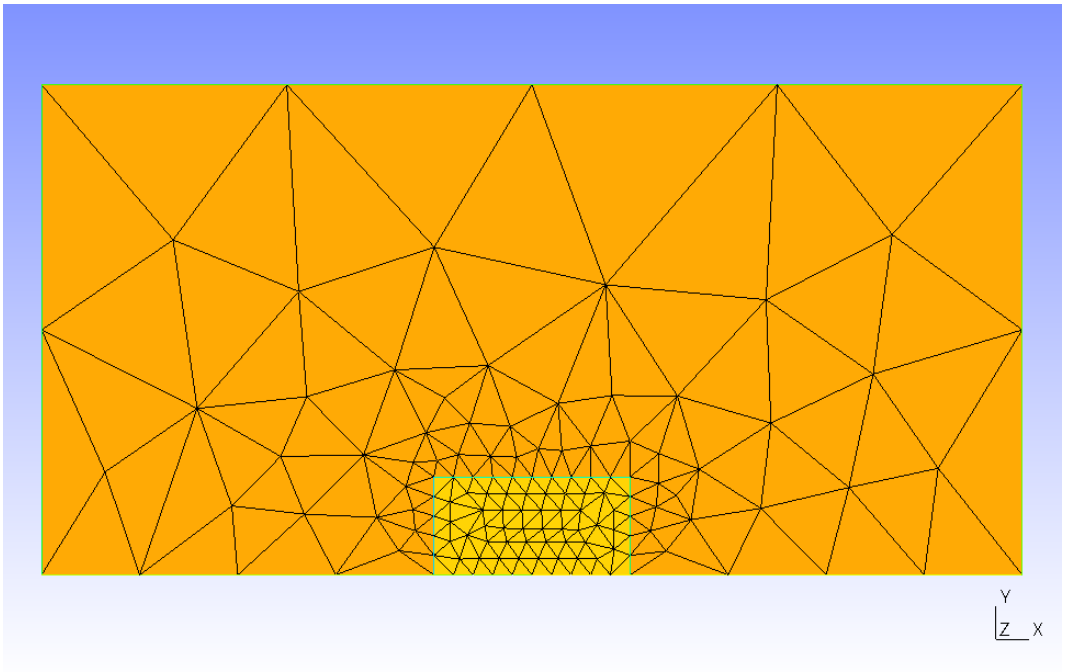
3 Modelization A

3.1 Characteristic of the modelization

The modelization is two-dimensional elastic with plane stresses C_PLAN.

3.2 Characteristics of the mesh

Number of nœuds136
Many SEG246
Many TRIA3244



3.3 Quantities tested and results

Field SING_ELEM :

Standard	identification	Time	Reference	Tolerance
DEGRE MAX	1.0	"ANALYTIQUE"		1.0.1.0E-04 %
DEGRE MIN	1.0	"ANALYTIQUE"		0.5.1.0E-04 %
RATIO MAX	1.0	"NON_REGRESSION"	7.3626	0.20%
MIN RATIO	1.0	"NON_REGRESSION"	1.0310	0.20%
TAILLE MAX	1.0	"NON_REGRESSION"	1.6918	0.20%
TAILLE MIN	1.0	"NON_REGRESSION"	0.0295	0.20%

Identification	Standard	Time	Reference	Tolerance
DEGRE M142	1.0	"NON_REGRESSION"	0.687	0.20%
DEGRE M143	1.0	"NON_REGRESSION"		0.5.1.0E-04 %
DEGRE M144	1.0	"NON_REGRESSION"		0.5.1.0E-04 %
DEGRE M145	1.0	"NON_REGRESSION"		0.5.1.0E-04 %
DEGRE M146	1.0	"NON_REGRESSION"	0.5842	1.5%
DEGRE M189	1.0	"NON_REGRESSION"	0.6870	0.20%
DEGRE M190	1.0	"NON_REGRESSION"	0.6870	0.20%
DEGRE M191	1.0	"NON_REGRESSION"	0.5842	1.5%

Field SING_ELNO :

Standard	identification	Time	Reference	Tolerance
DEGRE MAX	1.0	"ANALYTIQUE"		1.0.1.0E-04 %
DEGRE MIN	1.0	"ANALYTIQUE"		0.5.1.0E-04 %
RATIO MAX	1.0	"NON_REGRESSION"	7.3626	0.20%
RATIO MIN	1.0	"NON_REGRESSION"	1.0310	0.20%
TAILLE MAX	1.0	"NON_REGRESSION"	1.6918	0.20%
TAILLE MIN	1.0	"NON_REGRESSION"	0.0295	0.20%

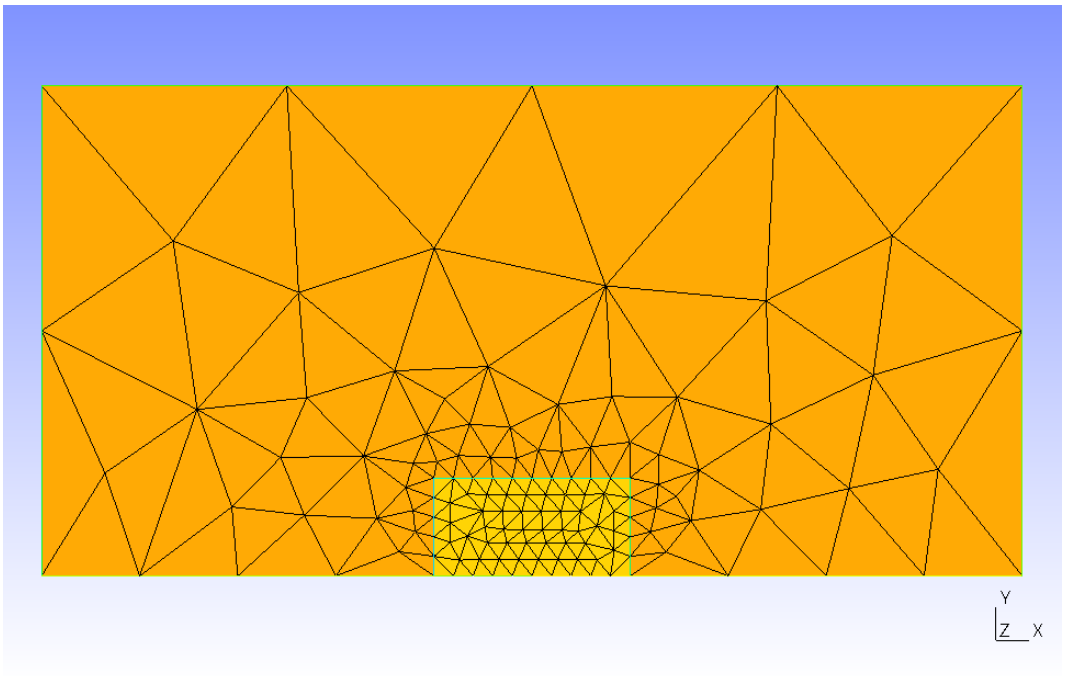
4 Modelization B

4.1 Characteristic of the modelization

The modelization is two-dimensional elastic with plane stresses C_PLAN.

4.2 Characteristics of the mesh

Number of nœuds515
Many SEG346
Many TRIA6244



4.3 Quantities tested and results

Field SING_ELEM :

Standard	identification	Time	Reference	Tolerance
DEGRE MAX	1.0	"ANALYTIQUE"	2.0	1.0%
DEGRE MIN	1.0	"ANALYTIQUE"	0.5	1.0%
RATIO MAX	1.0	"NON_REGRESSION"	2.52	2.0%
MIN RATIO	1.0	"NON_REGRESSION"	0.2150	2.0%
TAILLE MAX	1.0	"NON_REGRESSION"	4.48	2.0%
TAILLE MIN	1.0	"NON_REGRESSION"	0.0861	2.0%

Identification	Standard	Time	Reference	Tolerance
DEGRE M142	1.0	"NON_REGRESSION"	0.750	1.0%
DEGRE M143	1.0	"NON_REGRESSION"	0.5	1.0%
DEGRE M144	1.0	"NON_REGRESSION"	0.5	1.0%
DEGRE M145	1.0	"NON_REGRESSION"	0.5	1.0%
DEGRE M146	1.0	"NON_REGRESSION"	0.550	2.0%
DEGRE M189	1.0	"NON_REGRESSION"	0.750	1.0%
DEGRE M190	1.0	"NON_REGRESSION"	0.750	1.0%
DEGRE M191	1.0	"NON_REGRESSION"	0.550	2.0%
DEGRE M192	1.0	"NON_REGRESSION"	0.550	2.0%
DEGRE M193	1.0	"NON_REGRESSION"	0.550	2.0%

Field SING_ELNO :

Standard	identification	Time	Reference	Tolerance
DEGRE MAX	1.0	"ANALYTIQUE"	2.0	1.0%
DEGRE MIN	1.0	"ANALYTIQUE"	0.5	1.0%
RATIO MAX	1.0	"NON_REGRESSION"	2.52	2.0%
RATIO MIN	1.0	"NON_REGRESSION"	0.216	2.0%
TAILLE MAX	1.0	"NON_REGRESSION"	4.48	2.0%
TAILLE MIN	1.0	"NON_REGRESSION"	0.0860	0.20%

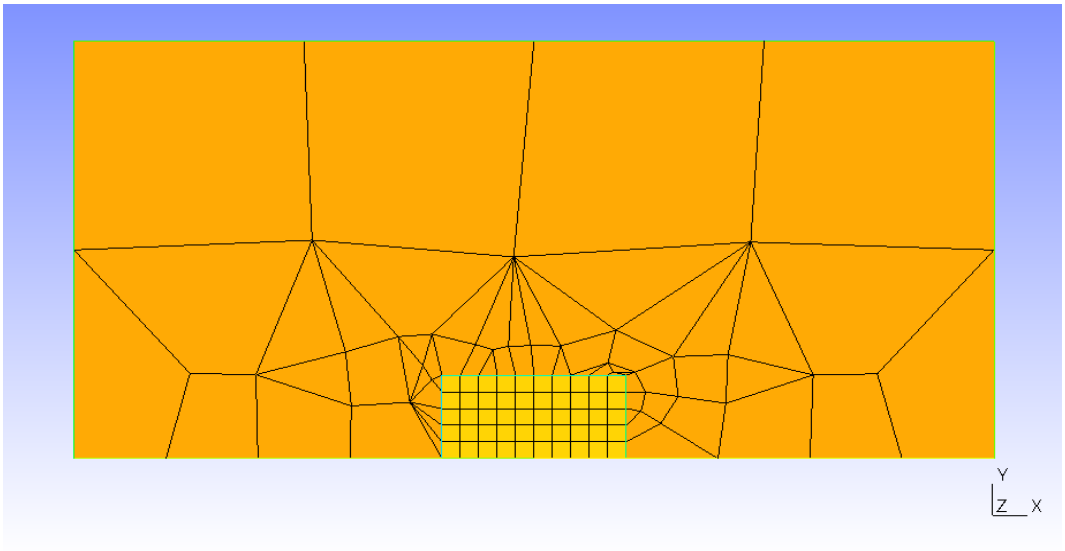
5 Modelization C

5.1 Characteristic of the modelization

The modelization is two-dimensional elastic with plane stresses C_PLAN.

5.2 Characteristics of the mesh

Number of nœuds111
Many SEG246
Many TRIA326
Many QUAD484



5.3 Quantities tested and results

Field SING_ELEM :

Standard	identification	Time	Reference	Tolerance
DEGRE MAX	1.0	"ANALYTIQUE"	1.0	1.0%
DEGRE MIN	1.0	"ANALYTIQUE"	0.5	1.0%
RATIO MAX	1.0	"NON_REGRESSION"	2.17	2.0%
MIN RATIO	1.0	"NON_REGRESSION"	0.167	2.0%
TAILLE MAX	1.0	"NON_REGRESSION"	4.59	2.0%
TAILLE MIN	1.0	"NON_REGRESSION"	0.2	2.0%

Identification	Standard	Time	Reference	Tolerance
DEGRE M30	1.0	"NON_REGRESSION"	0.750	1.0%
DEGRE M31	1.0	"NON_REGRESSION"	0.5	1.0%
DEGRE M32	1.0	"NON_REGRESSION"	0.5	1.0%
DEGRE M55	1.0	"NON_REGRESSION"	0.780	1.0%
DEGRE M56	1.0	"NON_REGRESSION"	0.6	1.0%
DEGRE M57	1.0	"NON_REGRESSION"	0.6	1.0%

Field SING_ELNO :

Standard	identification	Time	Reference	Tolerance
DEGRE MAX	1.0	"ANALYTIQUE"	1.0	1.0%
DEGRE MIN	1.0	"ANALYTIQUE"	0.5	1.0%
RATIO MAX	1.0	"NON_REGRESSION"	2.17	2.0%
RATIO MIN	1.0	"NON_REGRESSION"	0.167	2.0%
TAILLE MAX	1.0	"NON_REGRESSION"	4.59	2.0%
TAILLE MIN	1.0	"NON_REGRESSION"	0.205	2.0%

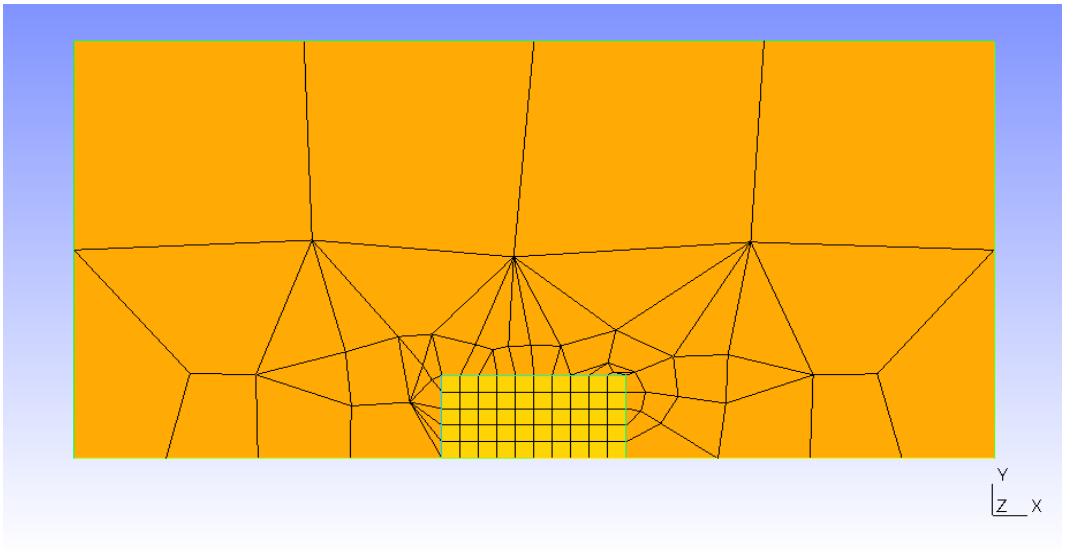
6 Modelization D

6.1 Characteristic of the modelization

The modelization is two-dimensional elastic with plane stresses C_PLAN.

6.2 Characteristics of the mesh

Number of nœuds331
Many SEG346
Many TRIA626
Many QUAD884



6.3 Quantities tested and results

Field SING_ELEM :

Standard	identification	Time	Reference	Tolerance
DEGRE MAX	1.0	"ANALYTIQUE"	2.0	1.0%
DEGRE MIN	1.0	"NON_REGRESSION"	0.51	1.0%
RATIO MAX	1.0	"NON_REGRESSION"	1.15	2.0%
MIN RATIO	1.0	"NON_REGRESSION"	0.16	2.0%
TAILLE MAX	1.0	"NON_REGRESSION"	4.56	2.0%
TAILLE MIN	1.0	"NON_REGRESSION"	0.259	2.0%

Identification	Standard	Time	Reference	Tolerance
DEGRE M30	1.0	"NON_REGRESSION"	0.81	1.0%
DEGRE M31	1.0	"NON_REGRESSION"	0.51	1.0%
DEGRE M32	1.0	"NON_REGRESSION"	0.51	1.0%
DEGRE M33	1.0	"NON_REGRESSION"	0.650	1.0%
DEGRE M55	1.0	"NON_REGRESSION"	0.84	1.0%
DEGRE M56	1.0	"NON_REGRESSION"	0.6	1.0%
DEGRE M57	1.0	"NON_REGRESSION"	0.6	1.0%
DEGRE M58	1.0	"NON_REGRESSION"	0.7	2.0%

Field SING_ELNO :

Standard	identification	Time	Reference	Tolerance
DEGRE MAX	1.0	"ANALYTIQUE"	2.0	1.0%
DEGRE MIN	1.0	"NON_REGRESSION"	0.51	1.0%
RATIO MAX	1.0	"NON_REGRESSION"	1.15	2.0%
RATIO MIN	1.0	"NON_REGRESSION"	0.16	2.0%
TAILLE MAX	1.0	"NON_REGRESSION"	4.56	2.0%
TAILLE MIN	1.0	"NON_REGRESSION"	0.259	2.0%

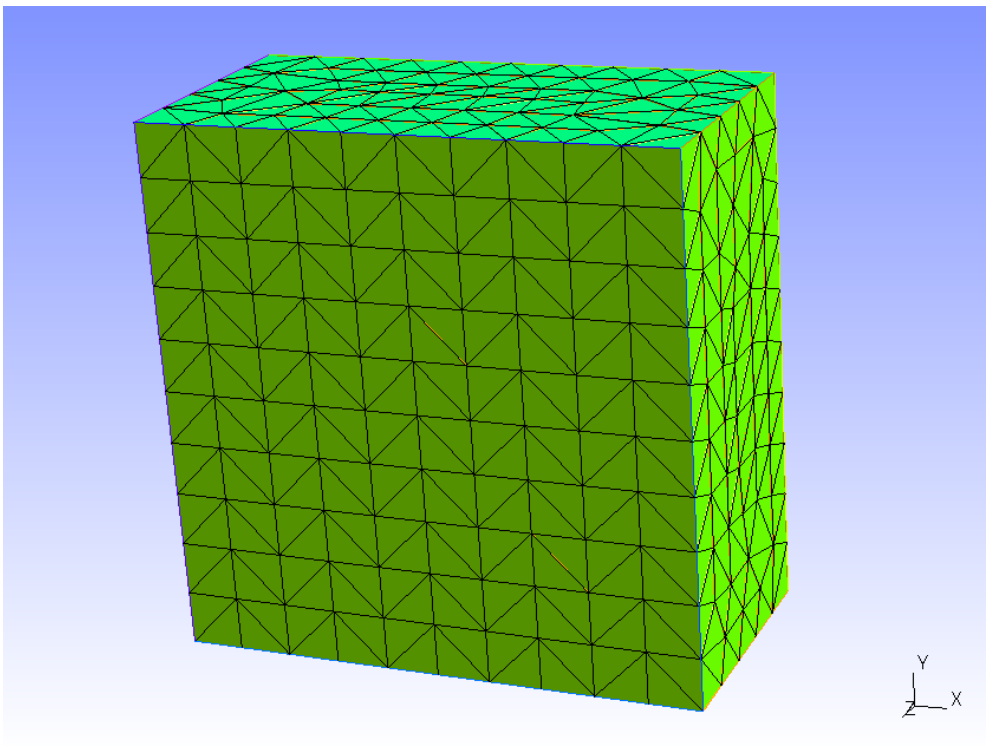
7 Modelization E

7.1 Characteristic of the modelization

The modelization is three-dimensional elastic 3D.

7.2 Characteristics of the mesh

Number of nœuds796
Many SEG2120
Many TRIA3868
Many TETRA43261



7.3 Quantities tested and results

Field SING_ELEM :

Standard	identification	Time	Reference	Tolerance
DEGRE MAX	1.0	"ANALYTIQUE"	1.0	1.0%
DEGRE MIN	1.0	"NON_REGRESSION"	0.56	1.0%
RATIO MAX	1.0	"NON_REGRESSION"	2.77	2.0%
MIN RATIO	1.0	"NON_REGRESSION"	0.155	2.0%
TAILLE MAX	1.0	"NON_REGRESSION"	9.1	2.0%
TAILLE MIN	1.0	"NON_REGRESSION"	0.44	2.0%

Field SING_ELNO :

Standard	identification	Time	Reference	Tolerance
DEGRE MAX	1.0	"ANALYTIQUE"	1.0	1.0%
DEGRE MIN	1.0	"NON_REGRESSION"	0.56	1.0%
RATIO MAX	1.0	"NON_REGRESSION"	2.77	1.0%
RATIO MIN	1.0	"NON_REGRESSION"	0.155	1.0%
CUTS MAX	1.0	"NON_REGRESSION"	2.77	1.0%
CUTS MIN	1.0	"NON_REGRESSION"	0.155	1.0%

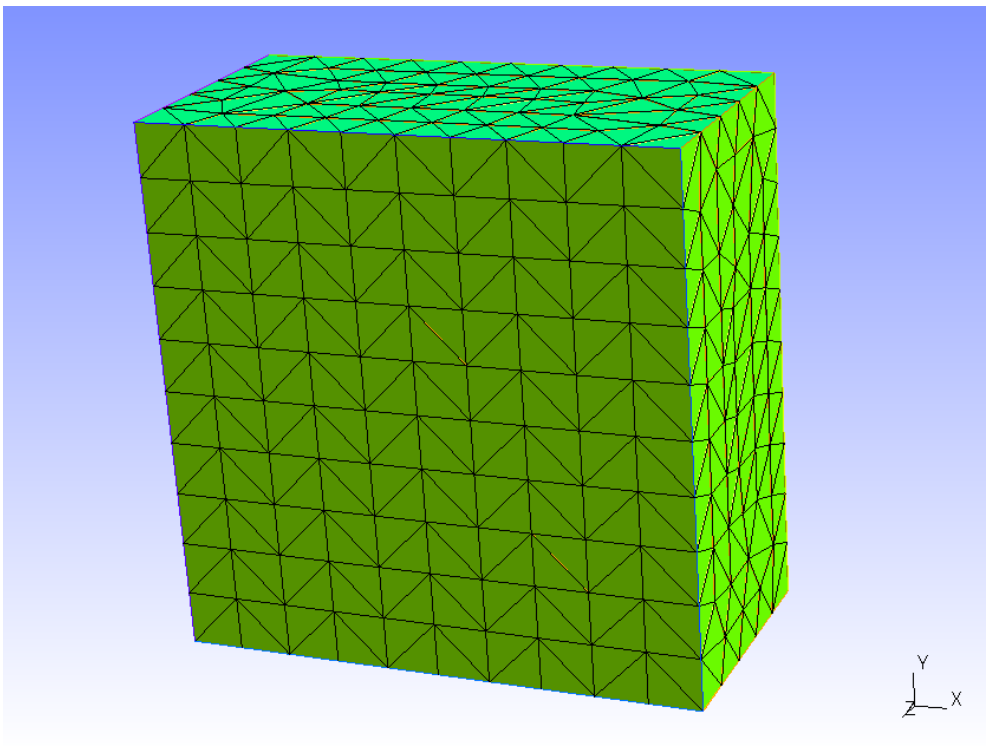
8 Modelization F

8.1 Characteristic of the modelization

The modelization is three-dimensional elastic 3D.

8.2 Characteristics of the mesh

Number of nœuds5286
Many SEG3120
Many TRIA6868
Many TETRA103261



8.3 Quantities tested and results

Field SING_ELEM :

Standard	identification	Time	Reference	Tolerance
DEGRE MAX	1.0	"ANALYTIQUE"		2.0.1.0E-04 %
DEGRE MIN	1.0	"NON_REGRESSION"	0.5604	1.0E-04%
RATIO MAX	1.0	"NON_REGRESSION"	2.65	1.0E-04%
MIN RATIO	1.0	"NON_REGRESSION"	0.2169	1.0E-04%
TAILLE MAX	1.0	"NON_REGRESSION"	6.5176	1.0E-04%
TAILLE MIN	1.0	"NON_REGRESSION"	0.4940	1.0E-04%

Field SING_ELNO :

Standard	identification	Time	Reference	Tolerance
DEGRE MAX	1.0	"ANALYTIQUE"		2.0.1.0E-04 %
DEGRE MIN	1.0	"NON_REGRESSION"	0.5604	1.0E-04%
RATIO MAX	1.0	"NON_REGRESSION"	2.6526	1.0E-04%
RATIO MIN	1.0	"NON_REGRESSION"	0.2169	1.0E-04%
TAILLE MAX	1.0	"NON_REGRESSION"	6.5176	1.0E-04%
TAILLE MIN	1.0	"NON_REGRESSION"	0.4940	1.0E-04%

9 Summary of the results

the got results are in coherence with the analytical solutions. They make it possible to validate this computation in $2D$ and in $3D$ for linear and quadratic elements.