SSNP128 - Validation of the element to discontinuity on a plane plate

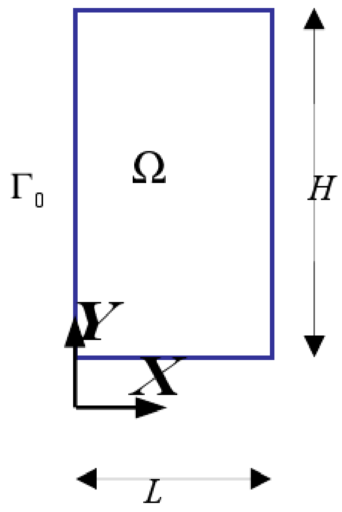
Abstract:

The goal of this test is to display an analytical solution in order to validate the quality of the element to discontinuity (confer to the documentation [R7.02.12] for details on this element). The purpose of this test is to check that this model led to a good prediction of the value of the jump of displacement along a crack. With this intention, one seeks an analytical solution presenting a nonconstant jump along a discontinuity which one compares with the solution obtained numerically. In addition when one seeks to validate a numerical method it is preferable to make sure of the unicity of the required solution. We will see that it is the case for the analytical solution presented if a condition relating to the maximum size of the field studied according to the parameters of the model is checked.

1 Problem of reference

1.1 Geometry

In the coordinate system Cartesian (x, y), let us consider an elastic rectangular plane plate noted $\Omega =]0, L[\times]0, H[$ (see [Figure 1.1-a]). Let us note $\Gamma_0 = \{0\} \times]0, H[$ the left face of the field and $\partial\Omega \setminus \Gamma_0$ the part complementary to edge.



Appear 1.1-a: Diagram of the plate

Dimensions of the field Ω :

$$L = 1\,\text{mm}, H = 2\pi\,\text{mm}$$

1.2 Material properties

the material is elastic with a critical stress and a tenacity arbitrarily chosen:

$$E = 10\,\text{MPa}, \nu = 0, \sigma_c = 1.1\,\text{MPa}, G_c = 0.9\,\text{N.mm}^{-1}$$

1.3 Boundary conditions and loadings

the boundary conditions are determined by the analytical solution presented in the following part so that they lead to a crack having a nonconstant jump along Γ_0. The loading corresponds to a displacement imposed on edges of the plate: (see [Figure 1.3-a]).

$$u = U(x, y) \quad \text{on } \Omega / \Gamma_0$$

$$u = U_0 \delta(y) - (y) \quad \text{on } \Gamma_0$$

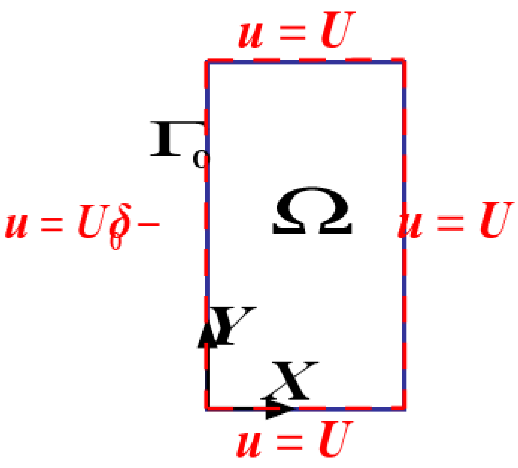


Figure 1.3-a: Diagram of the loading

the values U, U_0 and δ are defined during the construction of the reference solution in the following part.

2 Reference solution

In this part one displays an analytical solution with a nonconstant jump along Γ_0 , then a condition of unicity of the solution is given.

2.1 Analytical solution

the function of Airy $\Phi(x,y)$ controlled by the equation $\Delta\Delta\Phi=0$ on Ω , if the external forces null, are led to stresses satisfying the balance equations and of compatibility in elasticity (see Fung [bib1]). The components of the stress σ_{xx} , σ_{yy} and σ_{xy} derive from in the following way $\Phi(x,y)$:

$$\sigma_{xx}=\frac{\partial^2\Phi}{\partial y^2},\ \sigma_{yy}=\frac{\partial^2\Phi}{\partial x^2}\text{ and }\sigma_{xy}=\frac{\partial^2\Phi}{\partial x\partial y}\qquad\text{éq 2.1-1}$$

Let us choose a function bi--harmonic $\Phi(x,y)$ defined by:

$$\Phi(x,y)=\beta\frac{y^3}{6}+(\alpha x+\gamma)\frac{y^2}{2}+\eta x y$$

with α,β,γ and η arbitrary real constants. One from of deduced according to [éq 2.1-1] the stress field:

$$\begin{cases}\sigma_{xx} &= \alpha x+\beta y+\gamma\\ \sigma_{yy} &= 0\\ \sigma_{xy} &= -\alpha y-\eta\end{cases}\qquad\text{éq 2.1-2}$$

By integrating the elastic model, if one notes E the Young modulus and ν the Poisson's ratio (which one takes no one), one from of deduced the field from displacement in Γ checking the equilibrium:

$$\boldsymbol{u}=\begin{Bmatrix}u(x,y)\\ v(x,y)\end{Bmatrix}=\begin{Bmatrix}\frac{1}{E}\left(\alpha\left(\frac{x^2}{2}-y^2\right)+x(\beta y+\gamma)\right)\\ -\frac{1}{E}\left(\beta\frac{x^2}{2}+2\eta x\right)\end{Bmatrix}\qquad\text{éq 2.1-3}$$

respectively and the U_0 Let us note U displacements on Γ_0 and $\partial\Omega\setminus\Gamma_0$ given by [éq 2.1-3]. The latter correspond to the boundary conditions leading to the stress fields [éq 2.1-2]. From these data, it is easy to build a field of displacement with a discontinuity on edge Γ_0 . Indeed, knowing the normal stress σn on Γ_0 which one notes $F(y)$, one obtains the jump of displacement $\delta(y)$ by reversing the exponential constitutive law of Barenblatt type: `CZM_EXP` (confer to the documentation on the elements with internal discontinuity and their behavior: [R7.02.12]):

$$\delta(y)=-\frac{G_c F(y)}{\sigma_c\|F(y)\|}\ln\left(\frac{\|F(y)\|}{\sigma_c}\right)$$

for all y in $[0,H]$. Thus, the new displacement imposed on Γ_0 generating such a jump is equal to $U_0-\delta$. One thus built an analytical solution of the plane plate checking the balance equations and compatibility with a discontinuity in Γ_0 along which the jump of displacement δ is not constant. Let us point out the boundary conditions of problem:

$$\begin{cases} \boldsymbol{u}=U(x, y) & sur \quad \partial\Omega\setminus\Gamma_0 \\ \boldsymbol{u}=U_0(y)-\delta(y) & sur \quad \Gamma_0 \end{cases}$$ éq 2.1-4

2.2 Unicity of the solution

After having built an analytical solution it is important to make sure that the latter is single to be able to compare it with the numerical solution. One shows, to see [bib2], that unicity is guaranteed as soon as the following condition, on the geometry of the field like on the material parameters, is checked:

$$L<2\mu\frac{G_c}{\sigma_c^2}$$.éq 2.2-1

dimensions of the plate and the material parameters given previously check this condition.

2.3 Bibliographical references

[1] FUNG Y.C. : Foundation of Solid Mechanics, Prentice-Hall, (1979).

[2] LAVERNE J.: Energy formulation of the fracture by models of cohesive forces: numerical considerations theoretical and establishments, Doctorate of the University Paris 13, November 2004.

3 Modelization A

3.1 Characteristic of the modelization

the idea is to carry out a computational simulation corresponding to the problem presented in the preceding part and to compare the got results. The elements with discontinuity make it possible to represent crack along Γ_0 . The latter have as a modelization PLAN_ELDI and a behavior CZM_EXP. The other elements of the mesh are QUAD4 elastics in modelization D_PLAN.

The values of the parameters of the function of Airy for the construction of the analytical solution are taken arbitrarily:

$$\alpha=0\,\mathrm{MPa.mm^{-1}}, \beta=1/4\pi\,\mathrm{MPa.mm^{-1}}, \gamma=1/2\,\mathrm{MPa} \quad \mathrm{et} \quad \eta=0\,\mathrm{MPa}$$

3.2 Characteristics of the mesh

One meshes carries out a mesh of the plate structured in quadrangles with 20 in the width and 50 in the height. One has the elements with discontinuity along with dimensions Γ_0 with the norm directed according to $-X$. This is carried out using key word CREA_FISS of CREA_MAILLAGE (confer to the documentation [U4.23.02]).

3.3 Quantities tested and Quantity

results tested	Standard	Reference	Tolerance ($\%$)
Variable threshold: $VI1$ On element $MJ15$	"ANALYTIQUE"	0.49315	0.10
Variable threshold: $VI1$ On element $MJ45$	"ANALYTIQUE"	1.075	0.10
Normal stress: $VI6$ On element $MJ30$	"ANALYTIQUE"	0.4489	0.10

4 Summary of the results

These results enable us to conclude that the element with internal discontinuity led to a good approximation from the analytical solution. Moreover, one study on the dependence on the discretization was carried out in [bib2]. It is noted that the error made on the jump of displacement decrease when the mesh is refined. That makes it possible to conclude that, in spite of a constant jump by element, this model makes it possible to correctly reproduce a crack with a nonconstant jump by refining the mesh.