

## SSNP121 – Integration of the terms of contact in 2D and 3D

---

### Summarized:

This problem corresponds to a quasi-static analysis of a problem of mechanics with contact without friction. One is interested particularly here in integration of the terms of contact (*patch-test*). It is a question of studying two identical blocks with a grid differently and subjected to symmetric imposed displacements.

This test comprises six modelizations in 2D (linear elements SEG2):

- modelization a: METHODE= `CONTINUE'. The integration method by subelements also proposed by Bathe [bib1] is used with three subelements,
- modelization b: METHODE= `CONTRAINTE',
- modelization C: METHODE= `PENALIZATION',
- modelization G: METHODE= `CONTINUE'. An alternative of the modelization A where one tests the initial activation of the statute of contact.
- modelization L: METHODE= `GCP',
- modelization P: METHODE= `CONTINUE'. An alternative of the modelization G where one tests the algorithm of penalization of the contact.

a modelization in 2D (linear elements SEG2 in with respect to quadratic elements SEG3):

- modelization H: METHODE= `CONTINUE'. Contact surfaces are made up of elements SEG2 in with respect to elements SEG3,

six modelizations in 3D (quadratic elements):

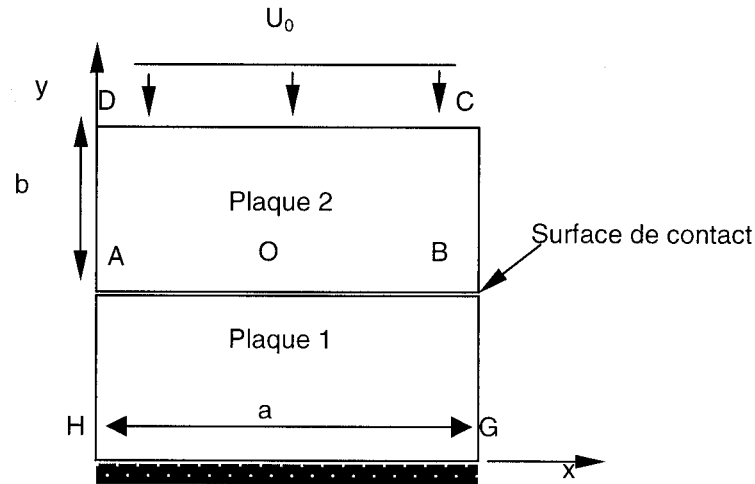
- modelization D: METHODE= `CONTINUE'. Contact surfaces are made up of elements QUAD8 in with respect to elements QUAD8,
- modelization E: METHODE= `CONTINUE'. Contact surfaces are made up of elements TRIA6 in with respect to elements TRIA6,
- modelization F: METHODE= `CONTINUE'. Contact surfaces are made up of elements TRIA6 in with respect to elements QUAD8.
- modelization Q: METHODE= `CONTINUE'. Contact surfaces are made up of elements QUAD9 in with respect to elements QUAD9, meshes 3D are HEXA27,
- modelization R: METHODE= `CONTINUE'. Contact surfaces are made up of elements TRIA6 in with respect to elements TRIA6, meshes 3D are PENTA18,

and four modelizations in 3D (linear elements in with respect to quadratic elements):

- modelization I: METHODE= `CONTINUE'. Contact surfaces are made up of elements QUAD4 in with respect to elements QUAD8,
- modelization J: METHODE= `CONTINUE'. Contact surfaces are made up of elements TRIA3 in with respect to elements TRIA6,
- modelization K: METHODE= `CONTINUE'. Contact surfaces are made up of elements TRIA3 in with respect to elements QUAD8.

## 1 Problem of reference

### 1.1 Geometry



Length  $a = 2 \text{ m}$ .

Width  $b = 1 \text{ m}$ .

$O$  not medium of the segment  $AB$  (origin of the reference).

### 1.2 Properties of the materials

**Plates 1 and 2:**

Poisson's ratio: 0.0

Young modulus:  $2 \cdot 10^6 \text{ N/m}^2$

### 1.3 Boundary conditions and loadings

plate 1 is blocked:

- On  $HG$   $DX = 0$  and  $DY = 0$ .

Plate 2 is subjected to an imposed displacement:

- On  $CD$ :  $DY = U_0 = -0.1 \text{ m}$  and  $DX = 0$ .

## 2 Reference solution

---

### 2.1 Method of calculating used for the reference solution

the reference solution, analytical, can be deduced from a very simple computation. The Poisson's ratio being null, the uniaxial strain is written  $\varepsilon = U_0 / HD = -0.1/2$ . the pressure is worth then  $E \varepsilon = -10^5 Pa$ .

### 2.2 Results of reference

the contact pressure is constant and equal to  $-10^5 Pa$  on all contact surface. In the same way vertical displacement (according to  $y$ ) is constant on contact surface and equal to  $U_0/2 = -0.05 m$ .

### 2.3 Bibliographical reference

1. N. EL-ABBASI and K.J. BATHE: "Stability and Patch Test Performance of Contact Discretizations and has New Algorithm Solution", Computers & Structures, 79,1473-1486, 2001

## 3 Modelization A

### 3.1 Characteristic of the modelization

One uses a modelization `D_PLAN` for the solid elements with the continuous method for the processing of the contact with integration of the type "`SIMPSON2`".

### 3.2 Characteristics of the mesh

Many nodes: 313  
Number of meshes and types: 265 `QUAD4` and 132 `SEG2`

the meshes of 2 contact surfaces are incompatible. 12 finite elements `SEG2` are laid out on initial contact surface of plate 1 and only 11 on contact surface of other surface.

By activating key word `SIMPSON2` three subelements are used for the integration of the terms of contact.

### 3.3 Quantities tested and results

Identification	Standard Value of reference	of reference	Tolerance
DY to the point <i>A</i>	- 0.05	"ANALYTIQUE"	1%
LAGS_C at the point <i>A</i>	- 1.E+5	"ANALYTIQUE"	1%
DY at the point <i>O</i>	- 0.05	"ANALYTIQUE"	1%
LAGS_C at the point <i>O</i>	- 1.E+5	"ANALYTIQUE"	1%
DY at the point <i>B</i>	- 0.05	"ANALYTIQUE"	1%
LAGS_C at the point <i>B</i>	- 1.E+5	"ANALYTIQUE"	1%

## 4 Modelization B

---

### 4.1 Characteristic of the modelization

One uses a modelization `D_PLAN` for the solid elements with the method `FORCED` for the processing of the contact .

### 4.2 Characteristics of the mesh

One uses the same mesh as for the preceding modelization.

Many nodes: 313  
Number of meshes and types: 265 QUAD4 and 132 SEG2

### 4.3 Quantities tested and results

Standard	Identification	Reference of reference	Tolerance
DY to the point <i>O</i>	- 0.05	"ANALYTIQUE"	1%
SIYY at the point <i>O</i>	- 1.E+5	"ANALYTIQUE"	1%
DY at the point <i>A</i>	- 0.05	"ANALYTIQUE"	1,7%
SIYY at the point <i>A</i>	- 1.E+5	"ANALYTIQUE"	6,1%
DY at the point <i>B</i>	- 0.05	"ANALYTIQUE"	1,7%
SIYY at the point <i>B</i>	- 1.E+5	"ANALYTIQUE"	6,1%

### 4.4 Remarks

the discrete formulations do not check *the patch-test* contrary to the continuous formulation, this is why the tolerances are looser.

## 5 Modelization C

---

### 5.1 Characteristic of the modelization

One uses of the contact a modelization D\_PLAN for the solid elements with the method PENALIZATION with a coefficient of penalization of 1.E7 for the processing.

### 5.2 Characteristics of the mesh

One uses the same mesh as for the preceding modelization.

Many nodes: 313  
Number of meshes and types: 265 QUAD4 and 132 SEG2

### 5.3 Quantities tested and results

Standard	Identification	Reference of reference	Tolerance
DY to the point <i>O</i>	- 0.05	"ANALYTIQUE"	2,1%
SIYY at the point <i>O</i>	- 1.E+5	"ANALYTIQUE"	2,1%
DY at the point <i>A</i>	- 0.05	"ANALYTIQUE"	0,7%
SIYY at the point <i>A</i>	- 1.E+5	"ANALYTIQUE"	7,5%
DY at the point <i>B</i>	- 0.05	"ANALYTIQUE"	0,7%
SIYY at the point <i>B</i>	- 1.E+5	"ANALYTIQUE"	7,5%

### 5.4 Remarks

the discrete formulations do not check *the patch-test* contrary to the continuous formulation, this is why the tolerances are looser.

## 6 Modelization D

---

### 6.1 Characteristic of the modelization

One 3D uses a modelization for the solid elements with the continuous method for the processing of the contact . Contact surfaces in opposite consist of elements QUAD8.

### 6.2 Characteristics of the mesh

Many nodes: 850  
Number of meshes and types: 128 HEXA20 and 64 QUAD8

### 6.3 Quantities tested and results

Standard	Identification	Reference of reference	Tolerance
DZ to the point <i>A</i>	- 0.05	"ANALYTIQUE"	0,1%
LAGS_C at the point <i>A</i>	- 1.E+5	"ANALYTIQUE"	0,1%
max (LAGS_C) on surface	- 1.E+5	"ANALYTIQUE"	0,1%
Min (LAGS_C) on surface	- 1.E+5	"ANALYTIQUE"	0,1%
DZ at the point <i>B</i>	- 0.05	"ANALYTIQUE"	0,1%
LAGS_C at the point <i>B</i>	- 1.E+5	"ANALYTIQUE"	0,1%



## 7 Modelization E

---

### 7.1 Characteristic of the modelization

One 3D uses a modelization for the solid elements with the continuous method for the processing of the contact . Contact surfaces in opposite consist of elements TRIA6.

### 7.2 Characteristics of the mesh

Many nodes: 1010  
Number of meshes and types: 256 PENTA15 and 128 TRIA6

### 7.3 Quantities tested and results

Standard	Identification	Reference of reference	Tolerance
DZ to the point <i>A</i>	- 0.05	"ANALYTIQUE"	0,1%
LAGS_C at the point <i>A</i>	- 1.E+5	"ANALYTIQUE"	0,1%
max (LAGS_C) on surface	- 1.E+5	"ANALYTIQUE"	0,1%
Min (LAGS_C) on surface	- 1.E+5	"ANALYTIQUE"	0,1%
DZ at the point <i>B</i>	- 0.05	"ANALYTIQUE"	0,1%
LAGS_C at the point <i>B</i>	- 1.E+5	"ANALYTIQUE"	0,1%

## 8 Modelization F

---

### 8.1 Characteristic of the modelization

One 3D uses a modelization for the solid elements with the continuous method for the processing of the contact . Contact surfaces in opposite consist of elements TRIA6 in with respect to meshes QUAD8.

### 8.2 Characteristics of the mesh

Many nodes: 930  
Number of meshes and types: 64 HEXA20, 128 PENTA15, 32 QUAD8 and 64 TRIA6

### 8.3 Quantities tested and results

Standard	Identification	Reference of reference	Tolerance
DZ to the point <i>A</i>	- 0.05	"ANALYTIQUE"	0,1%
LAGS_C at the point <i>A</i>	- 1.E+5	"ANALYTIQUE"	0,1%
max (LAGS_C) on surface	- 1.E+5	"ANALYTIQUE"	0,1%
Min (LAGS_C) on surface	- 1.E+5	"ANALYTIQUE"	0,1%
DZ at the point <i>B</i>	- 0.05	"ANALYTIQUE"	0,1%
LAGS_C at the point <i>B</i>	- 1.E+5	"ANALYTIQUE"	0,1%

### 8.4 Remarks

contact surface slave is with a grid in TRIA6 and contact surface Master is with a grid in QUAD8. A use with elements QUAD8 for surface slave and of the TRIA6 for surface Master does not satisfy the compatibility conditions necessary to the good integration of the terms with contact [R5.03.52]. In a general way, if one correctly does not apprehend this notion of compatibility, one advises with the user not informed to use the same elements for the mesh of surfaces main and slave.

## 9 Modelization G

### 9.1 Characteristic of the modelization

One uses a modelization `D_PLAN` for the solid elements with the continuous method for the processing of the contact with integration of the type `SIMPSON2`.

### 9.2 Characteristics of the mesh

Many nodes: 313  
Number of meshes and types: 265 QUAD4 and 132 SEG2

### 9.3 Quantities tested and results

Identification	Standard Value of reference	of reference	Tolerance
DY to the point <i>A</i>	- 0.05	"ANALYTIQUE"	1%
LAGS_C at the point <i>A</i>	- 1.E+5	"ANALYTIQUE"	1%
DY at the point <i>O</i>	- 0.05	"ANALYTIQUE"	1%
LAGS_C at the point <i>O</i>	- 1.E+5	"ANALYTIQUE"	1%
DY at the point <i>B</i>	- 0.05	"ANALYTIQUE"	1%
LAGS_C at the point <i>B</i>	- 1.E+5	"ANALYTIQUE"	1%

### 9.4 Remarks

In this modelization, the loading is a pressure imposed on the higher plate (and not a displacement). The 2 approaches are perfectly equivalent, however the use of a pressure forces here to block rigid body motions (according to *DZ*), that is made by declaring a contact initial (`CONTACT_INIT`).

## 10 Modelization H

---

### 10.1 Characteristic of the modelization

One uses a modelization `D_PLAN` for the solid elements with the continuous method for the processing of the contact between linear mixed elements and quadratic elements.

### 10.2 Characteristics of the mesh

Many nodes: 721  
Number of meshes and types: 144 QUAD4, 121 QUAD8, 48 SEG2 and 44 SEG3

the meshes are incompatible. 12 finite elements `SEG2` are laid out on initial contact surface of the plate slave and only 11 finite elements `SEG3` on contact surface Master.

By activating key word `NCOTES2`, a diagram of the Newton-Dimensions type coupled to a technique of subdivision in subelements was used for the integration of the terms of contact.

### 10.3 Quantities tested and results

Standard	Identification	Reference of reference	Tolerance
DY to the point <i>A</i>	- 0.05	"ANALYTIQUE"	0,5%
LAGS_C at the point <i>A</i>	- 1.E+5	"ANALYTIQUE"	0,5%
DY at the point <i>B</i>	- 0.05	"ANALYTIQUE"	0,5%
LAGS_C at the point <i>B</i>	- 1.E+5	"ANALYTIQUE"	0,5%

## 11 Modelization I

---

### 11.1 Characteristic of the modelization

One 3D uses a `modelization` for the solid elements with the continuous method for the processing of the contact between linear mixed elements and quadratic elements.

### 11.2 Characteristics of the mesh

Many nodes: 227  
Number of meshes and types: 16 HEXA20, 25 HEXA8, 32 QUAD8 and 50 QUAD4

the meshes are incompatible. Contact surfaces consist of 5 elements QUAD4 as screw - with-screw of 4 elements QUAD8. The diagram of integration used is of Newton-Dimensions type coupled to a technique of subdivision in subelements.

### 11.3 Quantities tested and results

Standard	Identification	Reference of reference	Tolerance
DZ to the point <i>A</i>	- 0.05	"ANALYTIQUE"	0,1%
LAGS_C at the point <i>A</i>	- 1.E+5	"ANALYTIQUE"	0,1%
max (LAGS_C) on surface	- 1.E+5	"ANALYTIQUE"	0,1%
Min (LAGS_C) on surface	- 1.E+5	"ANALYTIQUE"	0,1%
DZ at the point <i>B</i>	- 0.05	"ANALYTIQUE"	0,1%
LAGS_C at the point <i>B</i>	- 1.E+5	"ANALYTIQUE"	0,1%

## 12 Modelization J

### 12.1 Characteristic of the modelization

One 3D uses a modelization for the solid elements with the continuous method for the processing of the contact between linear mixed elements and quadratic elements.

### 12.2 Characteristics of the mesh

Many nodes: 630  
Number of meshes and types: 128 PENTA6, 128 PENTA15, 64 TRIA3 and 64 TRIA6

the meshes are compatible. Contact surfaces consist of elements TRIA3 in with respect to elements TRIA6. The diagram of integration used is of Newton-Dimensions type.

### 12.3 Quantities tested and results

Standard	Identification	Reference of reference	Tolerance
DZ to the point <i>A</i>	- 0.05	"ANALYTIQUE"	0,4%
LAGS_C at the point <i>A</i>	- 1.E+5	"ANALYTIQUE"	0,4%
max (LAGS_C) on surface	- 1.E+5	"ANALYTIQUE"	0,6%
Min (LAGS_C) on surface	- 1.E+5	"ANALYTIQUE"	0,6%
DZ at the point <i>B</i>	- 0.05	"ANALYTIQUE"	0,4%
LAGS_C at the point <i>B</i>	- 1.E+5	"ANALYTIQUE"	0,4%

## 13 Modelization K

### 13.1 Characteristic of the modelization

One 3D uses a `modelization` for the solid elements with the continuous method for the processing of the contact for the linear/quadratic mixed elements.

### 13.2 Characteristics of the mesh

Many nodes: 550  
Number of meshes and types: 64 HEXA20, 128 PENTA6, 32 QUAD8 and 64 TRIA3

the meshes are compatible. Contact surfaces consist of elements TRIA3 in with respect to elements QUAD8. The diagram of integration used is of Newton-Dimensions type.

### 13.3 Quantities tested and results

Standard	Identification	Reference of reference	Tolerance
DZ to the point <i>A</i>	- 0.05	"ANALYTIQUE"	0,1%
LAGS_C at the point <i>A</i>	- 1.E+5	"ANALYTIQUE"	0,1%
max (LAGS_C) on surface	- 1.E+5	"ANALYTIQUE"	0,1%
Min (LAGS_C) on surface	- 1.E+5	"ANALYTIQUE"	0,1%
DZ at the point <i>B</i>	- 0.05	"ANALYTIQUE"	0,1%
LAGS_C at the point <i>B</i>	- 1.E+5	"ANALYTIQUE"	0,1%

### 13.4 Remarks

contact surface slave is with a grid in TRIA3 and contact surface Master is with a grid in QUAD8. A use with elements QUAD8 for surface slave and of the TRIA3 for surface Master does not satisfy the compatibility conditions necessary to the good integration of the terms with contact [R5.03.52].

## 14 Modelization L

---

### 14.1 Characteristic of the modelization

One uses a modelization `D_PLAN` for the solid elements with method `GCP` for the processing of the contact .

### 14.2 Characteristics of the mesh

One uses the same mesh as for modelization A.

Nombre of nodes: 313  
Number of meshes and types: 265 QUAD4 and 132 SEG2

### 14.3 Quantities tested and results

Standard	Identification	Reference of reference	Tolerance
DY to the point <i>O</i>	- 0.05	"ANALYTIQUE"	1%
SIYY at the point <i>O</i>	- 1.E+5	"ANALYTIQUE"	1%
DY at the point <i>A</i>	- 0.05	"ANALYTIQUE"	1,7%
SIYY at the point <i>A</i>	- 1.E+5	"ANALYTIQUE"	6,1%
DY at the point <i>B</i>	- 0.05	"ANALYTIQUE"	1,7%
SIYY at the point <i>B</i>	- 1.E+5	"ANALYTIQUE"	6,1%

### 14.4 Remarks

the discrete formulations do not check *the patch-test* contrary to the continuous formulation, this is why the tolerances are looser.



## 15 Modelization P

### 15.1 Characteristic of the modelization

One uses a modelization `D_PLAN` for the solid elements with the continuous method for the processing of the contact with integration of the type `SIMPSON2`. The formulation of the contact is penalized, the coefficient of penalization is of `1.0E16`.

### 15.2 Characteristics of the mesh

Many nodes: 313  
Number of meshes and types: 265 QUAD4 and 132 SEG2

### 15.3 Quantities tested and results

Identification	Standard Value of reference	of reference	Tolerance
DY to the point <i>A</i>	- 0.05	"ANALYTIQUE"	1%
LAGS_C at the point <i>A</i>	- 1.E+5	"ANALYTIQUE"	1%
DY at the point <i>O</i>	- 0.05	"ANALYTIQUE"	1%
LAGS_C at the point <i>O</i>	- 1.E+5	"ANALYTIQUE"	1%
DY at the point <i>B</i>	- 0.05	"ANALYTIQUE"	1%
LAGS_C at the point <i>B</i>	- 1.E+5	"ANALYTIQUE"	1%

### 15.4 Remarks

In this modelization, the loading is a pressure imposed on the higher plate (and not a displacement). The 2 approaches are perfectly equivalent, however the use of a pressure forces here to block rigid body motions (according to  $DZ$ ), that is made by declaring a contact initial (`CONTACT_INIT`).

## 16 Modelization Q

---

### 16.1 Characteristic of the modelization

One 3D uses a modelization for the solid elements with the continuous method for the processing of the contact . Contact surfaces in opposite consist of elements QUAD9.

### 16.2 Characteristics of the mesh

Many nodes: 850  
Number of meshes and types: 128 HEXA27 and 64 QUAD9

### 16.3 Quantities tested and results

Standard	Identification	Reference of reference	Tolerance
DZ to the point <i>A</i>	- 0.05	"ANALYTIQUE"	0,1%
LAGS_C at the point <i>A</i>	- 1.E+5	"ANALYTIQUE"	0,1%
max (LAGS_C) on surface	- 1.E+5	"ANALYTIQUE"	0,1%
Min (LAGS_C) on surface	- 1.E+5	"ANALYTIQUE"	0,1%
DZ at the point <i>B</i>	- 0.05	"ANALYTIQUE"	0,1%
LAGS_C at the point <i>B</i>	- 1.E+5	"ANALYTIQUE"	0,1%

### 16.4 Remarks

In this modelization, one uses meshes HEXA27 which were transformed by the operator CREA\_MAILLAGE.

## 17 Modelization R

### 17.1 Characteristic of the modelization

One 3D uses a modelization for the solid elements with the continuous method for the processing of the contact . Contact surfaces in opposite consist of elements TRIA6.

### 17.2 Characteristics of the mesh

Many nodes: 1458  
Number of meshes and types: 256 PENTA18, 128 TRIA6

### 17.3 Quantities tested and results

Standard	Identification	Reference of reference	Tolerance
DZ to the point <i>A</i>	- 0.05	"ANALYTIQUE"	0,1%
LAGS_C at the point <i>A</i>	- 1.E+5	"ANALYTIQUE"	0,1%
max (LAGS_C) on surface	- 1.E+5	"ANALYTIQUE"	0,1%
Min (LAGS_C) on surface	- 1.E+5	"ANALYTIQUE"	0,1%
DZ at the point <i>B</i>	- 0.05	"ANALYTIQUE"	0,1%
LAGS_C at the point <i>B</i>	- 1.E+5	"ANALYTIQUE"	0,1%

### 17.4 Remarks

In this modelization, one uses meshes PENTA18 which were transformed by the operator CREA\_MALLAGE.

## 18 Summary of the results

---

One seeks on this example very simple (known under the name of *patch-test*) to test a new technique of integration of the terms of contact based on the subdivision by subelements available only for the formulation "CONTINUE". This technique aims to attenuate the amplitude of oscillation of the contact pressure. In the case studied here, the pressure is constant on all contact surface (let us notice that the Poisson's ratio is null).

One notes the discrete formulations as well as with "STRESS" and "PENALIZATION" the solution present of the nonphysical oscillations of about 6 at 7%. By means of the method "CONTINUE", with a diagram of integration adapted the oscillations disappear almost completely and the got results are very close to the reference solution (<0,5%).