FORMA20 - Mechanical adaptive mesh on a beam in bending

Summarized:

In this case, it is a question of making sure of **the NON-regression of the TP n°1 associated with the courses "Error indicators and mesh adaptation; Establishment and state of the art in *Code_Aster* "** of training "nonlinear Static analysis with *Code_Aster* ".

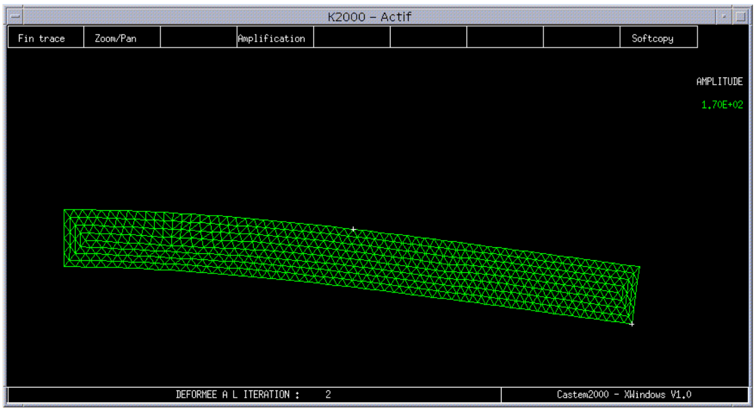
In fact, one distort **an elastic design on a metal beam in bending** in modelization plane stress. One makes it **converge uniformly** via the tools of refinement-coarsening HOMARD encapsulated in `MACR_ADAP_MAIL`, then **freely** by coupling the process with a card of spatial errors localised on each finite element.

From the point of view of **the data-processing validation**, this case test of course makes it possible to test the NON-regression of different coupling computations from card of errors/procedure of refinement-coarsening in mechanics, but also the options the "pre one and postprocessings" of these computations (lissage of the nodal stresses, transition of an error per element with an error with the nodes by element).

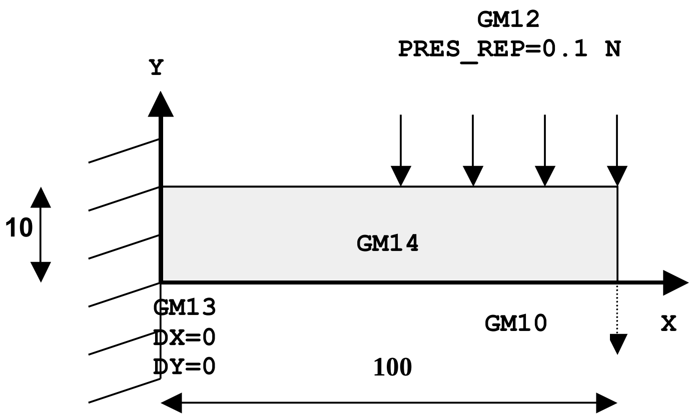
Each modelization is associated with question TP and one retranscribed **of it the main part of the elements of correction**. Entirety of the text of the TP being available on the http://www.code-aster.com/utilisation/formations `website`.

1 Problem of reference

1.1 Geometry



Appears 1.1-a: Deformed shape of the mesh



Appears 1.1-b: Diagram of the thermal loadings and the geometry

It acts of a metal beam (steel 16MND5 $E=210.10^3\,Mpa$, $\nu=0.2$) in bending. Elastic design (MECA_STATIQUE or STAT_NON_LINE) in modelization plane stresses (C_PLAN). Meshes in TRIA3/SEG2 (modelization A) and TRIA6/SEG3 (modelizations B and C).

The various key zones of computation are indicated: GM14 for all the voluminal part SORTED some, GM13 for fixed support (DDL_IMPO DX=DY=0 for all the points $(X=0, Y=0 \ldots 10)$), GM12 for the distributed pressure (PRES_REP=0.1N for all the points $(X=50 \ldots 100, Y=10)$) and GM10 (mesh-POINT $M1=N2$ at the point $(X=100, Y=0)$ on the level of which one will measure the deflection).

1.2 Material properties

On all structure (GROUP_MA GM14), one applies the characteristics material

$$E = 210000 \quad Mpa$$
$$\nu = 0.2$$

1.3 Boundary conditions and loadings

One can synthesize the decomposition of the loadings by zone in the shape of the following table:

Geometrical zones (GROUP_NO/GROUP_MA)	Loadings
GM13	DDL_IMPO DX = 0, DY = 0
GM12	PRES_REP = 0.1 N

2 Reference solution

2.1 Méthode de calcul used for the reference solutions

On such a case, it is not possible to exhume an analytical solution! The reference solution used for error analyses on the deflection and the potential energy of strain is in fact an approximate solution obtained after a series of four uniform refinements (on the same mesh but in TRIA6).

This procedure of uniform refinement can be controlled by a loop PYTHON and operator MACR_ADAP_MAIL option UNIFORME. The first two modelizations are precisely an illustration of this functionality.

2.2 Result of reference

Potential energy of strain $=0.102242\,J$
Marks with arrows $=-0.0614777\,m$

2.3 Uncertainty on the solutions

They acts only of approximate solutions obtained on a “quasi-converged” mesh.

2.4 Bibliographical references

1) X. DESROCHES “Estimators of error of Zhu-Zienkiewicz in elasticity 2D”. [R4.10.01], 1994.

2) X. DESROCHES “Estimator of error in residue”. [R4.10.02], 2000.

3) O. BOITEAU “Course and TP Error indicators & Mesh adaptation; Establishment and state of the art in *the Code_Aster*”. http://www.code - aster.com/utilisation/formations, 2002.

4) O. BOITEAU “FORMA05: Thermomechanical adaptive mesh on a fissured cylinder head”. [V6.03.120], 2002.

3 Modelization A

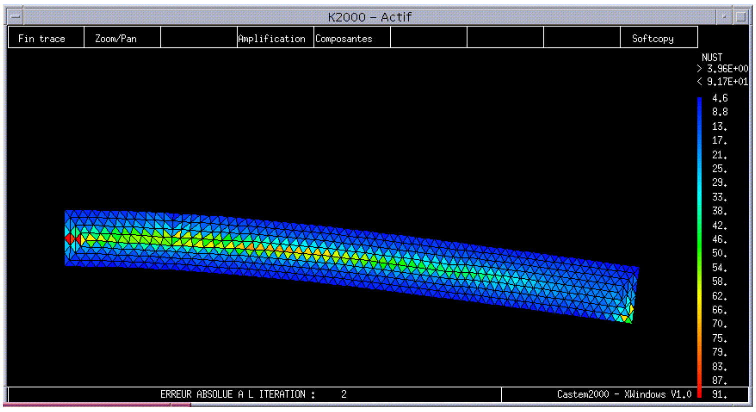
3.1 Characteristic of the modelization

The mesh is carried out with elements of the type TRIA3. The computation is made in linear elasticity with operator STAT_NON_LINE.

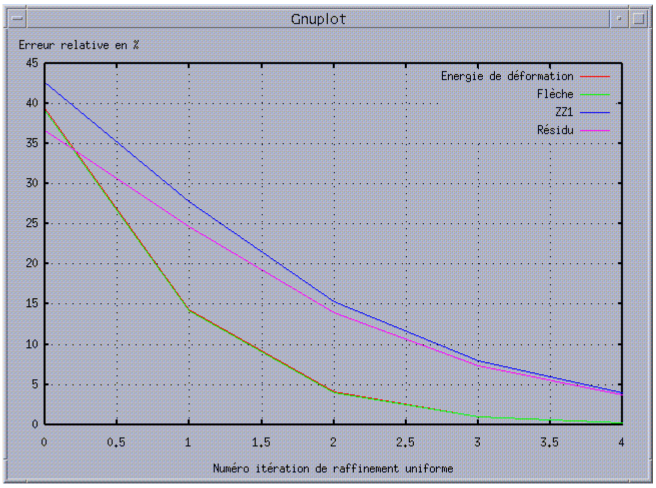
One calculates the cards of spatial errors of the indicator of Zhu-Zienkiewicz version 1 (ERZ1_ELEM) and of the indicator in pure residue (ERME_ELGA). Beforehand it is necessary to have calculated the stress field with nodes (SIGM_ELNO) and, post-to treat the card of error (via GIBI), it should be transformed of a CHAM_ELEM by element with a CHAM_ELEM with the nodes by element. One determines also the value of the deflection (POST_RELEVE_T) and the potential energy of strain (POST_ELEM).
The whole is placed in a loop PYTHON allowing the installation of a uniform procedure of refinement in nb_calc=4 levels (via MACR_ADAP_MAIL option UNIFORME=' RAFFINEMENT').

One can thus note the convergence of the values of deflection and energy, the increase of their errors relative compared to the errors provided by the indicators (they same into relative and on all structure), the variations of the indices of effectiveness of the indicators and their good checking of the assumption of saturation.

In order to illustrate advice of "good practice" for the quality of the studies, on the aspects geometry with a grid, mesh itself and standard of finite elements, one uses the options adhoc LIRE_MAILLAGE, MACR_ADAP_MAIL and MACR_INFO_MAIL.



Appear 3.1-a: Isovaleurs of the error in residue (component absolute ERREST) on the initial mesh.



Appear 3.1-b: Decreases of the relative errors of the energy of déformationet of the deflection compared with those of the relative total component of the indicators.

3.2 Characteristics of the mesh

Initially: 61 TRIA3, 15 SEG2, 48 nodes
After a uniform refinement: 244 TRIA3, 30 SEG2, 156 nodes
After two uniform refinements: 976 TRIA3, 60 SEG2, 555 nodes
After three uniform refinements: 3904 TRIA3, 120 SEG2, 2085 nodes
After four uniform refinements: 15616 TRIA3, 240 SEG2, 8073 nodes

3.3 Quantities tested and results

One tests the values of the relative errors out of deflection and potential energy of strain compared to the reference solutions (cf [§2.2]). And this, on the initial mesh and after four uniform refinements. The tests having to be multi-platforms, the relative tolerance, which on the initial errors is fixed at $10^{6\%}$, is voluntarily slackened on the errors after four refinements: 10-4%.
These tests are carried out on variables PYTHON (via TEST_FONCTION) inserted beforehand in functions ASTER (via FORMULA).

Identification	Values *Code_Aster*	Values of reference	Tolerance	relative Variation (in %)	Variable ASTER	Variable PYTHON
$E_p(0)$	39.406851%	idem	10-6%	$-1.26\ 10^{-12}$ | 0%	ERREEN0	eren0
$E_p(4)$	0.274116%	idem	10-4%	$1.5\ 10^{-12}$ | 0%	ERREEN4	eren4
Deflection (0)	39.244715%	idem	10-6%	$1.09\ 10^{-13}$ | 0%	ERREFL0	erfl0
Deflection (4)	0.270896%	idem	10-4%	$-2.25\ 10^{-13}$ | 0%	ERREFL4	erfl4

3.4 what it was necessary to retain of this part of the TP…

MACR_INFO_MAIL is thus complementary to LIRE_MAILLAGE (VERI_MAIL and INFO) and POST_ELEM . Their combined "forces" can thus allow:

to check the agreement of the mesh with the initial geometry (out of mass, dimension, the surface and volume),
to list the GROUP_MA and GROUP_NO, paramount for a good modelization of the boundary conditions,
to diagnose possible problems (symmetrization or connexity, elements of outline still present in the model, taken into account of boundary conditions on surfaces or lines of bad dimensions, interpenetration of elements),
to strictly evaluate the quality of the mesh from a point of view finite element.

$$\forall K \in T_h \quad \sigma_K = \frac{h_K}{\rho_K}$$ the possible close relation of 1

For example, an empirical criterion could be:

at least 50% of the finite elements with a quality standard below 1.5,
at least 90%, below 2.

The sequence "operators thermo-mécaniques/MACR_ADAP_MAIL OPTION "UNIFORME"" makes it possible to make converge properly, automatically and easily a mesh. It is however necessary to take care of the number of generated degrees of freedom which can quickly become prohibitory!

4 Modelization B

4.1 Characteristic of the modelization

Identical to the modelization A, but in TRIA6.

4.2 Characteristics of the mesh

Initially: 61 TRIA6, 15 SEG3, 156 nodes
After a uniform refinement: 244 TRIA6, 30 SEG3, 555 nodes
After two uniform refinements: 976 TRIA6, 60 SEG3, 2085 nodes
After three uniform refinements: 3904 TRIA6, 120 SEG3, 8073 nodes
After four uniform refinements: 15616 TRIA6, 240 SEG3, 31761 nodes

4.3 Quantities tested and results

One tests the values of the relative errors out of deflection and potential energy of strain compared to the reference solutions (cf [§2.2]). And this, on the initial mesh and after four uniform refinements. The tests having to be multi-platforms, the relative tolerance, which on the initial errors is fixed at 10 $^{6\%}$, is voluntarily slackened on the errors after four refinements: 10-4%.
These tests are carried out on variables PYTHON (via TEST_FONCTION) inserted beforehand in functions ASTER (via FORMULA).

Identification	Values *Code_Aster*	Values of reference	Tolerance	relative Variation (in %)	Variable ASTER	Variable PYTHON
$E_p(0)$	0.125637%	idem	10-6%	– 2.65 10 $^{-12}$ | 0%	ERREEN0	eren0
$E_p(4)$	7.015631 10 $^{-4\%}$	idem	10 $^{-4\%}$	4.71 10 $^{-13}$ | 0%	ERREEN4	eren4
Deflection (0)	0.106929%	idem	10 $^{-6\%}$	1.6 10 $^{-12}$ | 0%	ERREFL0	erfl0
Deflection (4)	1.546674 10 $^{-4\%}$	idem	10-4%	– 3.33 10-13 | 0%	ERREFL4	erfl4

4.4 what it was necessary to retain of this part of the TP…

The elements P_1 are disadvised in mechanics. **The good practice is rather: P_1 lumped in thermal and P_2** (possibly under-integrated) **mechanics** (not artificially not to privilege the thermal component of the strain field and to try to avoid space-time oscillations of the field of temperature and its violation of the principle of the maximum).

The choice of **the type of finite element bonus on the quality of meshes** on which are based this element.

5 Modelization C

5.1 Characteristic of the modelization

Identical to the modelization A with the following modifications:

mesh in TRIA6,
free refinement-coarsening (MACR_ADAP_MAIL option LIBRE=' RAFF_DERA') controlled by component NUEST of ERRE_ELGA_NORE (component relative of the indicator in residue). With as criteria CRIT_RAFF_PE=CRIT_DERA_PE=0.2 (one refines 20% of the worst elements and one déraffine 20% of best).

5.2 Characteristics of the mesh

Initially: 61 TRIA6, 15 SEG3, 156 nodes
After a free refinement: 107 TRIA6, 19 SEG3, 256 nodes
After two free refinements: 212 TRIA6, 26 SEG3, 479 nodes
After three free refinements: 404 TRIA6, 33 SEG3, 879 nodes
After four free refinements: 786 TRIA6, 39 SEG3, 1671 nodes

5.3 Quantities tested and results

One tests the values of the relative errors out of deflection and potential energy of strain compared to the reference solutions (cf [§2.2]). And this, on the initial mesh and after four uniform refinements. The tests having to be multi-platforms, the relative tolerance, which on the initial errors is fixed at 10 6% , is voluntarily slackened on the errors after four refinements: 10-4%.
These tests are carried out on variables PYTHON (via TEST_FONCTION) inserted beforehand in functions ASTER (via FORMULA).

Identification	Values *Code_Aster*	Values of reference	Tolerance	relative Variation (in %)	Variable ASTER	Variable PYTHON
$E_p(0)$	0.125637%	idem	10-6%	– 2.65 10-12 | 0%	ERREEN0	eren0
$E_p(4)$	1.245370 10 2%	idem	10-4%	– 2.27 10-12 | 0%	ERREEN4	eren4
Deflection (0)	0.106929%	idem	10-6%	1.6 10-12 | 0%	ERREFL0	erfl0
Deflection (4)	1.074923 10 – 2%	idem	10 – 4%	– 2.34 10-12 | 0%	ERREFL4	erfl4

5.4 what it was necessary to retain of this part of the TP…

The sequence "operators thermomechanical MACR_ADAP_MAIL OPTION "LIBRE"" makes it possible to make converge optimalement the mesh.

The quality of the elements is impacted little by the process of refinement/coarsening. Taking into account as of choices operated in HOMARD, it can even improve in 3D!

The type of indicator and its mode of standardization affect great the final mesh. Taking into account the type of standardization adopted for the indicators in mechanics,

$$\eta_{rel}(K) = 100 \times \frac{\eta(K)}{\sqrt{\eta(K)^2 + \|\sigma_h\|_{0,K}^{\ 2}}} \text{ (in \%)}$$

On problems with singularities (fixed support, discontinuity of curvature, returning corner, crack….), it is to better use the absolute component of these indicators. Because as for “our good old clamped beam”:

$$\eta_{rel}(K) \to 0 \text{ \% when } \|\sigma_h\|_{0,K} \to \infty \text{ (close to the fixed support)}$$

$$\eta_{rel}(K) \to 100 \text{ \% when } \|\sigma_h\|_{0,K} \to 0 \text{ (close to the deflection)}$$

and this, independently of the true values of the absolute indicator $\eta(K)$!

This does not call at all into question the great utility of these indicators. It is just necessary to take account of these elements to refine its diagnosis and “to possibly juggle” with these two components to refine in the zones of interest.

The problem does not arise in thermal, because the indicator in residue for the thermal problem is standardized differently. One can however compose with the components of the thermal indicator and the limiting conditions, “fictitious” or not, to direct the construction of a mesh refined or déraffiné by zones (cf [§6.3] [R4.10.03] and modelization A, _TP21 _, of documentation [V6.03.120]).

6 Summary of the results

In this case, it acts to make sure of **the NON-regression of the TP n°1 associated with the courses "Error indicators and mesh adaptation; Establishment and state of the art in *Code_Aster* "** of training "nonlinear Static analysis with *Code_Aster* ".

In fact, one "abuses" **an elastic design on a metal beam in bending** in modelization plane stress. One makes it **uniformly converge** via the tools of refinement-coarsening HOMARD encapsulated in `MACR_ADAP_MAIL`, then **freely** by coupling the process with a card of errors spatial localised on each finite element.

The purposes of this TP are multiple, it acts:

- to familiarize and put into practice the two dual problems: computation of card of error indicator and strategies of mesh adaptation. On standard cases, but also on pathological cases…,
- to detail the various parameter settings of the accused operators (`CALC_ERREUR`, `MACR_ADAP_MAIL`) and related operators who can appear particularly interesting for these problems (`INFO_MAILLAGE`, `MACR_INFO_MAIL`, `PROJ_CHAMP`…),
- to hammer advice of "good practice" for the quality of the studies and the use of the tools already available on the subject. One is interested only in the aspects geometry with a grid, mesh itself and standard of finite elements. One is not delayed here on the problems of time step, calibration of parameters numerical and on the aspects sensitivity with respect to the data,
- to illustrate the formidable potentialities and facilitated which allows the coupling "language `ASTER/PYTHON` " in the command file of a study (test, buckles, display, computation, personal macro-command, interactivity…). The official benchmarks being gauged to function in batch, some of these aspects "were thus commentarisés" in the command file.

From a data-processing **validation point of view**, this case test of course makes it possible to test the NON-regression of various couplings computations of card of errors/procedure of refinement-coarsening in mechanics, but also the options the "pre one and postprocessings" of these computations (lissage of the nodal stresses, transition of an error per element with an error with the nodes by element).

Each modelization is associated with question TP and one retranscribed **of it "substantial" the marrow of the elements of correction**. Entirety of the text of the TP being available on the http://www.code-aster.com/utilisation/formations `website`.