FORMA10 - Practical works of training “advanced Use”: way of Summarized

loading:

This test illustrates, on a material point, the influence of way of loading on the response of an elastoplastic behavior. It highlights the effects of discretization in time. It takes as a starting point the test SSNP15: a material point, made up of a plastic material with linear isotropic hardening, is subjected at the same time to a shears and tractive effort. The principal interest of this test lies in the nonradial character of the loading.

The modelization A corresponds to computation with imposed force, with behavior VMIS_ISOT_LINE, and illustrates the influence of a discretization in the time coarse by comparison with the solution obtained with time step finer. To obtain a solution with time step the finer, one forces the recutting of time step on a criterion based on the increment of cumulated plasticity (modelization B).

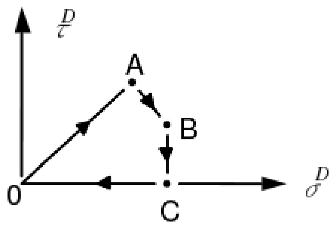
1 Problem of reference

1.1 Geometry

It acts of a material point: stress state and of strains homogeneous. This can be represented by only one finite element, with adapted boundary conditions, as in SSNP14 and SSNP15.

1.2 Boundary conditions and loadings

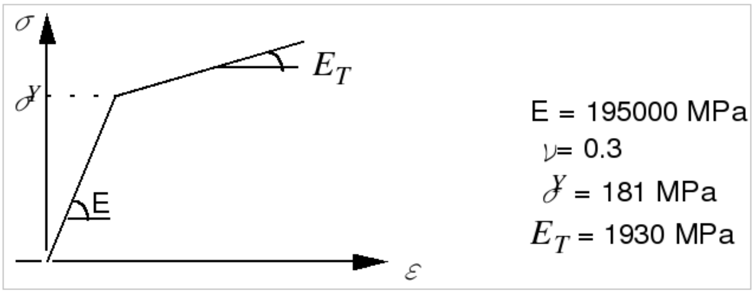
One imposes a way of loading defined by a component of tension and a component of shears, in the following way:



	σ^D (MPa)	τ^D (MPa)
A	151.2	93.1
B	257.2	33.1
C	259.3	0

1.3 Properties of the materials

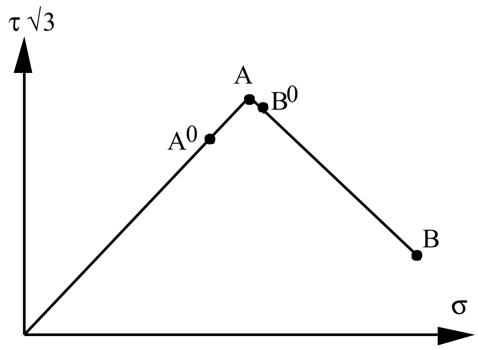
the behavior is elastoplastic of Von Mises, with isotropic hardening. Hardening is linear.



2 Reference solution

2.1 Method of calculating used for the reference solution

It is identical to that of test SSNP15.
In the plane $(\sigma, \tau\sqrt{3})$, the norm of von Mises results in the classical distance, so that one can immediately predict the phases of load and of discharge during the way of loading, since it is respectively the phases where the norm grows or decrease:

Trajet de chargement	Valeurs numériques $(\sigma \; ; \; \tau)$		Phases de chargement	
	A^0	(123,8 ; 76,23)	O-A^0	charge élastique
	A	(151,2 ; 93,1)	A^0-A	charge plastique
	B^0	(158,23 ; 89,12)	A-B^0	décharge
	B	(257,2 ; 33,1)	B^0-B	charge plastique

2.1.1 Approach of resolution

Mechanically, it acts of a test 0D controlled in stress, the material being elastoplastic with criterion of von Mises and linear isotropic hardening. For a loading controlled in stress, one easily determines the cumulated plastic strain:

$$F(\sigma, p)=\sigma_{eq}-\sigma^{Y}-R' p\leq 0\Rightarrow p=\frac{\sigma_{eq}-\sigma^{Y}}{R'} \text{ in chargeéq} \qquad 2.1.1\text{- } 2.1.1\text{-}1$$

the integration of the plastic strain is of course more delicate. The flow equation is written:

$$\dot{\varepsilon}^{p}=\frac{3}{2}\dot{p}\frac{\tilde{\sigma}}{\sigma_{eq}}\Rightarrow\dot{\varepsilon}^{p}=\frac{3}{2R'}\frac{\dot{\sigma}_{eq}}{\sigma_{eq}}\tilde{\sigma} \text{ in load} \qquad \text{éq } 2.1.1\text{- } 2.1.1\text{-}2$$

Lastly, one will deduce the strain via the relation from state:

$$\varepsilon=\varepsilon^{p}+E^{-1}:\sigma\Rightarrow\varepsilon_{xx}=\varepsilon_{xx}^{p}+\frac{\sigma}{E} \text{ and } \varepsilon_{xy}=\varepsilon_{xy}^{p}+\frac{\tau}{2\mu} \qquad \text{éq } 2.1.1\text{- } 2.1.1\text{-}3$$

2.1.2 Processing of the phase of radial loading

Let us notice that in phase of radial loading, the flow model [éq 2.1.2-1] is integrated directly:

$$\boldsymbol{\varepsilon}^{\mathbf{p}}=\frac{3}{2}p\frac{\tilde{\boldsymbol{\sigma}}}{\sigma_{eq}} \qquad \text{éq 2.1.2-1}$$

the cumulated plastic strain is then given by [éq 2.1.1-1], the plastic strain by [éq 2.1.2-1] and the total deflection by [éq 2.1.1-3]. With:

E $=195\ 000\ MPa$ 2μ $=150\ 000\ MPa$ R' $=1\ 949,29\ Mpa$

One obtains:

$p(A)$	$=2,0547\ 10^{-2}$	$\varepsilon_{xx}^{p}(A)$	$=1,4054\ 10^{-2}$	$\varepsilon_{xx}(A)$	$=1,4830\ 10^{-2}$
		$\varepsilon_{xy}^{p}(A)$	$=1,2981\ 10^{-2}$	$\varepsilon_{xy}(A)$	$=1,3601\ 10^{-2}$

2.1.3 Processing of the phase of nonradial loading

In the phase of nonradial loading $B0-B$, one can parameterize the way of stress by:

$$\boldsymbol{\sigma}(q)=\boldsymbol{\sigma}^{\mathbf{B^0}}+q\underbrace{\left(\boldsymbol{\sigma}^{\mathbf{B}}-\boldsymbol{\sigma}^{\mathbf{B^0}}\right)}_{\text{direction fixe}} \quad \text{avecéq} \quad 0\leq q\leq 1 \qquad 2.1.3\text{-}1$$

As the way of loading remains confined in the plane tension-shears (σ,τ) , one will may find it beneficial to represent the stress state by a complex number:

$$\Sigma=\sigma+i\sqrt{(3)}\tau\Rightarrow\sigma_{eq}=|(\Sigma)| \quad \text{and} \quad \Sigma(q)=\Sigma^{B^0}+q\underbrace{\left(\Sigma^{B^0}-\Sigma^{B}\right)}_{\text{direction fixe}} \qquad \text{éq 2.1.3-2}$$

the integration of the flow model [éq 2.1.1-2], followed by a integration by part, makes it possible to express the plastic strain:

$$\frac{2\mathrm{R}'}{3}\left[\boldsymbol{\varepsilon}^{\mathbf{p}}\right]_0^1=\int_0^1\frac{\dot{\sigma}_{eq}}{\sigma_{eq}}\tilde{\boldsymbol{\sigma}}\,dq=\left[\ln\left(\sigma_{eq}\right)\tilde{\boldsymbol{\sigma}}\right]_0^1-\frac{1}{2}\underbrace{\dot{\tilde{\boldsymbol{\sigma}}}}_{\tilde{\sigma}^B-\tilde{\sigma}^{B^0}}\int_0^1\ln\left(\sigma_{eq}^2\right)dq$$

The adoption of the complex plane allows an easy computation of the last integral:

$$\int_0^1\ln\left(\sigma_{eq}^2\right)dq=\int_0^1\ln\left(\Sigma\,\overline{\Sigma}\right)dq=\int_0^1\ln(\Sigma)\,dq+\int_0^1\ln\left(\overline{\Sigma}\right)dq=2\Re\left[\int_0^1\ln(\Sigma)\,dq\right]=2\Re\left[\frac{\Sigma\ln(\Sigma)-\Sigma}{\Sigma^B-\Sigma^{B^0}}\right]_0^1$$

Finally, the increment of plastic strain on the way $B0-B$ is worth:

$$\left[\boldsymbol{\varepsilon}^{\mathbf{p}}\right]_{B^0}^{B}=\frac{3}{2R'}\left[\ln\left(\sigma_{eq}\right)\tilde{\boldsymbol{\sigma}}\right]_{B^0}^{B}-\frac{3}{2R'}\Re\left[\frac{\Sigma\ln(\Sigma)-\Sigma}{\Sigma^B-\Sigma^{B^0}}\right]_{B^0}^{B}\left(\tilde{\sigma}^B-\tilde{\sigma}^{B^0}\right) \qquad \text{éq 2.1.3-4}$$

2.2 Results of reference

By calculating the plastic strain cumulated by [éq 2.1.1-1], the plastic strain by [éq 2.1.3-4] and the total deflection by [éq 2.1.1-3], one obtains:

$p(B)$	$=4,2329\ 10^{-2}$	$\varepsilon_{xx}^{p}(B)$	$=3,3946\ 10^{-2}$	$\varepsilon_{xx}(B)$	$=3,5265\ 10^{-2}$
One obtains:		$\varepsilon_{xy}^{p}(B)$	$=2,0250\ 10^{-2}$	$\varepsilon_{xy}(B)$	$=2,0471\ 10^{-2}$

One will be interested in the values of the stresses, the strains and the plastic strain cumulated at the points A and B the way of loading.

2.3 Bibliographical references

1) French company of the Mechanics. Guide validation of the software packages of structural analysis (VPCS). Technical AFNOR, 1990.

3 Modelization A

3.1 Characteristic of the modelization

It is operated of a test material point. One uses for that the command `SIMU_POINT_MAT`, which allows computation on only one element, with only one point of integration.

The way of loading between the point A and the point B is discretized into 5 time step.

3.2 Quantities tested and results

Identification	**Times**	**Reference**	***Aster***	**% difference**
ε_{xx}	A	1.4830 10-2	1.48297 10-2	– 0.002
ε_{xy}	A	1.3601 10-2	1.360110-2	0.003
ε_{xx}	B	3.5265 10-2	3.5686 10-2	1.2
ε_{xy}	B	2.0471 10-2	1.9577 10-2	– 4.3

4 Modelization B

4.1 Characteristic of the modelization

It is operated of a test material point. One uses for that the command `SIMU_POINT_MAT`, which allows computation on only one element, with only one point of integration.

The way of loading between the point A and the point B is discretized into 1 time step. But one uses a rather recent functionality which makes it possible Re-to cut out time step if an additional criterion is not checked with convergence. Here, one proposes to check that with each convergence, the increment of cumulated plastic strain does not exceed by 0,2 10-2. As soon as this criterion is not satisfied, then the automatic code Re-cutting time step and starts again until this criterion is satisfied.
As an indication, one thus carries out 25 computations between the point A and the point B, this discretization temporal is thus described as fine, compared with that used with modelization A.
It is pointed out that the point A corresponds to time $t=1\text{s}$ and the point B corresponds to time $t=2\text{s}$.

4.2 Quantities tested and results

Identification	Times	Reference	% difference
ε_{xx}	A	1.4830 10-2	– 0.002
ε_{xy}	A	1.3601 10-2	0.003
ε_{xx}	B	3.5265 10-2	0.39
ε_{xy}	B	2.0471 10-2	1.1

One can graphically note on Figure 4.2-1 the difference on the strain ε_{xy} between a fine temporal discretization (modelization B) and a coarse temporal discretization (5 time step, modelization A). This difference exists only for the nonradial part of the loading (beyond the point A). One sees clearly that for the modelization B, time step becomes rather small with the approach of the point B, right before time $t=2\text{s}$.

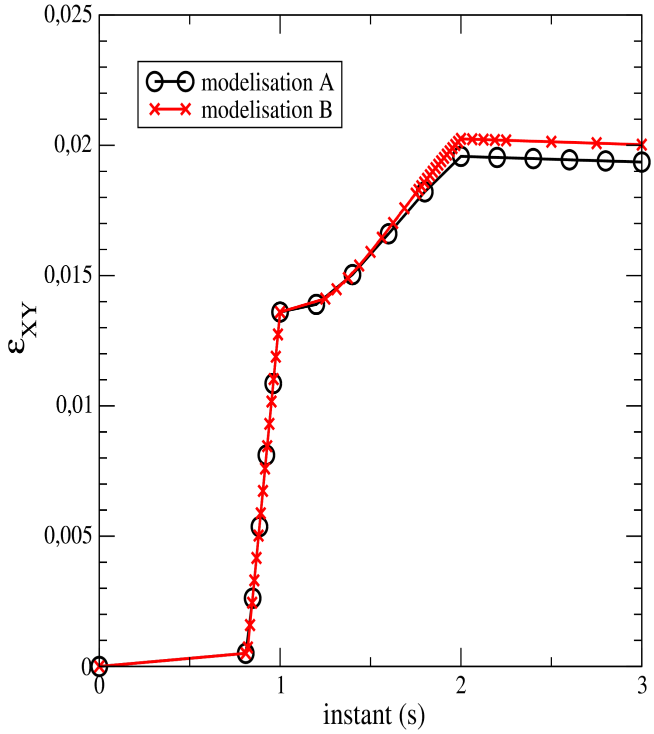


Figure 4.2-1: comparison between various temporal discretizations

5 Summary of the results

This test makes it possible to highlight, on a nonradial elastoplastic problem, the influence of the discretization in time.