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## SSNP110 - edge Crack in a rectangular plate finished in elastoplasticity

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### Summarized:

This test is a benchmark in nonlinear fracture mechanics.

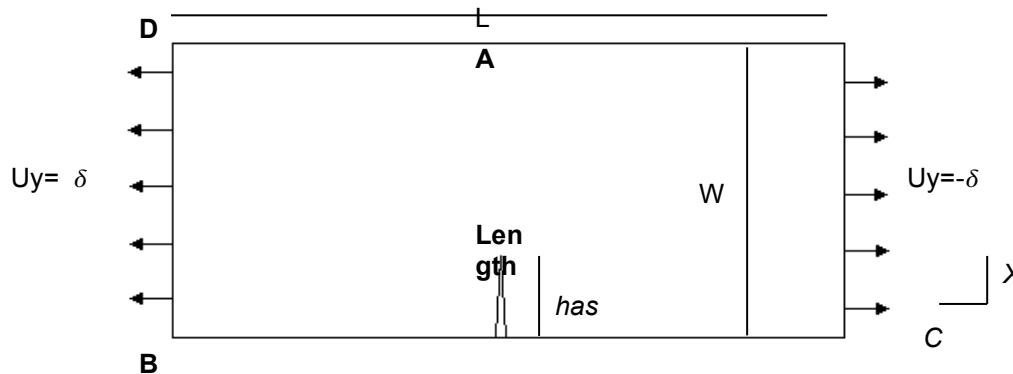
One considers a rectangular plate finished, fissured and subjected to a loading of tension. The constitutive law used is an elastoplastic model of Von Mises.

This benchmark understands five modelizations in 2D:

- the modelizations A and B, in plane stresses, aim at studying the influence of the taking into account or not of the terms of second order of the strains ( `DEFORMATION = "PETIT" or "GROT_GDEP"` in operator `STAT_NON_LINE` ) on the computation of the rate of refund of energy  $G$  ;
- the modelizations C and D, in plane stresses, aim at testing the method X-FEM (crack nonwith a grid) in nonlinear elasticity and elastoplasticity for computation of the field of displacement;
- the modelization E, in plane strains, validates the computation of  $G$  for the various methods of definition of the curve of hardening.

## 1 Problem of reference

### 1.1 Geometry



Width	$L = 50 \text{ mm}$
Depth	$W = 16 \text{ mm}$
of crack Properties	$a = 6 \text{ mm}$

### 1.2 of the material

the material is elastoplastic there of type Von Mises. For the modelizations A with D, there is no hardening. For the modelization E, various models of hardening are compared (linear hardening or power). The properties of the material are the following ones:

Young modulus	$E = 2,0601 \cdot 10^5 \text{ MPa}$
Poisson's ratio	$\nu = 0.3$
Yield stress	$\sigma_y = 808,34 \text{ MPa}$
Hardening modulus	$H = 0$ (modelization A with D) or $H = 6,867 \cdot 10^4 \text{ MPa}$ (modelization E)

### 1.3 Boundary conditions and loading

For the modelizations A, B and E, the model is restricted with half of structure, the plane of the crack vertical being a symmetry plane. For the modelizations C and D, the totality of structure is represented.

#### Boundary conditions

vertical Displacement  $UX = 0$  at the point  $B$

horizontal Displacement  $UY = 0$  in the ligament  $AB$  (condition of symmetry for the modelizations A, B and E)

#### Loading

horizontal Displacement imposed on the segment  $CD$  :  $UY = \delta$

## 2 Reference solution

No reference solution. This is a test of NON-regression.

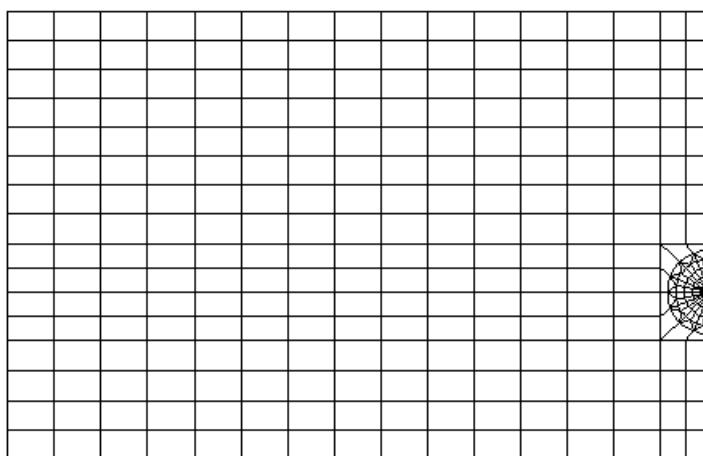
### 3 Modelization A

#### 3.1 Characteristic of the modelization

It acts of a computation in elastoplasticity under the assumption of small displacements, in plane stresses.

#### 3.2 Characteristics of the mesh

The mesh, built with an automatic procedure `gibi`, consists of 400 quadratic elements (1000 nodes). Tori are defined in crack tip in order to improve the accuracy of computation in fracture mechanics, cf [Figure 3.2-a] below. The radius of the largest torus is of  $1,5 \text{ mm}$ .



Appear 3.2-a: Mesh of the fissured rectangular plate

#### 3.3 Features tested

Computation of the rate of refund of energy by the method  $G-\theta$  in elastoplasticity.

#### 3.4 Quantities tested and results

the values of the rate of refund are tested for five values of imposed horizontal displacement  $\delta$ . One compares the results got for three different integration contours:

- crown 1:  $R_{inf}=0,15 \text{ mm}$  ;  $R_{sup}=0,6 \text{ mm}$
- crown 2:  $R_{inf}=0,3 \text{ mm}$  ;  $R_{sup}=0,9 \text{ mm}$
- crown 3:  $R_{inf}=0,9 \text{ mm}$  ;  $R_{sup}=1,5 \text{ mm}$

Displacement imposed $\delta$ ( mm )	$G(N/mm)$ contour 1	$G(N/mm)$ crowns 2	$G(N/mm)$ contour the 3
0,02	3.29	3.20	3.20
0,04	13.60	13.24	13.24
0,06	31.97	31.22	31.24
0,08	58.99	57.74	57.76
0,1	91.42	89.64	89.71

results are satisfactory: the maximum difference between the values of  $G$  obtained on three integration contours is lower than 2%.

## 4 Modelization B

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### 4.1 Characteristic of the modelization

It acts of a computation in elastoplasticity under the assumption of large displacements, in plane stresses.

The purpose of this benchmark is to examine the influence of the taking into account of large deformation in mechanical computation on the parameters of the fracture mechanics .

### 4.2 Characteristics of the mesh

The mesh is identical to that of modelization A.

### 4.3 Fonctionnalités tested

Computation of the rate of refund of energy by the method  $G-\theta$  in elastoplasticity.

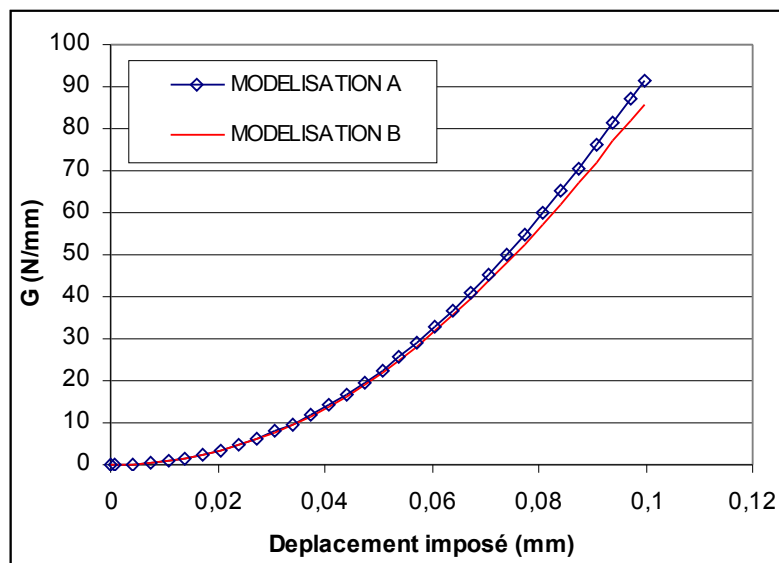
### 4.4 Quantities tested and results

the values of the rate of refund are tested for same integration contours as in modelization A.

Déplacement imposed $\delta$ ( mm )	$G$ ( N/mm ) contour 1	$G$ ( N/mm ) crowns 2	$G$ ( N/mm ) contour the 3
0,02	3.26	3.17	3.18
0,04	13.36	13.03	13.07
0,06	31.02	30.50	30.67
0,08	56.33	55.90	56.51
0,1	85.84	85.91	87.47

results are satisfactory: the maximum difference between the values of  $G$  obtained on three integration contours is lower than 2%.

The effect of the terms of second order in the strain is relatively weak: the difference between the results of the two modelizations is increasing with imposed displacement  $\delta$  and is worth to the maximum 6%, cf [Figure 4.4-a].



Appear 4.4-a: Comparison of rates of energy restitution of the two modelizations

## 5 Modelization C

### 5.1 Characteristic of the modelization

It acts of a computation in nonlinear elasticity under the assumption of small displacements, with the method X-FEM (crack nonwith a grid). The modelization is in plane stresses.

### 5.2 Characteristics of the mesh

The mesh is composed of linear elements (560 meshes QUA6 - 600 nodes). The refinement of the mesh is uniform (18 elements according to  $x$  and 31 elements according to  $y$ ).

### 5.3 Features tested

Computation of displacement for a model X-FEM in nonlinear elasticity.

### 5.4 Quantities tested and results

Several types of quantities are tested: indicator of radiality of the loading, displacements, and rate of energy restitution.

#### 5.4.1 Indicator of radiality of the loading

One on all the tests the maximum of component DCHA\_V of field DERA\_ELGA Gauss points. It is a test of regression.

Standard	identification of reference	Value of reference	Tolerance
MAX (DCHA_V)	"NON-REGRESSION"	0.58	10-6%

##### 5.4.1.1

#### 5.4.2 Displacements

One tests the component DY displacement at the two points where the crack emerges (points of each lip), at the last time of computation (imposed displacement  $0,1\text{ mm}$ ). This test is carried out on the field of displacement created by the operator POST\_CHAM\_XFEM.

The value of reference is the solution obtained in Standard modelization

A.	Identification of reference	Value of reference	Tolerance
Lip $y > 0$	"AUTRE_ASTER"	$9,876 \cdot 10^{-2}$	2.00%
Lip $y < 0$	"AUTRE_ASTER"	$-9,876 \cdot 10^{-2}$	2.00%

##### 5.4.2.1

#### 5.4.3 Rate of energy restitution

One tests the computation of rate of energy restitution  $G$  with 2 different contours:

-crown n°1 enters  $h$  and  $4h$ ,

-contour n°2 between  $2h$  and  $6h$

where  $h$  is the size of an element.

Standard	identification of	Value of reference	Tolerance
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reference

Crowns n°1: $G$ with $t=0,02$	"AUTRE_ASTER"	3,29	4.00%
Contour n°1: $G$ with $t=0,04$	"AUTRE_ASTER"	13,60	4.00%
Contour n°1: $G$ with $t=0,06$	"AUTRE_ASTER"	31,97	4.00%
Contour n°1: $G$ with $t=0,08$	"AUTRE_ASTER"	58.99	4.00%
Contour n°1: $G$ with $t=0,10$	"AUTRE_ASTER"	91.42	4.00%
Contour n°2: $G$ with $t=0,02$	"AUTRE_ASTER"	3,29	4.00%
Contour n°2: $G$ with $t=0,04$	"AUTRE_ASTER"	13,60	4.00%
Contour n°2: $G$ with $t=0,06$	"AUTRE_ASTER"	31,97	4.00%
Contour n°2: $G$ with $t=0,08$	"AUTRE_ASTER"	58.99	4.00%
Contour n°2: $G$ with $t=0,10$	"AUTRE_ASTER"	91.42	4.00%

5.4.3.1

## 6 Modelization D

### 6.1 Characteristic of the modelization

It acts of a computation in elastoplasticity under the assumption of small displacements, with the method X-FEM (crack nonwith a grid). One represents the opening of crack and his closing by activating the contact on the lips.

Compared to the modelization C one replaces nonlinear elasticity by elastoplasticity. Until time 0,1 the loading is monotonous and the results are thus identical between the modelizations C and D. In the modelization D, one carries out then a discharge and one applies an opposed loading, thus activating the contact to the lips of crack.

### 6.2 Characteristics of the mesh

The mesh is identical to that of the modelization C.

### 6.3 Functionalities tested

Computation of displacement for a model X-FEM in elastoplasticity.

### 6.4 Quantities tested and results

two quantities are tested:

- the component  $DY$  of displacement at the two points where the crack emerges (points of each lip), at time 0,1 of computation (imposed displacement  $0,1\text{ mm}$ ) and at the last moment 0,3 (imposed displacement  $-0,1\text{ mm}$ );
- values of the effective plastic strain ( $VI$ ) on the mesh  $M277$ , in item 10 and at times 0,1, 0,14 and 0,3.

For time 0,1, the value of reference of displacement is the solution obtained in modelization A. For the rest, the tests are of NON-regression.

Reference	displacemen t ( mm )	Code_Aster ( mm )	Difference
Lip $y > 0$ , $t = 0.1$	$9,876.10^{-2}$	$9,532.10^{-2}$	-3,5%
Lip $y < 0$ , $t = 0.1$	$-9,876.10^{-2}$	$-9,533.10^{-2}$	-3,5%
Lip $y > 0$ , $t = 0.3$	$1,782.10^{-2}$	NON-regression	
Lip $y < 0$ , $t = 0.3$	$-1,782.10^{-2}$	NON-regression	
<b>effective Plastic strain ( <math>VI</math> on <math>M277</math> )</b>			
$t = 0.1$	$1.270.10^{-2}$	NON-regression	
$t = 0.14$	$1.270.10^{-2}$	NON-regression	
$t = 0.3$	$2.451.10^{-2}$	NON-regression	

This test validates computation in elastoplasticity for models X-FEM.

## 7 Modelization E

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### 7.1 Characteristic of the modelization

It acts of a computation in nonlinear elasticity under the assumption of small displacements, in plane strains.

### 7.2 Characteristics of the mesh

The mesh is identical to that of modelization A.

### 7.3 Fonctionnalités tested

Computation of the rate of refund of energy by the THETA method in nonlinear elasticity. One compares the three methods to define hardening: TENSION, ECRO\_LINE and ECRO\_PUIS.

### 7.4 Quantities tested and results

the quantity tested is the rate of refund  $G$ , calculated for the first integration contour defined in the modelization A and for ten values of horizontal displacement imposed  $\delta$ . ( $\delta$  varying from 0 with 0,2mm ).

Two series distinct from computations are carried out: initially, one takes a constitutive law with linear hardening. Three computations are carried out with the various methods available to define the same curve of hardening (TENSION, ECRO\_LINE and ECRO\_PUIS). One compares between them the solutions obtained by these three computations; then a constitutive law with hardening out of model power is chosen and one compares the results got between ELAS\_VMIS\_TRAC and ELAS\_VMIS\_PUIS.

The test is of standard NON-regression. It is checked that: three various computations in linear hardening lead exactly to same result on  $G$ ; two computations in hardening out of model power lead to very close results (lower deviation than 0,3%). The variation decreases if one decreases time step by computation.

Imposed displacement $\delta$ ( mm )	$G(N/mm)$ - linear hardening:	$G(N/mm)$ - hardening linear:	$G(N/mm)$ - hardening linear:
	ELAS_VMIS_TRAC	ELAS_VMIS_LINE	ELAS_VMIS_PUIS
0,02	3,59	3,59	3,59
0,04	14,47	14,47	14,47
0,06	32,88	32,88	32,88
0,08	59,42	59,42	59,42
0,10	94,50	94,50	94,50
0,12	134,98	134,98	134,98
0,14	178,95	178,95	178,95
0,16	226,24	226,24	226,24
0,18	276,47	276,47	276,47
0,20	329,61	329,61	329,61

imposed Displacement $\delta$ ( mm )	$G(N/mm)$ - hardening power:	$G(N/mm)$ - hardening power:
	ELAS_VMIS_TRAC	ELAS_VMIS_PUIS
0,02	3,59	3,59
0,04	14,44	14,44
0,06	32,76	32,76
0,08	58,99	58,98
0,10	93,76	93,77
0,12	136,14	136,18
0,14	184,04	184,09
0,16	236,31	236,38
0,18	292,38	292,46
0,20	351,82	351,90

This test validate the computation of  $G$  in linear elasticity for the various methods of definition of the curve of hardening.

## 8 Summary of the results

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This case test aims at validating the computation of the rate of refund of the stresses in elastoplasticity:

- the modelizations A and B make it possible to make sure of the invariance of the computation of the rate of refund of energy by the method theta according to integration contours for constitutive laws of the elastic type - perfectly plastic. One also notes the weak contribution of the terms of second order in the strain;
- the modelization E validates the various methods of definition of the curve of hardening.

The modelizations C and D make it possible to validate the computation of the field of displacement for the models X-FEM.