SSNP109 - Cable of prestressing excentré in a Summarized concrete straight beam

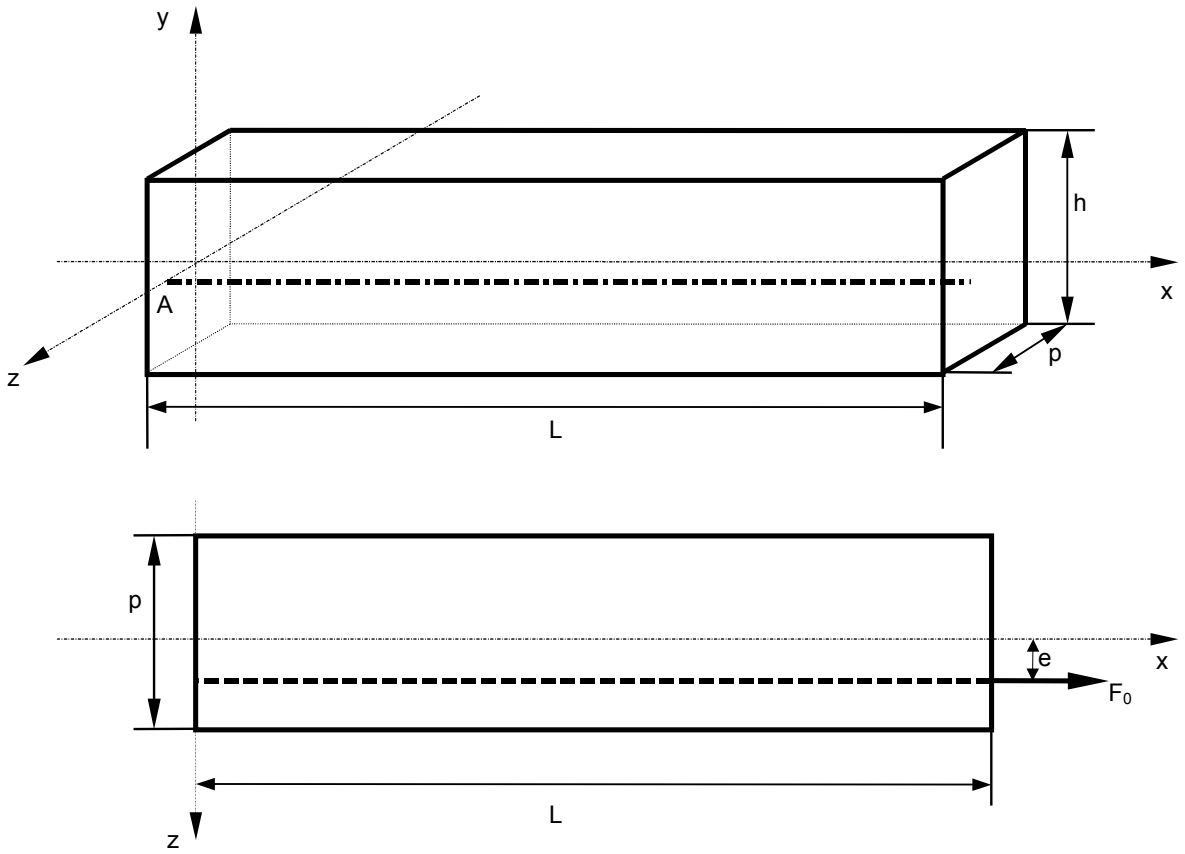
One considers a concrete straight beam, of rectangular section, crossed over its length by a cable of prestressed out of steel. The cable is right, parallel to average fiber of the beam, and passes to middle height of the section of the beam, while being excentré compared to the average plane. The left section of the beam and the left end of the cable are fixed. The cable is put in tension at its right end, in order to prestress the beam in bending-compression. The losses of tension along the cable are neglected.

The goal of this benchmark is to validate the method of calculating of the state of equilibrium of a structure of concrete prestressed by comparison with an analytical reference solution.

1 Problem of reference

1.1 Geometry

the beam out of concrete is right, of rectangular section.
Its dimensions are: $L \times h \times p = 10\,m \times 0{,}4\,m \times 0{,}2\,m$ ($y = h/2$).
The cable crosses the beam parallel with average fiber of the beam, with middle height. Its eccentricity compared to the average plane is $e = 0{,}05\,m$ ($z = e$).
The area of the straight section cable is worth $S_a = 1{,}5.10^{-4}\,m^2$.



1.2 Properties of the materials

Concrete material constituting beam: Young modulus $E_b = 3.10^{10}\,Pa$
Material steel constituting the cable: Young modulus $E_a = 2{,}1.10^{11}\,Pa$

the Poisson's ratio is taken equal to 0 for the two materials. One thus cancels the effects Fish in the directions y and z . Displacements have components only in the plane (x, z) .

The losses of tension in the cable being neglected, the various parameters being used for their estimate are fixed at 0.

1.3 Boundary conditions and loadings

the point A located in bottom of left edge of the beam, coordinates $(0\ ;-h/2\ ;0)$, is blocked in translation according to the three directions and rotation around the axis y .
The blocking of the degree of freedom of rotation DRY implies a slope null deformed shape of average fiber in $x = 0$.

The left end of the cable, of coordinates $(0\ ;0\ ;e)$, is blocked in translation according to the three directions.

One applies at the right end of the cable, coordinates $(L;0\ ;e)$, a normal force of tension $(F_0\ ;0\ ;0)$ where $F_0=2.10^5\,N$.

2 Reference solution

the analytical solution of reference is determined by the theory of the beams.
A embed-free beam is considered. The geometrical characteristics are those defined in paragraph [§2.1]. The prestressed cable applies at the loose lead a normal force of compression $(-F\,;0\,;0)$ and a bending moment $(0\,;-eF\,;0)$.

The solution of this problem is the following one:

Tensor of the stresses : $\sigma=\begin{bmatrix}\sigma_{xx} & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$ with $\sigma_{xx}=-\frac{F}{hp}\left(1+\frac{12ez}{p^2}\right)$

Displacements: $\begin{cases}u(x,y,z)=-\frac{F}{E_b hp}\left(1+\frac{12ez}{p^2}\right)x\\ v(x,y,z)=\frac{\nu_b F}{E_b hp}\left(1+\frac{12ez}{p^2}\right)\left(y+\frac{h}{2}\right)\\ w(x,y,z)=\frac{F}{E_b hp}\left(\nu_b z\ +\frac{6e}{p^2}\left(x^2-\nu_b\left(\left(y^2-\frac{h^2}{4}\right)-z^2\right)\right)\right)\end{cases}$

with **the boundary conditions** : $\begin{cases}u=v=w=0\\ \theta_y=0\end{cases}$ en $x=0$, $y=-\frac{h}{2}$, $z=0$

When the effects Fish are neglected ($\nu_b=0$), the solution in displacements is simplified as follows:

$$\begin{cases}u(x,y,z)=-\frac{F}{E_b hp}\left(1+\frac{12ez}{p^2}\right)x\\ v(x,y,z)=0\\ w(x,y,z)=\frac{F}{E_b hp}\times\frac{6ex^2}{p^2}\end{cases}$$

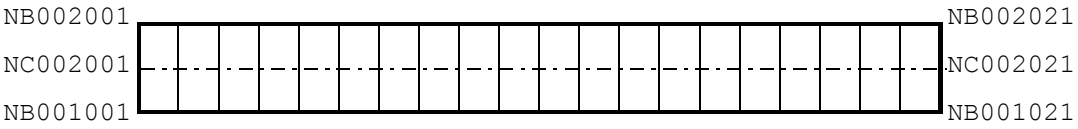
The numerical values of reference are calculated using the analytical statements above, by means of for F the value with the overall equilibrium of the normal force in the cable:

$$F=-F_0\,\frac{E_b hp}{E_b hp+E_a S_a\left[1+\frac{12e^2}{p^2}\right]}$$

3 Modelization A

3.1 Characteristic of the modelization

the figure below gives a simplified representation of the mesh of the beam.



The concrete beam is represented by 20 elements of the type DKT, supported per as many meshes quadrangles with 4 nodes.

One thickness $p=0,2m$ their is affected, as well as concrete material for which behaviors ELAS (Young modulus) are $E_b=3.10^{10}Pa$ defined and BPEL_BETON : the parameters characteristic of this relation are fixed at 0 because one neglects the losses of tension along the cable of prestressing.

The degrees of freedom DX DY, DZ and DRY of the node $NB001001$ are blocked.

The cable is represented by 20 elements MECA_BARRE, supported per as many meshes segments with 2 nodes. The ends left and right-hand side are respectively the nodes $NC001001$ and $NC001021$.

An area of cross-section $S_a=1,5.10^{-4}m^2$ is assigned to the elements, as well as a material steel for which behaviors ELAS (Young modulus) are $E_a=2,1.10^{11}Pa$ defined and BPEL_ACIER : the parameters characteristic of this relation are fixed at 0 (neglected losses of tension), except for the stress yield stress for which the value of $f_{prg}=1,77.10^9Pa$ is selected.

The degrees of freedom DX DY, and DZ of the node $NC001001$ are blocked.

The tension $F_0=2.10^5N$ is applied to the node $NC001021$. This value of tension is coherent with the values of section and yield stress, for a cable of prestressed of standard strand.

The computation of the state of equilibrium of the beam group and cable is carried out in only one step, the behavior being elastic. One carries out then two complementary computations allowing to determine the stresses in skins lower and higher ($z=\pm p/2$) of the beam.

3.2 Stages of computation and features tested

the main steps of computation correspond to the features which one wishes to validate:

- operator DEFI_MATERIAU : definition of behavior models BPEL_BETON and BPEL_ACIER, in the cas particulier where the losses of tension along the cable of prestressing are neglected (default values of the parameters);
- operator DEFI_CABLE_BP : determination of a constant profile of tension along the cable of prestressing, losses being neglected; computation of the coefficients of the kinematic relations between the degrees of freedom of the nodes of the cable and the degrees of freedom of the nodes “close” to the beam out of concrete, in the case of a excentré cable;
- operator AFFE_CHAR_MECA : definition of a loading of the type RELA_CINE_BP ;
- operator STAT_NON_LINE, option COMP_INCR : computation of the state of equilibrium by taking account of the loading of the type RELA_CINE_BP.

One uses finally operator CALC_CHAMP option SIGM_ELNO in order to calculate the stresses in lower skin then in higher skin of the beam.

3.3 Quantities tested and results

the value with the equilibrium of the normal force in the cable is $F = 1{,}9550910^5 N$. This value is used to compute: the numerical results of reference using the analytical statements clarified in paragraph [§3].

3.3.1 Displacements of the nodes of the concrete part

One compares the values extracted field DEPL resulting from STAT_NON_LINE with the theoretical values from reference corresponding to the plane $Z = 0$.
The tolerance of relative variation compared to the reference is worth 0,1%.

Node	Component	Value of reference	Computed value	relative Variation
NB001006	DX	– 2,036552.10-4 m	– 2,0365561834835.10-4 m	2,05.10-6%
NB002006	DX	– 2,036552.10-4 m	– 2,0365561835042.10-4 m	2,05.10-6%
NB001011	DX	– 4,073104.10-4 m	– 4,0731123669671.10-4 m	2,05.10-6%
NB002011	DX	– 4,073104.10-4 m	– 4,0731123670073.10-4 m	2,05.10-6%
NB001016	DX	– 6,109656.10-4 m	– 6,1096685504506.10-4 m	2,05.10-6%
NB002016	DX	– 6,109656.10-4 m	– 6,1096685505104.10-4 m	2,05.10-6%
NB001021	DX	– 8,146208.10-4 m	– 8,1462247339343.10-4 m	2,05.10-6%
NB002021	DX	– 8,146208.10-4 m	– 8,1462247340137.10-4 m	2,05.10-6%
NB001006	DZ	3,818535.10-3 m	3,8185428440476.10-3 m	2,05.10-6%
NB002006	DZ	3,818535.10-3 m	3,8185428440475.10-3 m	2,05.10-6%
NB001011	DZ	1,527414.10-2 m	1,5274171376197.10-2 m	2,05.10-6%
NB002011	DZ	1,527414.10-2 m	1,5274171376197.10-2 m	2,05.10-6%
NB001016	DZ	3,436682.10-2 m	3,4366885596448.10-2 m	1,91.10-6%
NB002016	DZ	3,436682.10-2 m	3,4366885596448.10-2 m	1,91.10-6%
NB001021	DZ	6,109656.10-2 m	6,1096695504804.10-2 m	2,05.10-6%
NB002021	DZ	6,109656.10-2 m	6,1096695504804.10-2 m	2,05.10-6%

3.3.2 linear Density of normal force on the average level of the concrete part (analyzes with the model plate)

One compare the values extracted field SIEF_ELNO resulting from STAT_NON_LINE with the theoretical values from reference.
The component to which the tests relate is N_{XX} ($N_{XX} = s_{xx} p$).
The tolerance of relative variation compared to the reference is worth 0,1%.

Node	Nets	Value of reference	Computed value	relative Variation
NB001001	QD001001	– 4,887725.105 N/m	– 4,8877348399136.105 N/m	2,01.10-6%
NB002001	QD001001	– 4,887725.105 N/m	– 4,8877348399728.105 N/m	2,01.10-6%
NB001011	QD001011	– 4,887725.105 N/m	– 4,8877348402090.105 N/m	2,01.10-6%
NB002011	QD001011	– 4,887725.105 N/m	– 4,8877348402511.105 N/m	2,01.10-6%
NB001021	QD001020	– 4,887725.105 N/m	– 4,8877348403607.105 N/m	2,01.10-6%
NB002021	QD001020	– 4,887725.105 N/m	– 4,8877348404039.105 N/m	2,01.10-6%

3.3.3 Normal stress on the lower skin (Z = - 0.1 m) of the concrete part

One compares the values extracted field SIGM_ELNO resulting from CALC_CHAMP with the theoretical values from reference.
The component to which the tests relate is SIXX.
The tolerance of relative variation compared to the reference is worth 0,1%.

Node	Nets	Value of reference	Computed value	relative Variation
NB001001	QD001001	1,221931.106 Pa	1,2219337100849.106 Pa	2,22.10-6%
NB002001	QD001001	1,221931.106 Pa	1,2219337101082.106 Pa	2,22.10-6%
NB001011	QD001011	1,221931.106 Pa	1,2219337101212.106 Pa	2,22.10-6%
NB002011	QD001011	1,221931.106 Pa	1,2219337100924.106 Pa	2,22.10-6%
NB001021	QD001020	1,221931.106 Pa	1,2219337100302.106 Pa	2,22.10-6%
NB002021	QD001020	1,221931.106 Pa	1,2219337101559.106 Pa	2,22.10-6%

3.3.4 Normal stress on the higher skin (z= 0.1 m) of the concrete part

One compares the values extracted field SIGM_ELNO resulting from CALC_CHAMP with the theoretical values from reference.
The component to which the tests relate is SIXX.
The tolerance of relative variation compared to the reference is worth 0,1%.

Node	Nets	Value of reference	Computed value	relative Variation
NB001001	QD001001	– 6,109656.106 Pa	– 6,1096685504454.106 Pa	2,05.10-6%
NB002001	QD001001	– 6,109656.106 Pa	– 6,1096685505156.106 Pa	2,05.10-6%
NB001011	QD001011	– 6,109656.106 Pa	– 6,1096685504816.106 Pa	2,05.10-6%
NB002011	QD001011	– 6,109656.106 Pa	– 6,1096685504999.106 Pa	2,05.10-6%
NB001021	QD001020	– 6,109656.106 Pa	– 6,1096685503914.106 Pa	2,05.10-6%
NB002021	QD001020	– 6,109656.106 Pa	– 6,1096685505642.106 Pa	2,05.10-6%

3.4 Remarks

the computed values correspond indeed to those theoretically expected. One obtains well a state of bending-compression for the concrete beam.

4 Synthesis

the got results are validated by comparison with an analytical solution of reference with a very good accuracy.

The particular features tested are the following ones:

- operator DEFI_MATERIAU : definition of the parameters characteristic of the materials steel and concrete allowing computation of the tension along the cable of prestressing, following the rules of the BPEL;
- operator DEFI_CABLE_BP : computation of the tension along the cable and the coefficients of the kinematic relations between the degrees of freedom of the nodes of the cable and the degrees of freedom of the nodes “close” to the concrete beam;
- operator AFFE_CHAR_MECA : definition of a loading of the type RELA_CINE_BP ;
- operator STAT_NON_LINE, option COMP_INCR : computation of the state of equilibrium by taking account of the loading of the type RELA_CINE_BP.