SSNP108 - Prestressed concrete element in compression

Summarized:

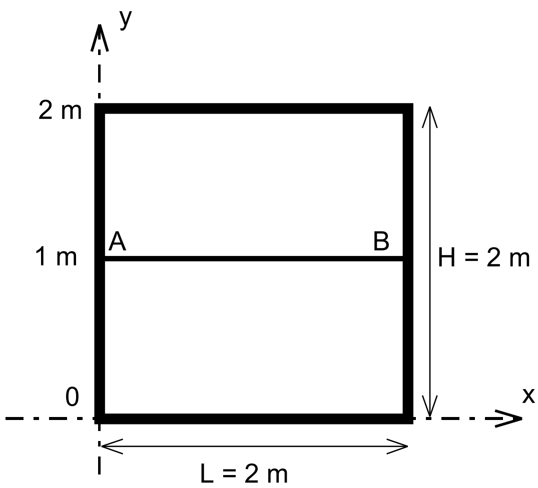
One considers an elementary structure made up of a square concrete plate crossed by a cable of prestressed whose neutral fiber is confused with the horizontal axis of symmetry of the plate. The left vertical edge of the plate is fixed. The cable is put in tension at its two ends in order to prestress the toggle plate. The losses of tension along the cable are neglected.

The goal of this benchmark is to validate, on a simple configuration, the method of calculating of the state of equilibrium of a prestressed structure of concrete. The results are validated by comparison with an analytical solution.

1 Problem of reference

1.1 Geometry

the concrete plate is square; the sides have even length $L=H=2m$.
The thickness of the plate is worth $e=0{,}6m$.
The cable crosses the plate horizontally, with middle height, without eccentricity in the thickness. The area of the straight section cable is worth $S_a=1{,}5.10^{-4}m^2$.



1.2 Properties of the materials

Concrete material constituting the plate:	Young modulus $E_b=3.10^{10}Pa$
Material steel constituting the cable:	Young modulus $E_a=2{,}1.10^{11}Pa$

the Poisson's ratio is taken equal to 0 for the two materials; one thus privileges the direction of normal load application (direction x).

The losses of tension being neglected, the various parameters being used for their estimate are fixed at 0.

1.3 Boundary conditions and loadings

the lower top of left edge of the plate, i.e. the node origin $(0;0)$, is clamped: all the degrees of freedom of translation and rotation are blocked.
The higher top of this same left edge, i.e. the node $(0;2)$, is supported bilaterally: the blocked degrees of freedom of translation are DX and DZ .

One applies at the two ends of the cable (which are fixed on the concrete in A and B) a normal force of tension: $(-F_0;0)$ with the node A $(0;1)$ and $(F_0;0)$ the node B $(2;1)$, with $F_0=2.10^5N$.

2 Reference solution

2.1 formal Solution

displacements in the cable and the concrete are continuous and homogeneous. Thus horizontal displacement on the interval $[0;L]$ is worth $u(x)=\frac{u_L}{L}x$

the normal stresses in the plate of concrete and the rope steel wire is written respectively (assumption of elasticity):

$$\begin{cases}\sigma_b = E_b\frac{\partial u}{\partial x} = E_b\frac{u_L}{L}\\ \sigma_a = E_a\frac{\partial u}{\partial x} + \sigma_0 = E_a\frac{u_L}{L} + \sigma_0\end{cases}$$

where $\sigma_0 = \frac{F_0}{S_a}$ is initial prestressing in the cable

and u_L is horizontal displacement with the X-coordinate L.

The equilibrium of the group plates and cables is written:
$\sigma_b S_b + \sigma_a S_a = 0 \Rightarrow u_L = \frac{-LF_0}{E_b eH + E_a S_a}$

The normal force in the cable is worth: $N_a = E_a S_a \frac{u_L}{L} + F_0 = F_0 \frac{E_b eH}{E_b eH + E_a S_a}$

The total force on the vertical section of the concrete plate is worth:

$$N_b = E_b eH \; \frac{u_L}{L} = -F0\frac{E_b eH}{E_b eH + E_a S_a}$$

One deducted the linear density of normal force on the concrete numerical Values

$$N_{xx} = -F0\frac{E_b e}{E_b eH + E_a S_a}$$

2.2 plate of reference

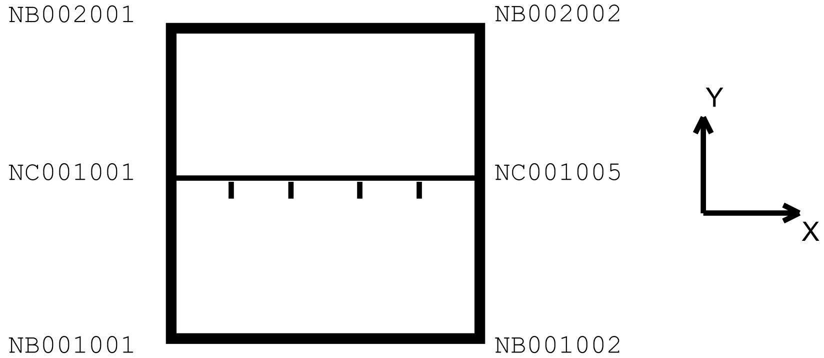
the numerical values of reference are:

$$u_L = -1{,}11013974.10^{-5} m$$
$$N_a = 1{,}99825153.10^{5} N$$
$$N_b = -1{,}99825153.10^{5} N$$
$$N_{xx} = -9{,}99125765.10^{4} N/m$$

3 Modelization A

3.1 Characteristic of the modelization

the figure below gives a simplified representation of the mesh.



The concrete plate is represented by an element DKT, supported by a mesh quadrangle with 4 nodes.

One thickness $e=0{,}6\,m$ is affected for him, as well as concrete material for which behaviors ELAS (Young modulus) are $E_b=3.10^{10}\,Pa$ defined and BPEL_BETON : the parameters characteristic of this relation are fixed at 0 because one neglects the losses of tension along the cable of prestressing.

The node $NB001001$ is clamped: DX, DY, DZ, DRX, DRY and DRZ are blocked. The node $NB002001$ is supported bilaterally: DX and DZ are blocked.

The cable is represented by 4 elements MECA_BARRE, supported by 4 meshes segments with 2 nodes.

An area of cross-section $S_a=1{,}5.10^{-4}\,m^2$ their is affected, as well as a material steel for which behaviors ELAS (Young modulus) are $E_a=2{,}1.10^{11}\,Pa$ defined and BPEL_ACIER : the parameters characteristic of this relation are fixed at 0 (neglected losses of tension), except for the stress yield stress for which a value zero is illicit ($f_{prg}=1{,}77.10^9\,Pa$).

The tension $F_0=2.10^5\,N$ is applied to the nodes $NC001001$ and $NC001005$. This value of tension is coherent with the values of section and yield stress, for a cable of prestressed of standard strand.

The computation of the state of equilibrium of the group plates and cable is carried out in only one step, the behavior being elastic.

3.2 Stages of computation and features tested

the main steps of computation correspond to the features which one wishes to validate:

operator DEFI_MATERIAU : definition of behavior models BPEL_BETON and BPEL_ACIER, in the cas particulier where the losses of tension along the cable of prestressing are neglected (default values of the parameters);
operator DEFI_CABLE_BP : determination of a constant profile of tension along the cable of prestressing, losses being neglected; computation of the coefficients of the kinematic relations between the degrees of freedom of the nodes of the cable and the degrees of freedom of the nodes “close” to the concrete plate;
operator AFFE_CHAR_MECA : definition of a loading of the type RELA_CINE_BP ;
operator STAT_NON_LINE, option COMP_INCR : computation of the state of equilibrium by taking account of the loading of the type RELA_CINE_BP.

4 Results of the modelization A

4.1 Values tested

4.1.1 linear Density of normal force on the vertical section of the concrete plate

One compares the values extracted field SIEF_ELNO resulting from STAT_NON_LINE with the theoretical values from reference. The extraction is done on the mesh $QD001001$ representing the concrete plate.
The component to which the tests relate is NXX.
The tolerance of relative variation compared to the reference is worth 10 $^{6\%}$.

Node	Value of reference	Computed value	relative Variation
NB001001	– 9,99125765.104 N/m	– 9,9912576495569.104 N/m	– 4,43.10- $^{11\%}$
NB001002	– 9,99125765.104 N/m	– 9,9912576495569.104 N/m	– 4,43.10- $^{11\%}$
NB002001	– 9,99125765.104 N/m	– 9,9912576495569.104 N/m	– 4,43.10- $^{11\%}$
NB002002	– 9,99125765.104 N/m	– 9,9912576495569.104 N/m	– 4,43.10- $^{11\%}$

4.1.2 horizontal Displacement of the nodes of the concrete plate

One compares the values extracted field DEPL resulting from STAT_NON_LINE with the theoretical values from reference.
The component to which the tests relate is DX.
The tolerance of relative variation compared to the reference is worth 10 $^{6\%}$.

Node	Value of reference	Computed value	relative Variation
NB001002	– 1,11013974.10-5 m	– 1,1101397388397.10-5 m	– 1,05.10- $^{9\%}$
NB002002	– 1,11013974.10 $^{-5}$ m	– 1,1101397388397.10 $^{-5}$ m	– 1,05.10- $^{9\%}$

4.1.3 normal Force in the cable

One compares the values extracted field SIEF_ELNO resulting from STAT_NON_LINE with the theoretical values from reference. The extraction is done on meshes $SG001001$ for the node $NC001001$, $SG001002$ the node $NC001002$, $SG001003$ the node $NC001003$, and $SG001004$ the nodes $NC001004$ and $NC001005$.
The component to which the tests relate is N.
The tolerance of relative variation compared to the reference is worth 10 $^{6\%}$.

Node	Value of reference	Computed value	relative Variation
$NC001001$	1,99825153.105 N	1,9982515299113.105 N	– 4,44.10- $^{11\%}$
$NC001002$	1,99825153.105 N	1,9982515299113.105 N	– 4,44.10- $^{11\%}$
$NC001003$	1,99825153.105 N	1,9982515299113.105 N	– 4,44.10- $^{11\%}$
$NC001004$	1,99825153.105 N	1,9982515299113.105 N	– 4,44.10- $^{11\%}$
$NC001005$	1,99825153.105 N	1,9982515299113.105 N	– 4,44.10- $^{11\%}$

4.1.4 horizontal Displacement of the nodes of the cable

One compares the values extracted field DEPL resulting from STAT_NON_LINE with the theoretical values from reference.
The component to which the tests relate is DX .
The tolerance of relative variation compared to the reference is worth 10 6% .

Node	Value of reference	Computed value	relative Variation
NC001002	– 2,77534935.10-6 m	– 2,7753493470988.10-6 m	– 1,05.10- 9%
NC001003	– 5,55069869.10-6 m	– 5,5506986941982.10-6 m	7,56.10- 10%
NC001004	– 8,32604804.10-6 m	– 8,3260480412974.10-6 m	1,56.10- 10%
NC001005	– 1,11013974.10-5 m	– 1,1101397388389.10-5 m	– 1,05.10- 9%

4.2 Remarks

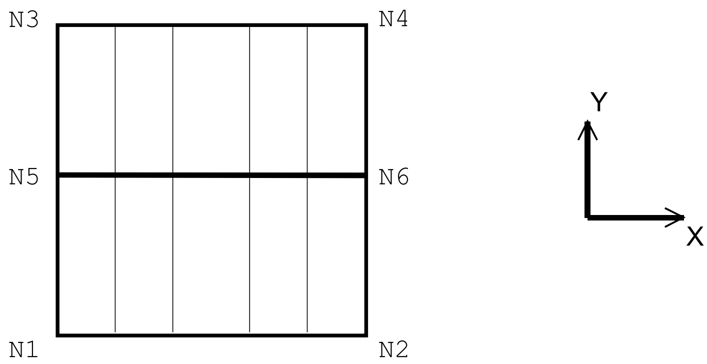
the computed values correspond indeed to those theoretically expected. One obtains well a compactness for the concrete plate.

One belonging to observes an infinitesimal difference between horizontal displacement with $NC001005$ the node cable and the horizontal displacement with the nodes $NB001002$ and $NB002002$ of the concrete plate. The recorded values should be identical, but the rounding errors appearing in the coefficients of the kinematic relations explain this infinitesimal difference.

5 Modelization B

5.1 Characteristic of the modelization

For this modelization, the nodes “cables” and “concrete” coincide.
The figure below gives a simplified representation of the mesh.



The concrete plate is represented by 10 elements `DKT`, supported by 10 meshes `QUAD4`.

One thickness $e=0,6\,m$ is affected for him, as well as concrete material for which behaviors ELAS (`Young` modulus) are $E_b=3.10^{10}\,Pa$ defined and `BPEL_BETON` : the parameters characteristic of this relation are fixed at 0 because one neglects the losses of tension along the cable of prestressing.

The nodes $N1$, $N5$ and $N3$ are clamped: DX, DY, DZ, DRX, DRY and DRZ are blocked.

The cable is represented by 5 elements `MECA_BARRE`, supported by 5 meshes `SEG2`.

An area of cross-section $S_a=1,5.10^{-4}\,m^2$ their is affected, as well as a material steel for which behaviors ELAS (`Young` modulus) are $E_a=2,1.10^{11}\,Pa$ defined and `BPEL_ACIER` : the parameters characteristic of this relation are fixed at 0 (neglected losses of tension), except for the stress yield stress for which a value zero is illicit ($f_{prg}=1,77.10^9\,Pa$).

The tension $F_0=2.10^5\,N$ is applied to the nodes $N5$ and $N6$. This value of tension is coherent with the values of section and yield stress, for a cable of prestressed of standard strand.

The computation of the state of equilibrium of the group plates and cable is carried out in only one step, the behavior being elastic.

5.2 Stages of computation and features tested

the main steps of computation correspond to the features which one wishes to validate:

operator `DEFI_MATERIAU` : definition of behavior models `BPEL_BETON` and `BPEL_ACIER`, in the cas particulier where the losses of tension along the cable of prestressing are neglected (default values of the parameters);
operator `DEFI_CABLE_BP` : determination of a constant profile of tension along the cable of prestressing, losses being neglected; computation of the coefficients of the kinematic relations between the degrees of freedom of the nodes of the cable and the degrees of freedom of the nodes “close” to the concrete plate;
operator `AFFE_CHAR_MECA` : definition of a loading of the type `RELA_CINE_BP` ;
operator `STAT_NON_LINE`, option `COMP_INCR` : computation of the state of equilibrium by taking account of the loading of the type `RELA_CINE_BP`.
operator `POST_RELEVE_T`, `NOM_CMP=` ` on all field `SIEF_ELNO`.

6 Results of the modelization B

6.1 Values tested

6.1.1 linear Density of normal force on the vertical section of the plate of concrete

Node	Value of reference	Computed value	relative Variation
$N1$	– 9,99125765.104 N/m	– 9,8387753336725.104 N/m	1.526%
$N3$	– 9,99125765.104 N/m	– 9,8387753336725.104 N/m	1.526%

6.1.2 normal Force in the cable

Node	Value of reference	Computed value	relative Variation
$N5$	1,99825153.105 N	1,9982248921222.105 N	– 0.001%
$N6$	1,99825153.105 N	1,9943932520206.105 N	– 0.193%

6.1.3 normal Force in the cable via command POST_RELEVE_T

Node	Value of reference	Computed value
$N5$	1,998224892.105 N	1,9982248921222.105 N
$N6$	1,994393252.105 N	1,9943932520206.105 N

6.2 Remarks

the computed values correspond indeed to those theoretically expected. One obtains well a compactness for the concrete plate.

7 Summary of the results

the got results are validated by comparison with an analytical solution of reference with a very good accuracy.

The particular features tested are the following ones:

operator DEFI_MATERIAU : definition of the parameters characteristic of the materials steel and concrete allowing computation of the tension along the cable of prestressing, following the rules of the BPEL;
operator DEFI_CABLE_BP : computation of the tension along the cable and the coefficients of the kinematic relations between the degrees of freedom of the nodes of the cable and the degrees of freedom of the nodes "close" to the concrete plate;
operator AFFE_CHAR_MECA : definition of a loading of the type RELA_CINE_BP ;
operator STAT_NON_LINE, option COMP_INCR : computation of the state of equilibrium by taking account of the loading of the type RELA_CINE_BP.