

SSNP15 - Plate in tension-shears - Von Mises (isotropic hardening)

Summarized:

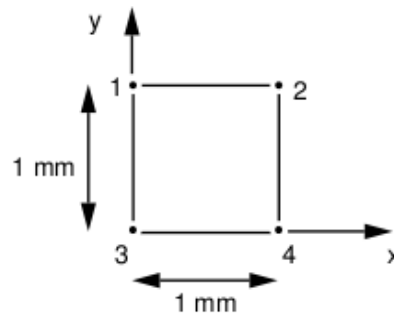
This test 2D plane stresses quasi-static, from guide VPCS [1], enters the frame of the elastoplastic validation of the behavior models. A volume element, made up of a plastic material with linear isotropic hardening, is subjected to a tractive effort and a shearing force.

The principal interest of this test lies in the nonradial character of the loading.

1 Problem of reference

1.1 Geometry

the stresses and strains are homogeneous in the volume element. This one can be represented by a plane or voluminal element, for example:



1.2 Material properties

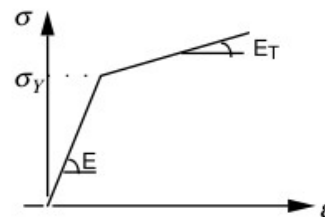
Elastoplastic constitutive law with linear kinematic hardening.

$$E = 195000 \text{ MPa}$$

$$\nu = 0.3$$

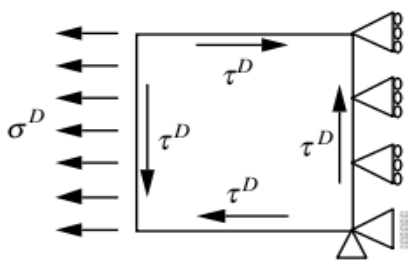
$$\sigma_y = 181 \text{ MPa}$$

$$E_T = 1930 \text{ MPa}$$



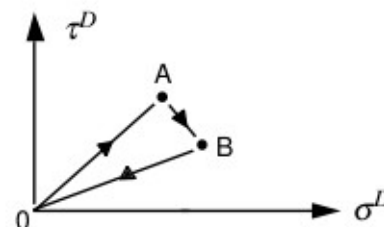
1.3 Boundary conditions and loadings

the volume element is blocked according to Ox along the side $[2,4]$ while being subjected to a tension σ^D and a shearing force τ^D .



The way of loading is the following:

	σ^D [MPa]	τ^D [MPa]
A	151.2	93.1
B	257.2	33.1



2 Reference solution

2.1 Method of calculating used for the reference solution

In the plane $(\sigma, \tau\sqrt{3})$, the norm of von Mises results in the classical distance in the octahedral plane between the projection of the stress state and hydrostatic line, so that one can immediately predict the phases of load and of discharge during the way of loading, since it is respectively the phases where the norm grows or decrease:

Way of loading	numerical Values (σ, τ) [MPa]	Phases of loading
	A^0 (123.8; 76.23)	$O - A^0$ elastic Load
	A (151.2; 93.10)	$A^0 - A$ Charges plastic
	B^0 (158.23; 89.12)	$A - B^0$ Discharge
	B (257.2; 33.10)	$B^0 - B$ Charges plastic

the loading is done according to a curve parameterized by time:

- Phase 1: plasticization of the point O at the point A (times 0,0 to 1,0).
- Phase 2: discharge from the point A at the point B (times 1,0 to 2,0).
- Phase 3: total discharge of the point B at the point C (times 2,0 to 3,0).

Urgent	σ [MPa]	τ [MPa]	Number of steps
0,0 – Point O	0	0.0,1	
			1.0,9
			10.1,0
– Point A	151,2	93,1	1.2,0
– Point B	257,2	33,1	40.3,0
– Point C	0	0	1

2.1.1 Approach of resolution

Mechanically, it acts of a test 0D controlled in stresses, the material being elastoplastic with criterion of von Mises and linear isotropic hardening. For a loading controlled in stress, one easily determines the cumulated plastic strain:

$$F(\sigma, p) = \sigma_{\acute{e}q} - \sigma_y - R' p \leq 0 \quad \Rightarrow \quad p = \frac{\sigma_{\acute{e}q} - \sigma_y}{R'} \quad \text{en charge} \quad \text{éq 2.1.1-1}$$

the integration of the plastic strain is of course more delicate. The flow equation is written:

$$\dot{\varepsilon}^p = \frac{3}{2} \dot{p} \frac{\tilde{\sigma}}{\sigma_{\dot{\varepsilon}q}} \Rightarrow \dot{\varepsilon}^p = \frac{3}{2 R'} \cdot \frac{\sigma_{\dot{\varepsilon}q}}{\sigma_{\dot{\varepsilon}q}} \cdot \tilde{\sigma} \quad \text{en charge} \quad \text{éq 2.1.1-2}$$

Lastly, one will deduce the strain via the relation from state:

$$\varepsilon = \varepsilon^p + E^{-1} : \sigma \Rightarrow \varepsilon_{xx} = \varepsilon_{xx}^p + \frac{\sigma}{E} \quad \text{et} \quad \varepsilon_{xy} = \varepsilon_{xy}^p + \frac{\tau}{2\mu} \quad \text{éq 2.1.1-3}$$

2.1.2 Processing of the phase of radial loading

Let us notice that in phase of radial loading, the flow model [éq 2.1.2-1] is integrated directly:

$$\varepsilon^p = \frac{3}{2} p \frac{\tilde{\sigma}}{\sigma_{\dot{\varepsilon}q}} \quad \text{éq 2.1.2-1}$$

the cumulated plastic strain is then given by [éq 2.1.1-1], the plastic strain by [éq 2.1.2-1] and the total deflection by [éq 2.1.1-3]. With:

E	$= 195\,000\text{ MPa}$	2μ	$= 150\,000\text{ MPa}$	R'	$= 1\,949.29\text{ MPa}$
One obtains:					
$p(A)$	$= 2.0547 \cdot 10^{-2}$	$\varepsilon_{xx}^p(A)$	$= 1.4054 \cdot 10^{-2}$	$\varepsilon_{xx}(A)$	$= 1.4830 \cdot 10^{-2}$
		$\varepsilon_{xy}^p(A)$	$= 1.2981 \cdot 10^{-2}$	$\varepsilon_{xy}(A)$	$= 1.3601 \cdot 10^{-2}$

2.1.3 Processing of the phase of nonradial loading

In the phase of nonradial loading $B^0 - B$, one can parameterize the way of stress by:

$$\sigma(q) = \sigma^{B^0} + q \underbrace{(\sigma^B - \sigma^{B^0})}_{\text{direction fixe}} \quad \text{avec} \quad 0 \leq q \leq 1 \quad \text{éq 2.1.3-1}$$

As the way of loading remains confined in the plane tension-shears (σ, τ) , one will may find it beneficial to represent the stress state by a complex number:

$$\Sigma = \sigma + i\sqrt{(3)}\tau \Rightarrow \sigma_{\dot{\varepsilon}q} = |\Sigma| \quad \text{et} \quad \Sigma(q) = \Sigma^{B^0} + q \underbrace{(\Sigma^B - \Sigma^{B^0})}_{\text{direction fixe}} \quad \text{éq 2.1.3-2}$$

the integration of the flow model [éq 2.1.1-2], followed by a integration by part, makes it possible to express the plastic strain:

$$\frac{2R'}{3} [\varepsilon^p]_0^1 = \int_0^1 \frac{\dot{\sigma}_{\dot{\varepsilon}q}}{\sigma_{\dot{\varepsilon}q}} \tilde{\sigma} dq = \left[\ln(\sigma_{\dot{\varepsilon}q}) \tilde{\sigma} \right]_0^1 - \frac{1}{2} \underbrace{\tilde{\sigma}}_{\tilde{\sigma}^B - \tilde{\sigma}^{B^0}} \int_0^1 \ln(\sigma_{\dot{\varepsilon}q}^2) dq$$

The adoption of the complex plane allows an easy computation of the last integral:

$$\int_0^1 \ln(\sigma_{\text{éq}}^2) dq = \int_0^1 \ln(\Sigma \bar{\Sigma}) dq = \int_0^1 \ln(\Sigma) dq + \int_0^1 \ln(\bar{\Sigma}) dq = 2 \operatorname{Re} \left[\int_0^1 \ln(\Sigma) dq \right] = 2 \operatorname{Re} \left[\frac{\Sigma \ln(\Sigma) - \Sigma}{\Sigma^B - \Sigma^{B^0}} \right]_0^1$$

Finally, the increment of plastic strain on the way $B^0 - B$ is worth:

$$[\varepsilon^p]_{B^0}^B = \frac{3}{2R'} [\ln(\sigma_{\text{éq}}) \tilde{\sigma}]_{B^0}^B - \frac{3}{2R'} \operatorname{Re} \left[\frac{\Sigma \ln(\Sigma) - \Sigma}{\Sigma^B - \Sigma^{B^0}} \right]_{B^0}^B (\tilde{\sigma}^B - \tilde{\sigma}^{B^0}) \quad \text{éq 2.1.3-4}$$

2.2 Results of reference

By calculating the plastic strain cumulated by [éq 2.1.1-1], the plastic strain by [éq 2.1.3-4] and the total deflection by [éq 2.1.1-3], one obtains:

$$\begin{array}{llll} p(B) & = 4.2329 \cdot 10^{-2} & \varepsilon_{xx}^p(B) & = 3.3946 \cdot 10^{-2} & \varepsilon_{xx}(B) & = 3.5265 \cdot 10^{-2} \\ \text{One} & & \varepsilon_{xy}^p(B) & = 2.0250 \cdot 10^{-2} & \varepsilon_{xy}(B) & = 2.0471 \cdot 10^{-2} \\ \text{obtains:} & & & & & \end{array}$$

One will be interested in the values of the stresses, the strains and the plastic strain cumulated at the points A and B the way of loading.

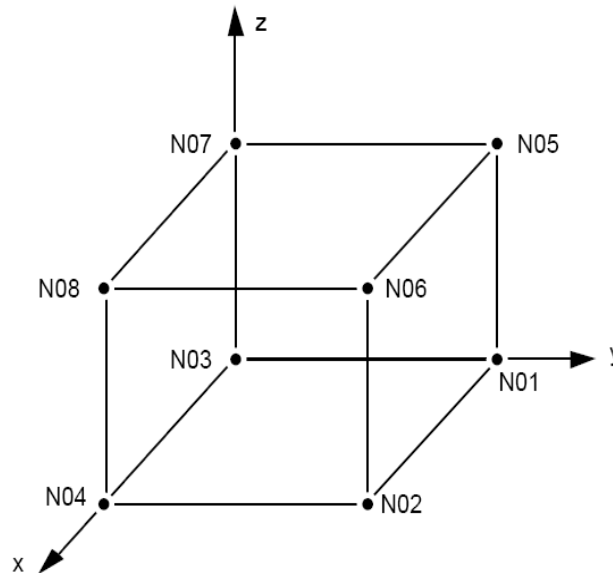
2.3 Bibliographical references

- 1) French company of the Mechanics. Guide validation of the software packages of structural analysis (VPCS). Technical AFNOR, 1990.

3 Modelization A

3.1 Characteristic of the modelization

Modelization 3D



the loading and the boundary conditions are modelled by:

- Condition of Dirichlet (key word `DDL_IMPO`):
 - Node *N04* $x = y = 0$,
 - Node *N08* $x = y = z = 0$,
 - Node *N02* $x = 0$,
 - Node *N06* $x = 0$.
- Condition of Neumann, surface forces (key word `FORCE_FACE`):
 - on the sides (meshes of skin): $(1,5,6,2)$ $(1,5,7,3)$, $(3,4,8,7)$ and $(4,8,6,2)$.

3.2 Characteristics of the mesh

Many nodes: 8

Number of meshes and types: 1 HEXA8, 4 QUAD4

3.3 Quantities tested and Case

3.3.1 results of Standard

VMIS_ISOT_LINE	Identification of reference	Value of reference	Tolerance
σ_{xx} to time <i>A</i>	"ANALYTIQUE"	1,512E+002	0,1%
σ_{xy} to time <i>A</i>	"ANALYTIQUE"	9,310E+001	0,1%
p to time <i>A</i>	"ANALYTIQUE"	2,0547E-002	0,1%
Rate of triaxiality <i>TRIAx</i> to time <i>A</i>	"ANALYTIQUE"	2,2800E-001	0,1%
ε_{xx} to time <i>A</i>	"ANALYTIQUE"	1,48297E-002	0,1%
ε_{xy} to time <i>A</i>	"ANALYTIQUE"	1,36014E-002	0,1%
ε_{xx}^p to time <i>A</i>	"ANALYTIQUE"	1,40543E-002	0,1%
ε_{xy}^p to time <i>A</i>	"ANALYTIQUE"	1,29807E-002	0,1%
p to time <i>B</i>	"ANALYTIQUE"	4,23293E-002	1,0%
Rate of triaxiality <i>TRIAx</i> to time <i>B</i>	"ANALYTIQUE"	3,25349E-001	0,1%
ε_{xx} to time <i>B</i>	"ANALYTIQUE"	3,5265E-002	1,0%
ε_{xy} to time <i>B</i>	"ANALYTIQUE"	2,0471E-002	1,0%
ε_{xx}^p to time <i>B</i>	"ANALYTIQUE"	3,3946E-002	1,0%
ε_{xy}^p to time <i>B</i>	"ANALYTIQUE"	2,0250E-002	1,0%

as well as the indicators of load-discharge:

Standard	identification of reference	Value	Tolerance
INDIC_ENER to time <i>A</i>	"ANALYTIQUE"	0	0,1%
INDIC_ENER at time <i>B</i>	"ANALYTIQUE"	3,26E-002	3,0%
INDIC_SEUIL at time <i>A</i>	"ANALYTIQUE"	0	0,1%
INDIC_SEUIL at time <i>B</i>	"ANALYTIQUE"	3,26E-002	3,0%
INDIC_ENER at time <i>C</i> (discharge supplements)	"ANALYTIQUE"	4,69E-002	3,0%
INDIC_SEUIL at time <i>C</i> (discharge supplements)	"ANALYTIQUE"	1,0	1,0%
DERA_ELNO/RADI_V at time 0,1	"ANALYTIQUE"	0,0	1,0%

3.3.2 Case of VMIS_ECMI_LINE

One calculates only energies, and one compares compared to case VMIS_ISOT_LINE :

Standard	identification of reference	Value	Tolerance
ETOT_ELGA/TOTALE to time 0,1	"AUTRE_ASTER"	1,16403E-03	0,1%
ETOT_ELGA/TOTALE at time 0,9	"AUTRE_ASTER"	1,84340	0,1%
ETOT_ELGA/TOTALE at time 2,0	"AUTRE_ASTER"	9,58487	0,1%
ETOT_ELGA/TOTALE at time 3,0	"AUTRE_ASTER"	9,40794	0,1%
ETOT_ELNO/TOTALE at time 0,1	"AUTRE_ASTER"	1,16403E-03	0,1%
ETOT_ELNO/TOTALE at time 0,9	"AUTRE_ASTER"	1,84340	0,1%
ETOT_ELNO/TOTALE at time 2,0	"AUTRE_ASTER"	9,58487	0,1%
ETOT_ELNO/TOTALE at time 3,0	"AUTRE_ASTER"	9,40794	0,1%
ETOT_ELEM/TOTALE at time 0,1	"AUTRE_ASTER"	1,16403E-03	0,1%
ETOT_ELEM/TOTALE at time 0,9	"AUTRE_ASTER"	1,84340	0,1%
ETOT_ELEM/TOTALE at time 2,0	"AUTRE_ASTER"	9,58487	0,1%
ETOT_ELEM/TOTALE at time 3,0	"AUTRE_ASTER"	9,40794	0,1%
ETOT_NOEU/TOTALE at time 3,0	"AUTRE_ASTER"	9,40650	0,1%

3.3.3 Case of DERA_ELxx

One test the indicators of discharge DCHA_V and loss of radiality DCHA_R in the mesh *CUBE* :

- at first Gauss point (DERA_ELGA),
- node N_2 (DERA_ELNO).

Standard	identification of reference	Value	Tolerance
DERA_ELGA/DCHA_V to the increment 2	"NON_REGRESSION"	3.07692E-1	0.10%
DERA_ELNO/DCHA_V with the increment 2	"NON_REGRESSION"	3.07692E-1	0.10%
DERA_ELGA/RADI_V with the increment 2	"NON_REGRESSION"	0.0	0.10%

One tests in the mesh *CUBE* at the point of gauss n°1:

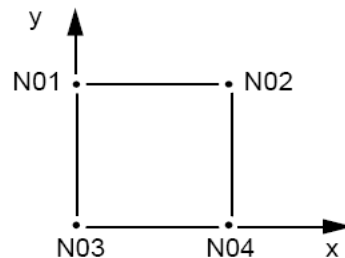
- The indicator of discharge `IND_DCHA` which makes it possible to know if the discharge remains elastic or if there would be a risk of plasticization if a pure kinematic hardening were used,
- the indicator `VAL_DCHA` which indicates the proportion of output of the criterion.

	Standard	identification of reference	Value	Tolerance
DERA_ELGA	IND_DCHA to the increment 10	"NON_REGRESSION"	2	0.10%
	VAL_DCHA with the increment 10	"NON_REGRESSION"	0,0	0.001
	IND_DCHA with the increment 12	"NON_REGRESSION"	-1	0.10%
	VAL_DCHA with the increment 12	"NON_REGRESSION"	0,0	0.001
	IND_DCHA with the increment 14	"NON_REGRESSION"	-2	0.10%
	VAL_DCHA with the increment 14	"NON_REGRESSION"	1.057898	0.10%
	IND_DCHA with the increment 52	"NON_REGRESSION"	-2	0.10%
	VAL_DCHA with the increment 52	"NON_REGRESSION"	1.057898	0.10%

4 Modelization B

4.1 Characteristic of the modelization

Modelization in plane stresses: C_PLAN



the loading and the boundary conditions are modelled by:

- Condition of Dirichlet (key word DDL_IMPO):
 - Node *N04* $x=y=0$,
 - Node *N02* $x=0$.
- Condition of Neumann, surface forces (key word FORCE_CONTOUR):
 - on the sides (meshes of skin): (1, 2), (2, 4), (4, 3) and (3, 1).

4.2 Characteristics of the mesh

Many nodes: 4

Number of meshes and types: 1 QUAD4, 4 SEG2

4.3 Quantities tested and results

Identification	Times	Reference	% Tolerance
σ_{xx}	<i>A</i>	151.2	0.1
σ_{xy}	<i>A</i>	93.1	0.1
ε_{xx}	<i>A</i>	1.4830 10-2	0.1
ε_{xy}	<i>A</i>	1.3601 10-2	0.1
p	<i>A</i>	2.055 10-2	0.1
Rates of triaxiality <i>TRIAx</i>	<i>A</i>	2.28 10-1	0.1
ε_{xx}	<i>B</i>	3.5265 10-2	1.0
ε_{xy}	<i>B</i>	2.0471 10-2	1.0
p	<i>B</i>	4.2329 10-2	1.0
Rates of triaxiality <i>TRIAx</i>	<i>B</i>	3.25349 10-1	0.1
ε_{xx}^p	<i>B</i>	3.3946 10-2	1.0
ε_{xy}^p	<i>B</i>	2.0250 10-2	1.0

as well as the indicators of load-discharge:

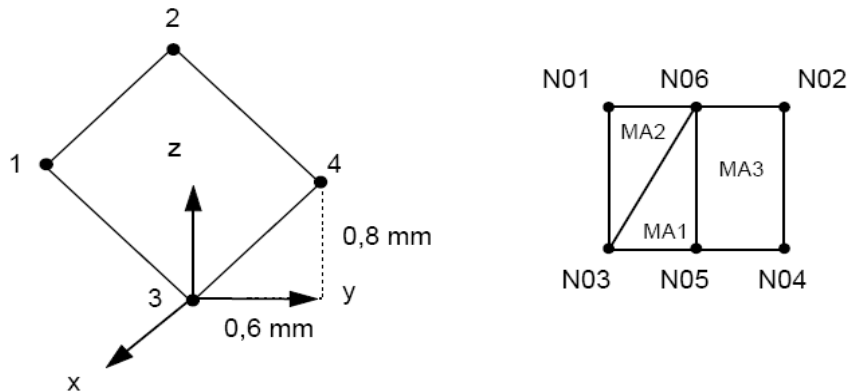
Identification	Times	Reference	% tolerance
INDIC_ENER	<i>A</i>	0.	0.1
INDIC_SEUIL	<i>A</i>	0.	0.1
INDIC_ENER	<i>B</i>	3.26 10 ⁻²	3.00
INDIC_SEUIL	<i>B</i>	9.71 10 ⁻²	1.00

5 Modelization C

5.1 Characteristic of the modelization

Modelization shell DKT-DKQ

5.2 Geometry



dimensions of structure do not change compared to the problem of reference, only differs its directional sense.

Coordinates of the nodes:

Nodes	x	y	z
N01	0	-0.8	0.6
N02	0	-0.2	1.4
N03	0	0	0
N04	0.0.6.0.8		
N05	0.0.3.0.4		
N06	0	-0.5	1

Boundary conditions:

```
DDL_IMPO= (_F (NOEUD=' NO4', DY=0.0, DZ=0.0, ),
            _F (TOUT=' OUI', DX=0.0, ), ),
LIAISON_DDL=_F (NOEUD= ("NO2", "NO2", ),
                DDL= ("DY", "DZ", ),
                COEF_MULT= (0.75, 1.0, ),
                COEF_IMPO=0.0, ), );
```

Loading:

One imposes surface forces (key word FORCE_ARETE) on the sides (meshes of skin SEG2) (1,2) (2,4), (4,3) and (3,1).

Specificity DKT and DKQ:

Two layers in the thickness for plasticity.

5.3 Characteristics of the mesh

Many nodes: 6
Number of meshes and types: 2 TRIA3 and 1 QUAD4

5.4 Quantities tested and results

displacements tested are those of the problem of reference by taking account of the rotation of structure.

The strains, the stresses and the generalized forces are tested in the reference user defined by the command ANGL_REP. The values are thus those given by the problem of reference.

The values are tested at the point A of the way of loading OA . One tests as follows:

Displacements (DEPL). He result easily from the reference solution since the strain is homogeneous.

Identification	Reference	% Tolerance
DY N01	1.86722 10 ⁻²	1.00E-004
DZ N01	- 3.25413 10 ⁻²	1.00E-004
DY N06	1.224 10 ⁻²	1.00E-004
DZ N06	- 1.88485 10 ⁻²	1.00E-004
DY N02	5.80782 10 ⁻³	1.00E-004
DZ N02	- 4.35586 10 ⁻³	1.00E-004

stresses (SIGM_ELNO).

Identification	Reference	% Tolerance
SIXX MA2 N01	1.512 102.1.0	
SIXY MA2 N01	93.1	1.0
SIXX MA1 N03	1.512 102.1.0	
SIXY MA1 N03	93.1	1.0
SIXX MA2 N03	1.512 102.1.0	
SIXY MA2 N03	93.1	1.0
SIXX MA3 N02	1.512 102.1.0	
SIXY MA3 N02	93.1	1.0

forces generalized by elements to nodes (EFGE_ELNO).

Identification	Reference	% tolerance
NXX MA2 N01	3.024 102.1.0	
NXY MA2 N01	1.862 102.1.0	
NXX MA1 N03	3.024 102.1.0	
NXY MA1 N03	1.862 102.1.0	
NXX MA2 N03	3.024 102.1.0	
NXY MA2 N03	1.862 102.1.0	
NXX MA3 N02	3.024 102.1.0	
NXY MA3 N02	1.862 102.1.0	

strains per element to the nodes starting from displacements (EPSI_ELNO).

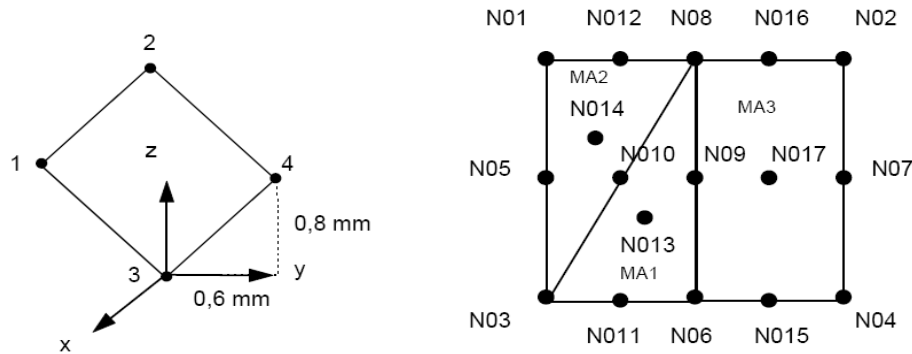
Identification	Reference	% tolerance
<i>EPXX MA2 N01</i>	1.48297 10-2	0.01
<i>EPYY MA2 N01</i>	- 7.25977 10-3	0.01
<i>EPXY MA2 N01</i>	1.36014 10-2	0.01
<i>EPXX MA1 N03</i>	1.48297 10-2	0.01
<i>EPYY MA1 N03</i>	- 7.25977 10-3	0.01
<i>EPXY MA1 N03</i>	1.36014 10-2	0.01
<i>EPXX MA2 N03</i>	1.48297 10-2	0.01
<i>EPYY MA2 N03</i>	- 7.25977 10-3	0.01
<i>EPXY MA2 N03</i>	1.36014 10-2	0.01
<i>EPXX MA3 N02</i>	1.48297 10-2	0.01
<i>EPYY MA3 N02</i>	- 7.25977 10-3	0.01
<i>EPXY MA3 N02</i>	1.36014 10-2	0.01
<i>EPXX MA3 N04</i>	1.48297 10-2	0.01
<i>EPYY MA3 N04</i>	- 7.25977 10-3	0.01
<i>EPXY MA3 N04</i>	1.36014 10-2	0.01

6 Modelization D

6.1 Characteristic of the modelization

Modelization shell COQUE_3D

6.2 Geometry



dimensions of structure do not change compared to the problem of reference, only differs its directional sense.

Coordinates of the nodes:

Nodes	x	y	z
N01	0	- 0.8	0.6
N02	0	- 0.2	1.4
N03	0	0	0
N04	0.0.6.0.8		
N05	0	- 0.4	0.3
N06	0.0.3.0.4		
N07	0.0.2.1.1		
N08	0	- 0.5	1
N09	0	- 0.1	0.7
N010	0	- 0.25	0.5
N011	0	0.15	0.2
N012	0	- 0.65	0.8
N013	0	- 0.06666	0.466666
N014	0	- 0.433333	0.533333
N015	0	0.45	0.6
N016	0	- 0.35	1.2
N017	0	0.05	0.9

Boundary conditions:

```
DDL_IMPO= ( _F (NOEUD=' NO4', DX=0., DY=0., DZ=0., DRX=0., DRY=0., DRZ=0.),
            _F (NOEUD=' NO2', DRX=0., DRY=0., DRZ=0.),
            _F (NOEUD=' NO7', DRX=0., DRY=0., DRZ=0.)),

LIAISON_DDL= ( _F (NOEUD= ("NO2", "NO2",),
                    DDL= ("DY", "DZ",),
                    COEF_MULT= (0.75, 1. ,),
                    COEF_IMPO=0.),
              _F (NOEUD= ("NO7", "NO7",),
                    DDL= ("DY", "DZ",),
                    COEF_MULT= (0.75, 1. ,),
                    COEF_IMPO=0.),),)
```

Loading:

One imposes surface forces (key word `FORCE_ARETE`) on the sides (meshes of skin `SEG3`) (1,2) (2,4), (4,3) and (3,1).

6.3 Characteristics of the mesh

Many nodes: 17

Number of meshes and types: 2 `TRIA7` and 1 `QUAD9`

6.4 Quantities tested and results

displacements tested are those of the problem of reference by taking account of the rotation of structure.

The strains, the stresses and the generalized forces are tested in the reference user defined by the command `ANGL_REP`. The values are thus those given by the problem of reference.

The values are tested at the point *A* of the way of loading *OA*. One tests as follows:

Displacements (`DEPL`). He result easily from the reference solution since the strain is homogeneous.

Identification	Reference	% tolerance
<i>DY N01</i>	1.86722 10-2	1.00E-004
<i>DZ N01</i>	- 3.25413 10-2	1.00E-004
<i>DY N08</i>	1.224 10-2	1.00E-004
<i>DZ N08</i>	- 1.84485 10-2	1.00E-004
<i>DY N02</i>	5.80782 10-3	1.00E-004
<i>DZ N02</i>	- 4.35586 10-3	1.00E-004

forces generalized by elements to nodes (`EFGE_ELNO`).

Identification	Reference	% difference
<i>NXX MA1 N01</i>	3.024 102.1.0	
<i>NXY MA1 N01</i>	1.862 102.1.0	
<i>NXX MA1 N03</i>	3.024 102.1.0	
<i>NXY MA1 N03</i>	1.862 102.1.0	
<i>NXX MA2 N03</i>	3.024 102.1.0	
<i>NXY MA2 N03</i>	1.862 102.1.0	
<i>NXX MA3 N02</i>	3.024 102.1.0	

NXY MA3 N02 1.862 102.1.0

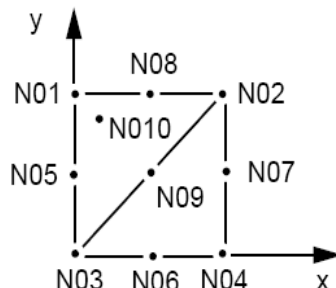
strains per element to the nodes starting from displacements (EPSI_ELNO).

Identification	Reference	% tolerance
<i>EPXX MA1 N01</i>	1.48297 10-2	0.01
<i>EPYY MA1 N01</i>	- 7.25977 10-3	0.01
<i>EPXY MA1 N01</i>	1.36014 10-2	0.01
<i>EPXX MA1 N03</i>	1.48297 10-2	0.01
<i>EPYY MA1 N03</i>	- 7.25977 10-3	0.01
<i>EPXY MA1 N03</i>	1.36014 10-2	0.01
<i>EPXX MA2 N03</i>	1.48297 10-2	0.01
<i>EPYY MA2 N03</i>	- 7.25977 10-3	0.01
<i>EPXY MA2 N03</i>	1.36014 10-2	0.01
<i>EPXX MA3 N02</i>	1.48297 10-2	0.01
<i>EPYY MA3 N02</i>	- 7.25977 10-3	0.01
<i>EPXY MA3 N02</i>	1.36014 10-2	0.01
<i>EPXX MA3 N04</i>	1.48297 10-2	0.01
<i>EPYY MA3 N04</i>	- 7.25977 10-3	0.01
<i>EPXY MA3 N04</i>	1.36014 10-2	0.01

7 Modelization E

7.1 Characteristic of the modelization

Modelization COQUE_3D (MEC3TR7H)



Boundary conditions:

```
DDL_IMPO: (THE NODE IS OUTSIDE THE FIELD OF DEFINITION
WITH A RIGHT PROFILE OF THE EXCLU TYPE NODE: N04, DX: 0. , DY: 0.)
          (THE NODE IS OUTSIDE THE FIELD OF DEFINITION
WITH A RIGHT PROFILE OF THE EXCLU TYPE NODE: N02, DX: 0.)
          (THE NODE IS OUTSIDE THE FIELD OF DEFINITION
WITH A RIGHT PROFILE OF THE EXCLU TYPE NODE: N07, DX: 0.)
          (THE NODE IS OUTSIDE THE FIELD OF DEFINITION
WITH A RIGHT PROFILE OF THE EXCLU TYPE NODE: (N01, N02, N03), DZ: 0.)
```

Loading

```
FORCE_NODALE: (THE NODE IS OUTSIDE THE FIELD OF DEFINITION
WITH A RIGHT PROFILE OF THE EXCLU TYPE NODE: N01, FX: -9.683333, FY: -
15.516666)
          (THE NODE IS OUTSIDE THE FIELD OF DEFINITION
WITH A RIGHT PROFILE OF THE EXCLU TYPE NODE: N02, FX: 15.516666, FY:
15.516666)
          (THE NODE IS OUTSIDE THE FIELD OF DEFINITION
WITH A RIGHT PROFILE OF THE EXCLU TYPE NODE: N03, FX: - 40.716666, FY: -
15.516666)
          (THE NODE IS OUTSIDE THE FIELD OF DEFINITION
WITH A RIGHT PROFILE OF THE EXCLU TYPE NODE: N04, FX: - 15.516666, FY:
15.516666)
          (THE NODE IS OUTSIDE THE FIELD OF DEFINITION
WITH A RIGHT PROFILE OF THE EXCLU TYPE NODE: N05, FX: - 100.8, FY: -
62.066666)
          (THE NODE IS OUTSIDE THE FIELD OF DEFINITION
WITH A RIGHT PROFILE OF THE EXCLU TYPE NODE: N06, FX: - 62.066666)
          (THE NODE IS OUTSIDE THE FIELD OF DEFINITION
WITH A RIGHT PROFILE OF THE EXCLU TYPE NODE: N07, FX: 62.066666)
          (THE NODE IS OUTSIDE THE FIELD OF DEFINITION
WITH A RIGHT PROFILE OF THE EXCLU TYPE NODE: N08, FX: 62.066666)
```

7.2 Characteristic of the mesh

Many nodes: 11, Number of meshes and types: 2 TRIA7

7.3 Quantities tested and results

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.

the displacement of the node $N01$ results easily from the reference solution since the strain is homogeneous. It is tested not in A but also in a point A^0 ($\sigma^D = 123.8$ $\tau^D = 76.2$) of the way OA . The cumulated plastic strain is also tested.

Identification	Times	Reference	% tolerance
DX $N01$	A^0	$- 6.349 \cdot 10^{-4}$	0.5
DY $N01$	A^0	$- 1.207 \cdot 10^{-3}$	0.5
DY $N01$	A	$- 3.431 \cdot 10^{-2}$	0.5
p	A	$2.055 \cdot 10^{-2}$	1.0

7.4 Remark

Only the portion OA of the way of the loading is actually tested.

8 Modelization F

8.1 Characteristic of the modelization

This modelization is identical to modelization A. the only difference is at the level of the management of time step. The selected temporal discretization is minimal: 0,1 ; 0,9 ; 1 ; 2 ; 3 .

One causes a recutting of time step so with convergence, the maximum increment of cumulated plastic strain exceeds 0,1 % .

8.2 Quantities tested and Standard

Identification	results of reference	Value	Tolerance
σ_{xx} to time A	"AUTRE_ASTER"	151.2	0,1%
σ_{xy} at time A	"AUTRE_ASTER"	93.1	0,1%
ε_{xx} at time A	"AUTRE_ASTER"	1.48297E-2	0,1%
ε_{xy} at time A	"AUTRE_ASTER"	1.36014E-2	0,1%
p at time A	"AUTRE_ASTER"	2.05473E-2	0,1%
ε_{xx}^p at time A	"AUTRE_ASTER"	1.4054E-2	0,1%
ε_{xy}^p at time A	"AUTRE_ASTER"	1.2981E-2	0.10%

Moreover, in one second series of computations, one test the error indicator due to nonthe radiality of the loading: from a coarse temporal discretization, like previously, one activates the subdivision of time step if the error due to nonthe radiality exceeds 2% (RESI_RADIAL_REL=0.02). This test is carried out for 2 equivalent behaviors: VMIS_CINE_LINE, VMIS_ECMI_LINE. The results are identical for the two behaviors:

Standard	identification of reference	Value	Tolerance
σ_{xx} to time A	"AUTRE_ASTER"	151.2	0,1%
σ_{xy} at time A	"AUTRE_ASTER"	93.1	0,1%
ε_{xx} at time A	"AUTRE_ASTER"	1.48297E-2	0,1%
ε_{xy} at time A	"AUTRE_ASTER"	1.36014E-2	0,1%
p at time A	"AUTRE_ASTER"	2.05473E-2	0,1%
ε_{xx}^p at time A	"AUTRE_ASTER"	1.4054E-2	0,1%
ε_{xy}^p at time A	"AUTRE_ASTER"	1.2981E-2	0.10%

One tests moreover indicators of loss of radiality DERA_ELGA :

Standard	identification of reference	Value	Tolerance
----------	-----------------------------	-------	-----------

DERA_ELGA/ERR_RADI to time 1,	"NON_REGRESSION"	0	0.00%
DERA_ELGA/ERR_RADI at time 1,5	d'NON_REGRESSION "	9.50E-003	0.00%

9 Modelization G

9.1 Characteristic of the modelization

This modelization is similar to modelization A. the principal difference is at the level of the management of time step. One the 1st computation is carried out with a temporal discretization approximately 2 times coarser than that of modelization A. Ensuite, one extracts from result the list from the times really calculated (by taking account of possible under-cuttings of time step during the 1st computation) and one creates one 2nd list of times, 2 times finer than this extracted. To finish, one carries out one the 2nd computation, identical to 1st, but with the finer list of times.

9.2 Quantities tested and results

the tests relate to the strains at the end of the load $\varepsilon_{xx}(B)$ and $\varepsilon_{xy}(B)$ for 2 computations. The tolerances of the 1st computation are 2 times looser than those of the 2nd computation.

9.2.1 Computation with the coarse list

Standard	Identification of reference	Value	Tolerance
ε_{xx} to time A	"ANALYTIQUE"	$3,5265 \cdot 10^{-2}$	0.4%
ε_{xy} at time A	"ANALYTIQUE"	$2,0471 \cdot 10^{-2}$	1.2%

These doubled tests of tests of NON-regression.

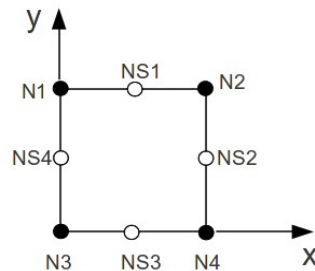
9.2.2 Computation with the refined list

Standard	Identification of reference	Value	Tolerance
ε_{xx} to time A	"ANALYTIQUE"	$3,5265 \cdot 10^{-2}$	0.2%
ε_{xy} at time A	"ANALYTIQUE"	$2,0471 \cdot 10^{-2}$	0.6%

10 Modelization H

10.1 Characteristic of the modelization

Modelization in plane stresses with under-integration: C_PLAN_SI
the loading and the boundary conditions are modelled by:



- Condition of Dirichlet (key word DDL_IMPO):
 - Node *N04* , $x = y = 0$,
 - Nodes *N02* , *NS2* $x = 0$.
- Condition of Neumann, surface forces (key word FORCE_CONTOUR):
 - on the sides (meshes of skin): (1, 2), (2, 4), (4, 3) and (3, 1).

10.2 Characteristics of the mesh

Many nodes: 8
Number of meshes and types: 1 QUAD8, 4 SEG3

10.3 Quantities tested and Case

10.3.1 results of standard

VMIS_ISOT_LINE	Identification of reference	Value of reference	Tolerance
σ_{xx} at time <i>A</i>	"ANALYTIQUE"	1,512E+002	0,1%
σ_{xy} at time <i>A</i>	"ANALYTIQUE"	9,310E+001	0,1%
<i>p</i> at time <i>A</i>	"ANALYTIQUE"	2,0547E-002	0,1%
Rate of triaxialitéformule <i>TRIAx</i> at time <i>A</i>	"ANALYTIQUE"	2,2800E-001	0,1%
ε_{xx} at time <i>A</i>	"ANALYTIQUE"	1,48297E-002	0,1%
ε_{xy} at time <i>A</i>	"ANALYTIQUE"	1,36014E-002	0,1%
plastic Indicator <i>V2</i> at time <i>A</i>	"ANALYTIQUE"	1.0	0,1%
<i>p</i> at time <i>B</i>	"ANALYTIQUE"	4,23293E-002	1,0%
Rate of triaxialitéformule <i>TRIAx</i> at time <i>B</i>	"ANALYTIQUE"	3,25349E-001	0,1%
ε_{xx} at time <i>B</i>	"ANALYTIQUE"	3,5265E-002	1,0%

ε_{xy} at time <i>B</i>	"ANALYTIQUE"	2,0471E-002	1,0%
ε_{xx}^p at time <i>B</i>	"ANALYTIQUE"	3,3946E-002	1,0%
ε_{xy}^p at time <i>B</i>	"ANALYTIQUE"	2,0250E-002	1,0%

as well as the indicators of load-discharge:

Standard	identification of reference	Value	Tolerance
INDIC_ENER to time <i>A</i>	"ANALYTIQUE"	0	0,1%
INDIC_ENER at time <i>B</i>	"ANALYTIQUE"	3,26E-002	3,0%
INDIC_SEUIL at time <i>A</i>	"ANALYTIQUE"	0	0,1%
INDIC_SEUIL at time <i>B</i>	"ANALYTIQUE"	3,26E-002	3,0%
INDIC_ENER at time <i>C</i> (discharge supplements)	"ANALYTIQUE"	4,69E-002	3,0%
INDIC_SEUIL at time <i>B</i>	"ANALYTIQUE"	9,71E-002	1,0%
INDIC_SEUIL at time <i>C</i> (discharge supplements)	"ANALYTIQUE"	1,0	1,0%
DERA_ELNO/RADI_V at time 0,1	"ANALYTIQUE"	0,0	1,0%

10.3.2 Case of VMIS_ECMI_LINE

One calculates energies in the course of resolution and one compares compared to case VMIS_ISOT_LINE :

Standard	identification of reference	Value	Tolerance
ETOT_ELGA/TOTALE to time 0,1	"AUTRE_ASTER"	1,16403E-03	0,1%
ETOT_ELGA/TOTALE at time 0,9	"AUTRE_ASTER"	1,84340	0,1%
ETOT_ELGA/TOTALE at time 2,0	"AUTRE_ASTER"	9,58487	0,1%
ETOT_ELGA/TOTALE at time 3,0	"AUTRE_ASTER"	9,40794	0,1%
ETOT_ELNO/TOTALE at time 0,1	"AUTRE_ASTER"	1,16403E-03	0,1%
ETOT_ELNO/TOTALE at time 0,9	"AUTRE_ASTER"	1,84340	0,1%
ETOT_ELNO/TOTALE at time 2,0	"AUTRE_ASTER"	9,58487	0,1%
ETOT_ELNO/TOTALE at time 3,0	"AUTRE_ASTER"	9,40794	0,1%
ETOT_ELEM/TOTALE at time 0,1	"AUTRE_ASTER"	1,16403E-03	0,1%
ETOT_ELEM/TOTALE at time 0,9	"AUTRE_ASTER"	1,84340	0,1%

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.

ETOT_ELEM/TOTALE at time 2,0	"AUTRE_ASTER"	9,58487	0,1%
ETOT_ELEM/TOTALE at time 3,0	"AUTRE_ASTER"	9,40794	0,1%

One recomputes energies by POST_ELEM and one compares compared to case VMIS_ISOT_LINE :

Standard	identification of reference	Value	Tolerance
ENER_TO2/TOTALE to time 0,1	"AUTRE_ASTER"	1,16403E-03	0,1%
ENER_TO2/TOTALE at time 0,9	"AUTRE_ASTER"	1,84340	0,1%
ENER_TO2/TOTALE at time 2,0	"AUTRE_ASTER"	9,58487	0,1%
ENER_TO2/TOTALE at time 3,0	"AUTRE_ASTER"	9,40794	0,1%
ENER_TO3/TOTALE at time 0,1	"AUTRE_ASTER"	1,16403E-03	0,1%
ENER_TO3/TOTALE at time 0,9	"AUTRE_ASTER"	1,84340	0,1%
ENER_TO3/TOTALE at time 2,0	"AUTRE_ASTER"	9,58487	0,1%
ENER_TO3/TOTALE at time 3,0	"AUTRE_ASTER"	9,40794	0,1%
ENER_TO4/TOTALE at time 0,1	"AUTRE_ASTER"	1,16403E-03	0,1%
ENER_TO4/TOTALE at time 0,9	"AUTRE_ASTER"	1,84340	0,1%
ENER_TO4/TOTALE at time 2,0	"AUTRE_ASTER"	9,58487	0,1%
ENER_TO4/TOTALE at time 3,0	"AUTRE_ASTER"	9,40794	0,1%

One validates the indicators of load-discharge :

Standard	identification of reference	Value	Tolerance
INDIC_ENER to time A	"ANALYTIQUE"	0	0,1%
INDIC_ENER at time B	"ANALYTIQUE"	3,26E-002	3,0%
INDIC_SEUIL at time A	"ANALYTIQUE"	0	0,1%
INDIC_ENER at time C (discharge supplements)	"ANALYTIQUE"	4,69E-002	3,0%
INDIC_SEUIL at time B	"ANALYTIQUE"	9,71E-002	1,0%
INDIC_SEUIL at time C (discharge supplements)	"ANALYTIQUE"	1,0	1,0%
DERA_ELNO/RADI_V at time 0,1	"ANALYTIQUE"	0,0	1,0%

11 Summary of the results

the results are identical whatever the selected modelization. The results are close to the reference solution since the variations are overall lower than 0.6% .