SSNL502 - Beam in Summarized

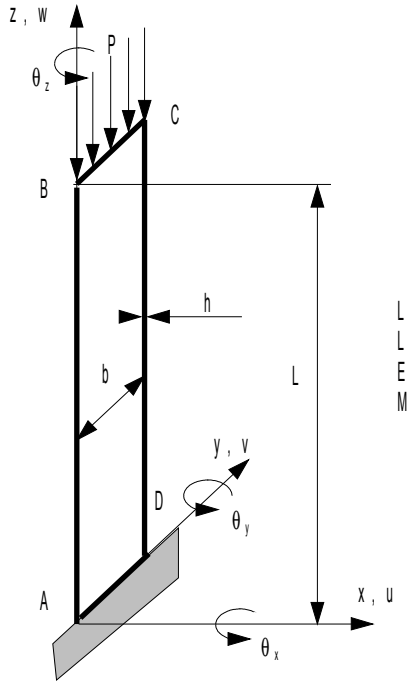
buckling:

This test represents a computation of stability of a cantilever beam subjected to a compressive force at an end. It makes it possible to validate the modelizations finite elements COQUE_3D with meshes the TRIA7 and QUAD9, modelizations POU_D_T_GD and POU_D_TGM with meshes the SEG2 in the nonlinear quasi-static field in large displacements and large rotations in the presence of instability (Buckling of Eulerian).

Displacements and the moments obtained are compared with an analytical reference solution.

1 Problem of reference

1.1 Geometry



Longueur : L = 0.5 m
Largeur : b = 0.075 m
Epaisseur : h = 0.0045 m
Moment d'inertie $I_y = 5.7\ 10^{-10}\ m^4$ ($bh^3/12$)

1.2 Properties of the material

the properties of the material constituting the plate are:

$E = 2.10^{11} Pa$ Modulus Young

$\nu = 0.3$ Poisson's ratio

1.3 Boundary conditions and loadings

Boundary conditions: Clamped AD side

One seeks the successive states of equilibrium under the loading imposed on the side BC:

$$p(t) = p_{cr}\, t$$

with t pseudo_temps

p_{cr} critical load of Eulerian

the load applied corresponds to the critical load of Eulerian $p_{cr} = \frac{\pi E I}{4 L^2} = 1124{,}21\, N$

1.4 Initial conditions

Without Reference solution

2 object

2.1 Method of calculating used for the reference solution

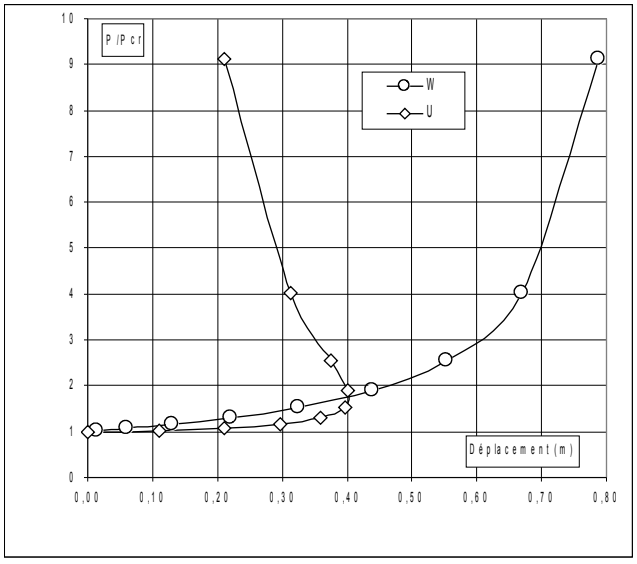
the solution of the problem known as of "" the élastique' "is presented in [bib1] by making L" assumption of nonextension of the average axis. The analytical solution is obtained by considering elliptic integrals.

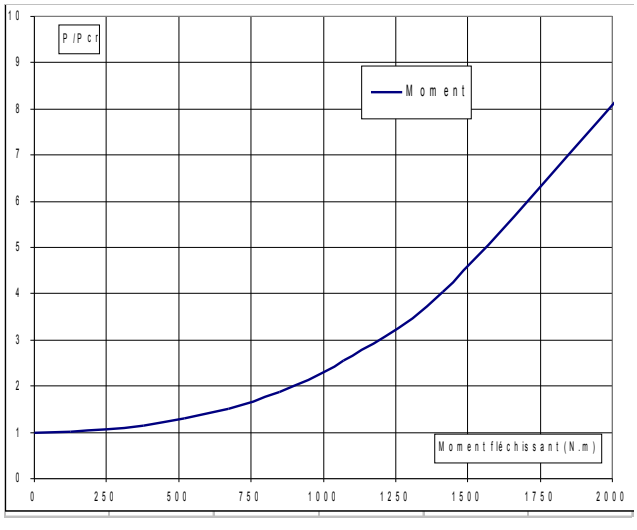
2.2 Results of reference

the results of reference retained for the checks are indicated in bold characters in the table below. Displacements are defined in the reference of definition of the geometry [§1.1].

P/P_{cr}	u_B/L	w_B/L	$\theta°$	M_AL/EI
1,015	**0,220**	**0,030**	**20°**	**0,56**
1,063	0,422	0,119	40°	1,09
1,152	**0,593**	**0,259**	**60°**	**1,67**
1,293	0,719	0,440	80°	2,28
1,518	**0,792**	**0,651**	**100°**	**2,96**
1,884	0,803	0,877	120°	3,73
2,541	**0,750**	**1,107**	**140°**	**4,70**
4,029	0,625	1,340	160°	6,20
9,116	**0,421**	**1,577**	**176°**	**9,44**

Charge P (N)	u_B (m)	w_B (m)	M_A $(N.M)$
1141.07	**0.1100**	**0.0150**	**127.58**
1195.03	0.2110	0.0595	248.32
1295.09	**0.2965**	**0.1295**	**380.45**
1453.60	0.3595	0.2200	519.41
1706.55	**0.3960**	**0.3255**	**674.33**
2118.01	0.4015	0.4385	849.74
2856.62	**0.3750**	**0.5535**	**1070.72**
4529.44	0.3125	0.6700	1412.44
10248.29	**0.2105**	**0.7885**	**2150.55**





2.3 Uncertainties on the analytical

solution Solution

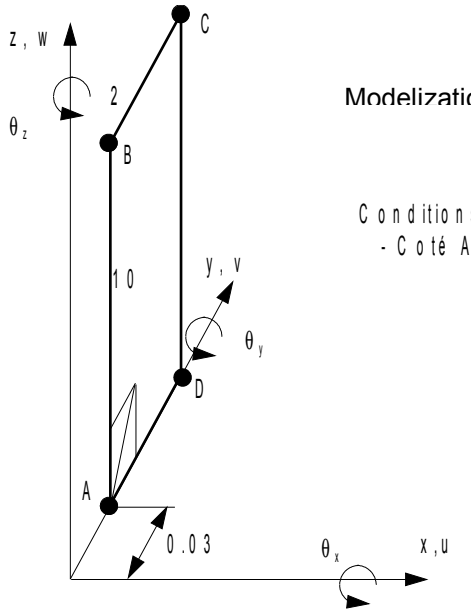
2.4 bibliographical References

1.S.P. TIMOSHENKO, J.M. MANAGES: Theory of elastic stability, second edition, DUNOD 1966.

2.J.L. BATOZ: Large displacements and large rotations of thin beams elastic, mechanical Department of engineering University of Technology of Compiegne 1981.

3 Modelization A

3.1 Characteristic of the modelization



Modelization COQUE_3D (TRIA7)

Conditions aux limites :
- Coté AD : u = v = w = θ_x = θ_y = θ_z = 0

3.2 Characteristic of the mesh

Many nodes: 145
Number of meshes and type: 40 TRIA7

3.3 Quantities tested and results

the strategy of computation used breaks up into two stages:

- **Imposed loading** : one imposes a disturbing load of $1/1000$ critical load following X to reveal the mode of buckling. This load is applied for $P/P_{cr}=0.98$ and until $P/P_{cr}=1.015$.
- **Imposed displacement** : beyond 1.01, the structure became very flexible, one forces an increase in displacement DZ (option DDL_IMPO in STAT_NON_LINE) to determine the behavior postbuckling.

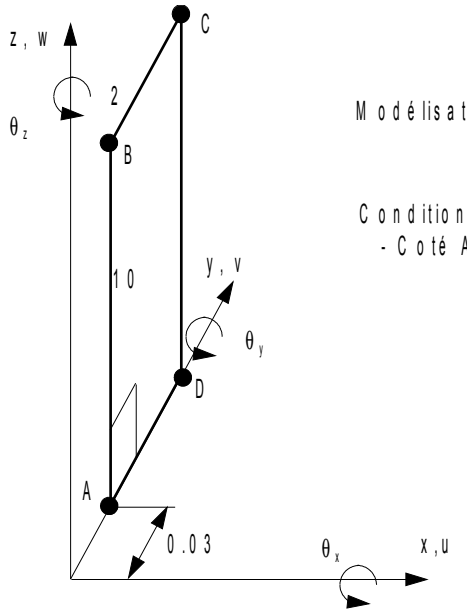
The use of the technique of length of arc makes difficult the definition of the value of reference to be introduced into command TEST_RESU, since these values cannot be imposed. To define the values of reference, we searched possible values of DZ the closest to those listed in the table of [§2.2] and we deferred the values of the parameter of control and DX which one was to obtain for the values of DZ in question.

DZ	Identification	Times	Reference
– 0.0150	DX	1.04532	0.1100
	DZ	1.04532	– 0.0150
	ETA_PILOTAGE	1.04532	1.015
– 0.0595	DX	1.09778	0.2110
	DZ	1.09778	– 0.0595
	ETA_PILOTAGE	1.09778	1.063

– 0.22	DX	1.20824	0.3595
	DZ	1.20824	– 0.22
	ETA_PILOTAGE	1.20824	1.293
– 0.3255	DX	1.26646	0.396
	DZ	1.26646	– 0.3255
	ETA_PILOTAGE	1.26646	1.518
– 0.5535	DX	1.38521	0.375
	DZ	1.38521	– 0.5535
	ETA_PILOTAGE	1.38521	2.541
– 0.67	DX	1.46121	0.3125
	DZ	1.46121	– 0.67
	ETA_PILOTAGE	1.46121	4.029

4 Modelization B

4.1 Characteristic of the modelization



Modélisation COQUE_3D (QUAD9)

Conditions aux limites :
- Coté AD : $u = v = w = \theta_x = \theta_y = \theta_z = 0$

4.2 Characteristics of the mesh

Many nodes: 105
Number of meshes and type: 20 QUAD9

4.3 Quantities tested and results

the strategy of computation used breaks up into two stages:

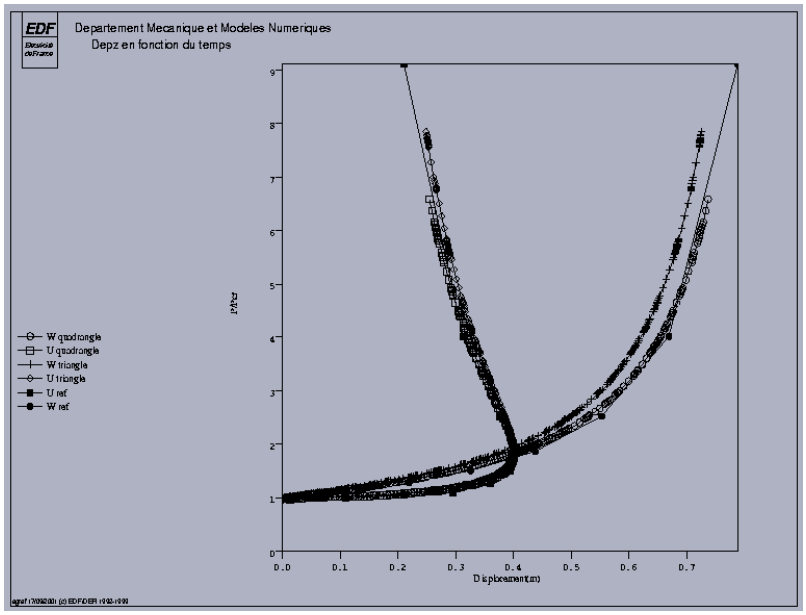
- **Imposed loading** : one imposes a disturbing load of $1/1000$ critical load following X to reveal the mode of buckling. This load is applied for $P/P_{cr}=0.98$ and until $P/P_{cr}=1.015$.
- **Imposed displacement** : beyond 1.01, the structure became very flexible, one forces an increase in displacement DZ (option DDL_IMPO in STAT_NON_LINE) to determine the behavior postbuckling.

The use of the technique of length of arc makes difficult the definition of the value of reference to be introduced into command TEST_RESU, since these values cannot be imposed. To define the values of reference, we searched possible values of DZ the closest to those listed in the table of [§2.2] and we deferred the values of the parameter of control and DX which one was to obtain for the values of DZ in question.

DZ	Identification	Times	Reference
– 0.0150	DX	1.03356	0.1100
	DZ	1.03356	– 0.0150
	ETA_PILOTAGE	1.03356	1.015

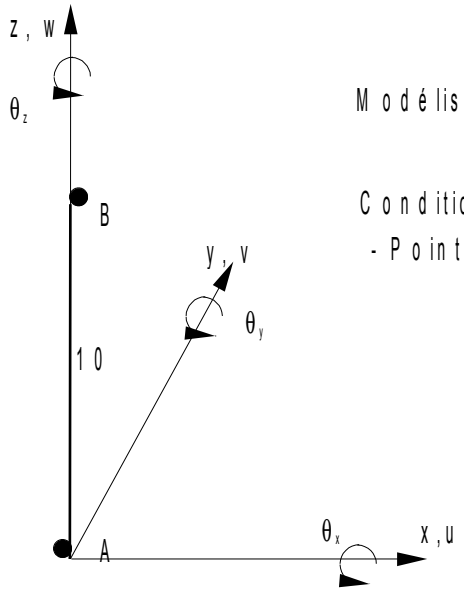
– 0.0595	DX	1.08921	0.2110
	DZ	1.08921	– 0.0595
	ETA_PILOTAGE	1.08921	1.063
– 0.22	DX	1.20259	0.3595
	DZ	1.20259	– 0.22
	ETA_PILOTAGE	1.20259	1.293
– 0.3255	DX	1.25521	0.396
	DZ	1.25521	– 0.3255
	ETA_PILOTAGE	1.25521	1.518
– 0.5535	DX	1.37521	0.375
	DZ	1.37521	– 0.5535
	ETA_PILOTAGE	1.37521	2.541
– 0.67	DX	1.45321	0.3125
	DZ	1.45321	– 0.67
	ETA_PILOTAGE	1.45321	4.029

5 graphic Results of the modelizations A and B



6 Modelization C

6.1 Characteristic of the modelization



Modélisation POU_D_T_GD

Conditions aux limites
- Point A : u = v = w = θx = θy = θz = 0

6.2 Characteristics of the mesh

Many nodes: 11
Number of meshes and type: 10 SEG2

6.3 Quantities tested and Values

6.3.1 results tested

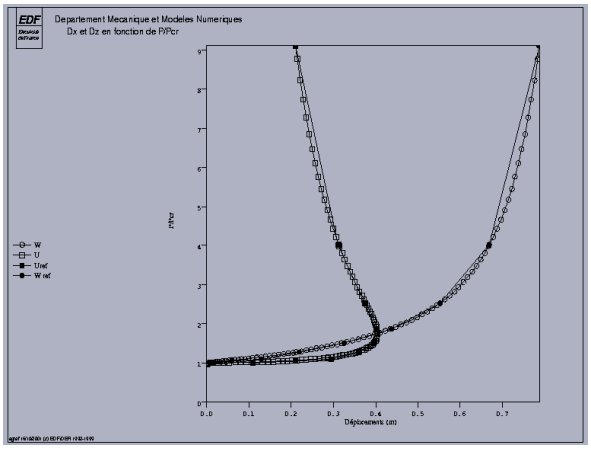
the strategy of computation used breaks up into two stages:

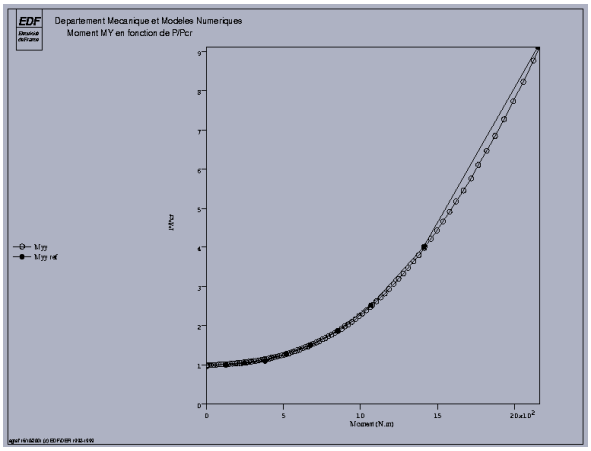
- **Imposed loading** : one imposes a disturbing load of $1/1000$ critical load following X to reveal the mode of buckling. This load is applied for $P/P_{cr}=0.98$ and until $P/P_{cr}=1.015$.
- **Imposed displacement** : beyond 1.01, the structure became very flexible, one forces an increase in displacement DZ (option DDL_IMPO in STAT_NON_LINE) to determine the behavior postbuckling.
- The results are in good adequacy with the reference solution from ETA_PILOTAGE = 1.293. Before this value, the disturbing load (necessary to obtain buckling) degrades the solution, and the variations with the analytical solution are important (up to 80%). The corresponding values are the object of tests of non regression. But this variation is only related to the disturbing load, since by increasing the top-load, one finds the good solution.

DZ	Identification	Times	Reference
– 0.22	DX	1.18684	0.3595
	DZ	1.18684	– 0.22
	ETA_PILOTAGE	1.18684	1.293

	MYY	1.18684	519.41
– 0.3255	DX	1.24521	0.396
	DZ	1.24521	– 0.3255
	ETA_PILOTAGE	1.24521	1.518
	MYY	1.24521	674.3
– 0.4385	DX	1.30521	0.4015
	DZ	1.30521	– 0.4385
	ETA_PILOTAGE	1.30521	1.884
	MYY	1.30521	849.74

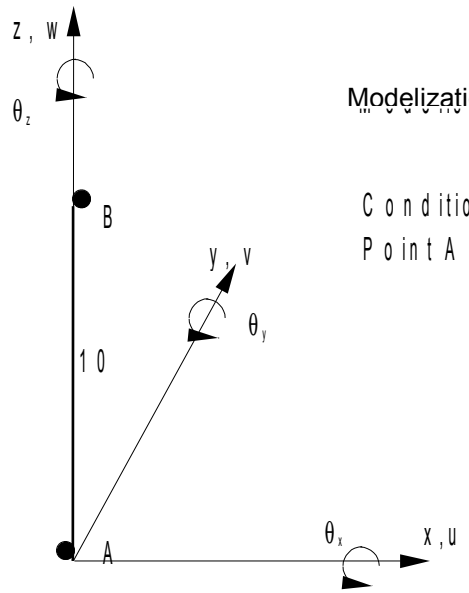
6.4 graphic Results of the modelization C





7 Modelization D

7.1 Characteristic of the modelization



Modelization POU_D_TGM (SEG2)

Conditions aux limites :
Point A : u = v = w = x = θ_x = θ_y = θ_z =

7.2 Characteristic of the mesh

Many nodes: 11
Number of meshes and type: 10 SEG2 uniformly distributed in the length

7.3 Characteristics of the mesh of the cross-sectional area (fibers)

Many fibers: 50 (5 in the width and 10 in the thickness)
Number of meshes and type: 50 QUA4

7.4 Quantities tested and Values

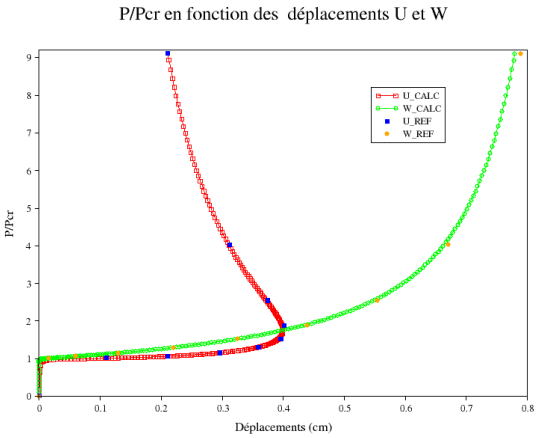
7.4.1 results tested

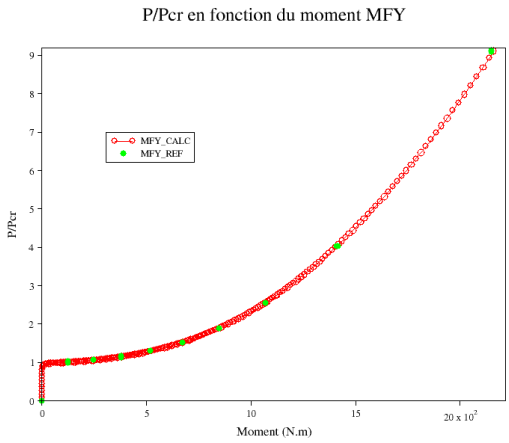
the strategy of computation used breaks up into two stages:

- **Imposed loading** : There is predeformed structure according to its first mode of buckling and with a very low amplitude in front of the length of the beam (about $5.10^{-4} m$). One applies the loading until $P/P_{cr}=0.95$.
- **Imposed displacement** : beyond 0.95, the structure starts to undergo a strong side displacement for a very weak increase in the loading, one thus controls structure in length of arc to determine the behavior postbuckling.
- The results are in good adequacy with the reference solution from ETA_PILOTAGE = 1.152. Before this value, the predeformation (necessary to obtain buckling) degrades the solution, and the variations with the analytical solutions are important (up to 60%). The corresponding values are the object of tests of non regression. But this variation is only related to the initial predeformation (arbitrary), since by increasing the top-load, one finds the good solution.

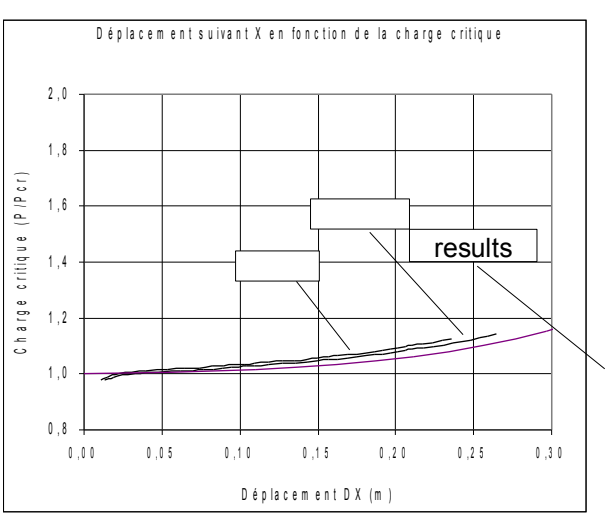
DZ	Identification	Times	Reference
– 0.22	DX	3.8	0.3595
	DZ	3.8	– 0.22
	ETA_PILOTAGE	3.8	1.293
	MYY	3.8	519.41
– 0.3255	DX	4.56	0.396
	DZ	4.56	– 0.3255
	ETA_PILOTAGE	4.56	1.518
	MYY	4.56	674.3
– 0.4385	DX	5.358	0.4015
	DZ	5.358	– 0.4385
	ETA_PILOTAGE	5.358	1.884
	MYY	5.358	849.74

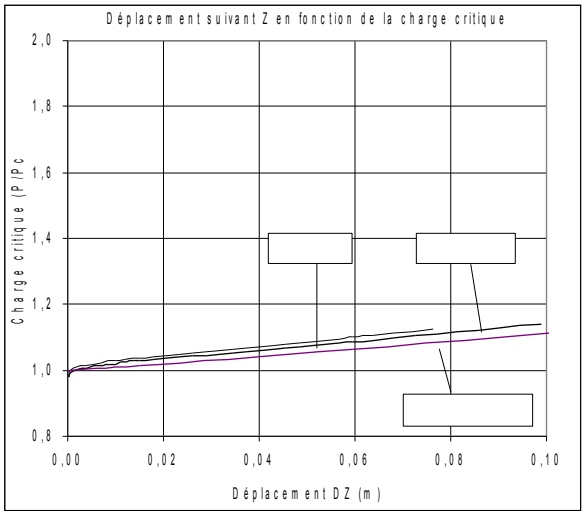
7.4.2 Results graphic of the modelization D





8 Summary of the RéférenceRéférenceTRIA7QUAD9TRIA7QUAD9





the critical load is well detected. The first two results corresponding to the loads $P/P_{cr}=1.015$ and 1.063 are correct, the maximum error is of 3.5% for mesh TRIA7 and 2.2% for mesh QUAD9. Mesh QUAD9 gives better results.

If one continues computations with the shell elements, continuous mesh QUAD9 to give better results. In the zone where displacements in DZ are most important, the mistake made on the load reaches 9% on the quadrangles and goes up to 30% on the triangles. The errors increase in this region because of the slopes of the curves.

The coefficient of correction of transverse shears A_CIS was put at 0.833, corresponding to the thick shells. The value ($9000=10^6\times H/L$) which should have been taken into account does not make it possible to carry out computations. It introduces a bad conditioning of the stiffness matrixes by increasing their disparities.

The solution beam of the code, that it is modelization POU_D_T_GD or POU_D_TGM, provides very good results compared to the analytical solution of reference. However, the computing times are different, the modelization POU_D_T_GD which exact is geometrically led to a computing time almost 40% inferior compared to the modelization POU_D_TGM. Indeed the strain "PETIT_REAC" used for the model multifibre is only one approximation of large displacements and especially of large rotations, it requires to make small increments of loading. On the other hand, it should be recalled that one is able to treat behaviors other than elastic with POU_D_TGM.