

## SSNL501 - Beam fixed at the two ends subjected to a uniform pressure

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### Abstract:

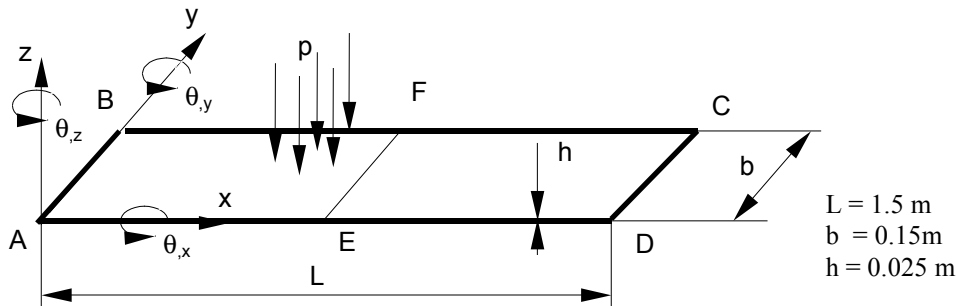
This test represents a quasi-static computation of a clamped beam subjected to a uniform pressure, made up of a perfectly plastic elastic material. This test makes it possible to validate the following modelizations finite elements:

- COQUE\_C\_PLAN (SEG3),
- DKT (TRIA3, QUAD4),
- COQUE\_3D (TRIA7, QUAD9),
- POU\_D\_TGM (SEG2).

The limiting pressure is compared with an analytical reference solution.

## 1 Problem of reference

### 1.1 Geometry



### 1.2 Properties of the material

the properties of the material constituting the beam are:

$$E = 2.10^{11} \text{ Pa} \quad \text{Young's modulus}$$

$$\nu = 0.3 \quad \text{Poisson's ratio}$$

the material follows a perfectly plastic elastic constitutive law:

$$\sigma_e = 2.3510^8 \text{ Pa} \quad \text{Yield stress}$$

$$\varepsilon_e = 1.17510^{-3} \quad \text{Elastic strain limits}$$

### 1.3 Boundary conditions and loadings

- Boundary conditions: Dimensioned  $AB$  and  $CD$  embedded
- following Displacement imposed  $Z$  in  $E$  ( $x=L/2$ ):

$$DZ_e = 6.60910^{-3} \text{ m} \quad \left( DZ_e = \frac{q_e L^4}{384EI} \right)$$

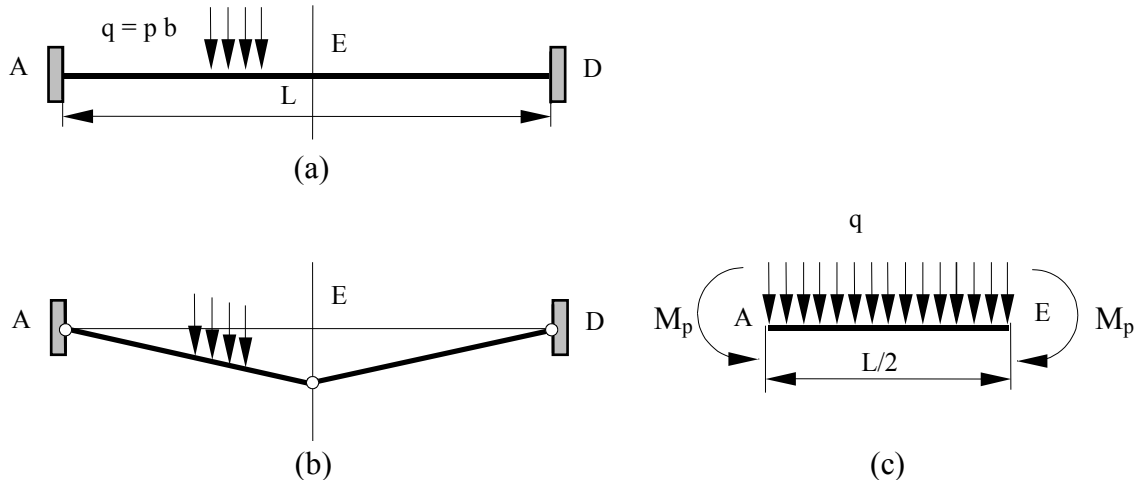
$$DZ(E) \text{ from with } 0 \text{ Initial conditions } 30 DZ_e$$

### 1.4 Without

Reference solution object

## 2 Method of calculating

### 2.1 used for the reference solution



failure of the beam varies appears when there are plastic hinges at the points  $A$ ,  $D$  and  $E$  (figure b). The static equilibrium of the left half of the beam, makes it possible to determine the pressure limits (figure c)

$$\sum M_A = 2M_p - q_L \left(\frac{L}{2}\right) \left(\frac{L}{4}\right) = 0 \quad \Rightarrow \quad q_L = \frac{16M_p}{L^2}$$

where:

$q_L$  represent the limiting pressure

$M_p$  represents the plastic moment ( $M_p = \sigma_e \frac{bh^2}{4}$ )

the appearance of the first plastic point on fiber external of the beam takes place at the points  $A$  and  $D$ , the other fibers being in elastic mode. The pressure yield stress is of  $q_e = 2\sigma_e \frac{bh^2}{L^2}$ .

### 2.2 Results of reference

Pressure limits  $q_L = 39166.67 \text{ N/m}$

### 2.3 Uncertainties on the analytical

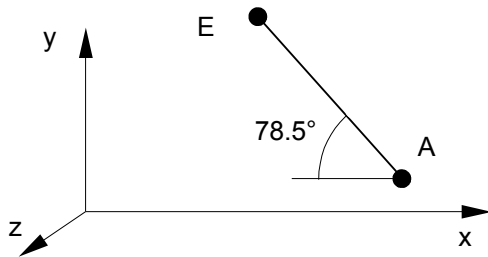
solution Solution

### 2.4 bibliographical References

- 1) WILLIAM A. NASH: Theory and problems of Strength of material, Schaum' S outline series, 2/ed, McGRAW-HILL

### 3 Modelization A

#### 3.1 Characteristic of the modelization



Modélisation COQUE\_C\_PLAN

Point A (2.1 ; 0.7) (nœud n1)

Conditions aux limites : Point A :  $u = v = \theta_z = 0$

Conditions de symétrie : Point E :  $\theta_z = 0$

la poutre étant inclinée à  $78.5^\circ$ , la valeur du déplacement imposé est alors :  $DX_e = 6.609 \cdot 10^{-3} \text{ m} \cdot \sin(78.5^\circ)$

#### 3.2 Characteristics of the mesh

Many nodes: 21

Number of meshes and type: 10 SEG3

#### 3.3 Values tested

$DX(E)/DX_e$	Identification	Times	Reference	Aster	% difference
10	ETA_PILOTAGE	10.1.0		1.0699	6.9
12	ETA_PILOTAGE	12.1.0		1.0699	6.9

#### 3.4 Remarks

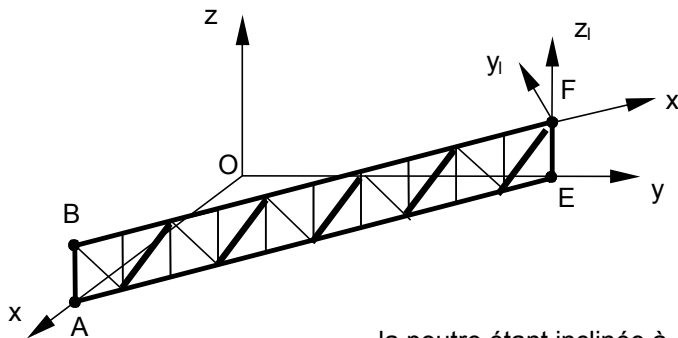
the width for modelization COQUE\_C\_PLAN are imposed on 1 in Code\_Aster. Consequently, we multiplied by  $1/b$  the loading to take account of the real width of the beam.

In this analysis one uses to find the solution, a technique of resolution of type imposed displacement ("DDL\_IMPO"). This method provides for each value of imposed displacement, a multiplying coefficient of the loading ("ETA\_PILOTAGE"). The value of the loading imposed in "AFFE\_CHAR\_MECA" is equal to the limiting pressure, consequently the value of reference of parameter "ETA\_PILOTAGE" is equal to 1.

The reference being taken compared to the solution beam with 20 elements, one observes an improvement of results COQUE\_C\_PLAN when the mesh is refined.

## 4 Modelization B

### 4.1 Characteristic of the modelization



Modélisation DKT (TRIA3)

$$OE = OA = L / 2\sqrt{2}$$

AB // EF // axe Z

Conditions aux limites (repère global)

- côté AB:  $u = v = w = \theta_x = \theta_y = \theta_z = 0$

Conditions de symétrie

- côté EF:  $u = 0$  (repère local  $x_1 y_1 z_1$ )

- côté EF:  $\theta_z = 0$  (repère global)

la poutre étant inclinée à 45°, la valeur du déplacement imposé est alors :  $DX_e = 6.609 \cdot 10^{-3} \text{ m} \cdot \sin(45^\circ)$

### 4.2 Characteristics of the mesh

Many nodes: 43

Number of meshes and type: 20 TRIA3

### 4.3 Values tested

$DX(E)/DX_e$	Identification	Times	Reference	Aster	% difference
5	ETA_PILOTAGE	5.1.0		1.11133	11.11
25	ETA_PILOTAGE	15.1.0		1.142	11.42

### 4.4 Remarks

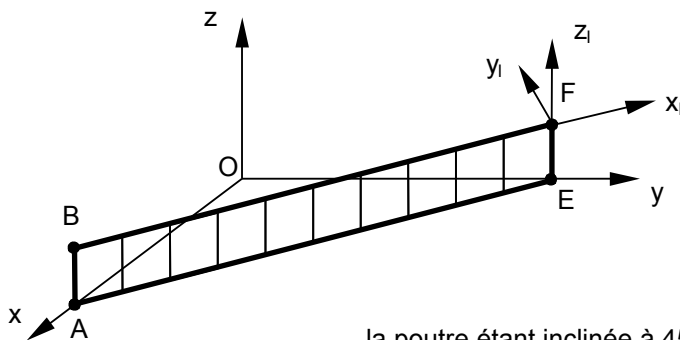
In this analysis, one use to find the solution, a technique of resolution of type imposed displacement ("DDL\_IMPO"). This method provides for each value of imposed displacement, a multiplying coefficient of the loading ("ETA\_PILOTAGE"). The value of the loading imposed in "AFFE\_CHAR\_MECA" is equal to the limiting pressure, consequently the value of reference of parameter "ETA\_PILOTAGE" is equal to 1.

Computations were stopped when the value of parameter ETA\_PILOTAGE was stabilized.

The reference being taken compared to the solution beam with 50 elements, one observes an improvement of results DKT TRIA3 when the mesh is refined.

## 5 Modelization C

### 5.1 Characteristic of the modelization



Modélisation DKT (QUAD4)

$$OE = OA = L / 2\sqrt{2}$$

AB // EF // axe Z

Conditions aux limites (repère global)

- côté AB:  $u = v = w = \theta_x = \theta_y = \theta_z = 0$

Conditions de symétrie

- côté EF:  $u = 0$  (repère local  $x_1 y_1 z_1$ )

- côté EF:  $\theta_z = 0$  (repère global)

la poutre étant inclinée à 45°, la valeur du déplacement imposé est alors :  $DX_e = 6.609 \cdot 10^{-3} \text{ m} \cdot \sin(45^\circ)$

### 5.2 Characteristics of the mesh

Many nodes: 43

Number of meshes and type: 10 QUAD4

### 5.3 Values tested

$DX(E)/DX_e$	Identification	Times	Reference	Aster	% difference
5	ETA_PILOTAGE	5.1.0		1.0837	8.37
25	ETA_PILOTAGE	25.1.0		1.0998	9.98

### 5.4 Remarks

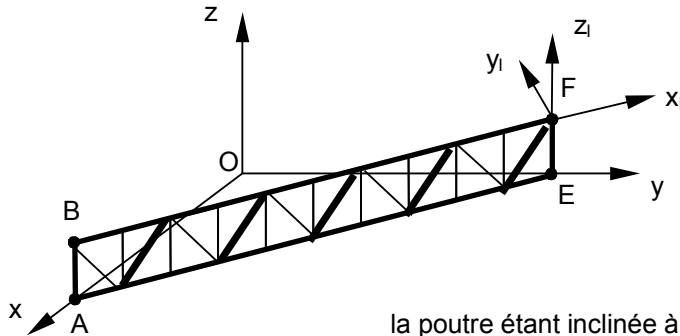
In this analysis, one use to find the solution, a technique of resolution of type imposed displacement ("DDL\_IMPO"). This method provides for each value of imposed displacement, a multiplying coefficient of the loading ("ETA\_PILOTAGE"). The value of the loading imposed in "AFFE\_CHAR\_MECA" is equal to the limiting pressure, consequently the value of reference of parameter "ETA\_PILOTAGE" is equal to 1.

Computations were stopped when the value of parameter ETA\_PILOTAGE was stabilized.

The reference being taken compared to the solution beam with 50 elements, one observes an improvement of results DKT QUAD4 when the mesh is refined.

## 6 Modelization D

### 6.1 Characteristic of the modelization



Modélisation COQUE\_3D (TRIA7)

$$OE = OA = L / 2\sqrt{2}$$

AB // EF // axe Z

Conditions aux limites (repère global)

- côté AB:  $u = v = w = \theta_x = \theta_y = \theta_z = 0$

Conditions de symétrie

- côté EF:  $u = 0$  (repère local  $x_1 y_1 z_1$ )

- côté EF:  $\theta_z = 0$  (repère global)

la poutre étant inclinée à 45°, la valeur du déplacement imposé est alors :  $DX_e = 6.609 \cdot 10^{-3} \text{ m} \cdot \sin(45^\circ)$

### 6.2 Characteristics of the mesh

Many nodes: 83

Number of meshes and type: 20 TRIA7

### 6.3 Values tested

$DX(E)/DX_e$	Identification	Times	Reference	Aster	% difference
5	ETA_PILOTAGE	5.1.0		1.1143	11.43
15	ETA_PILOTAGE	15.1.0		1.1682	16.82

### 6.4 Remarks

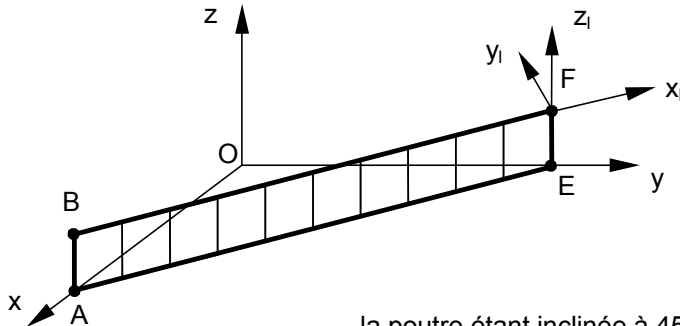
In this analysis, one use to find the solution, a technique of resolution of type imposed displacement ("DDL\_IMPO"). This method provides for each value of imposed displacement, a multiplying coefficient of the loading ("ETA\_PILOTAGE"). The value of the loading imposed in "AFFE\_CHAR\_MECA" is equal to the limiting pressure, consequently the value of reference of parameter "ETA\_PILOTAGE" is equal to 1.

Computations were stopped when the value of parameter ETA\_PILOTAGE was stabilized.

The reference being taken compared to the solution beam with 50 elements, one observes 3D an improvement of the results shells TRIA7 when the mesh is refined.

## 7 Modelization E

### 7.1 Characteristic of the modelization



Modélisation COQUE\_3D (QUAD9)

$$OE = OA = L / 2\sqrt{2}$$

AB // EF // axe Z

Conditions aux limites (repère global)

- côté AB:  $u = v = w = \theta_x = \theta_y = \theta_z = 0$

Conditions de symétrie

- côté EF:  $u = 0$  (repère local  $x_1 y_1 z_1$ )

- côté EF:  $\theta_z = 0$  (repère global)

la poutre étant inclinée à  $45^\circ$ , la valeur du déplacement imposé est alors :  $DX_e = 6.609 \cdot 10^{-3} \text{ m} \cdot \sin(45^\circ)$

### 7.2 Characteristics of the mesh

Many nodes: 54

Number of meshes and type: 10 QUAD9

### 7.3 Values tested

$DX(E)/DX_e$	Identification	Times	Reference	Aster	% difference
5	ETA_PILOTAGE	5.1.0		1.0978	9.78
25	ETA_PILOTAGE	25.1.0		1.1085	10.85

### 7.4 Remarks

In this analysis, one use to find the solution, a technique of resolution of type imposed displacement ("DDL\_IMPO"). This method provides for each value of imposed displacement, a multiplying coefficient of the loading ("ETA\_PILOTAGE"). The value of the loading imposed in "AFFE\_CHAR\_MECA" is equal to the limiting pressure, consequently the value of reference of parameter "ETA\_PILOTAGE" is equal to 1.

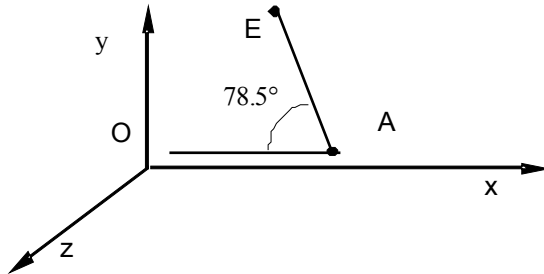
Computations were stopped when the value of parameter ETA\_PILOTAGE was stabilized.

The reference being taken compared to the solution beam with 50 elements, one observes 3D an improvement of the results shells QUAD9 when the mesh is refined.



## 8 Modelization F

### 8.1 characteristic of the modelization



Modélisation POU\_D\_TGM

A= 2.1 0.7 0.

AE =0.75 m

Conditions aux limites (repère global)

- noeud A:  $u = v = w = \theta_x = \theta_y = \theta_z = 0$

Conditions de symétrie

- noeud E:  $u = 0, \theta_z = 0$

la poutre étant inclinée à  $78.5^\circ$ , la valeur du déplacement imposé est alors :  $DX_e = 6.609 \cdot 10^{-3} \text{ m} \cdot \sin(78.5^\circ)$

### 8.2 characteristic of the mesh

- mesh of the beam

Many nodes: 21

Number of meshes and type: 10 SEG2

- mesh of the section



Many nodes: 355

Number of meshes and type: 280 QUAD4

### 8.3 Values tested

$DX(E)/DX_e$	Identification	Times	Reference	% Tolerance
5	ETA_PILOTAGE	10.1.0		14.
5.24	ETA_PILOTAGE	12.1.0		14.

In this analysis, one uses to find the solution, a technique of resolution of type imposed displacement ("DDL\_IMPO"). This method provides for each value of imposed displacement, a multiplying coefficient of the loading ("ETA\_PILOTAGE"). The value of the loading imposed in "AFFE\_CHAR\_MECA" is equal to the limiting pressure, consequently the value of reference of parameter "ETA\_PILOTAGE" is equal to 1.

Computations were stopped when the value of parameter ETA\_PILOTAGE was stabilized.

The solution beam improves appreciably when the mesh is refined.

## 9 Summary of the results

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Concerning the evolution of normal displacement in the center of the plate according to the parameter of control, one observes that:

- the modelizations comprising of the quadrangles give better results, compared to meshes the triangles.

Computations were stopped when the value of parameter `ETA_PILOTAGE` was stabilized, or when computation was not possible any more. Taking into account the meshes used, the got results are satisfactory. The errors observed are for the modelizations:

- `COQUE_C_PLAN` : 6.9% (A) ,
- `DKT` : 11.4% for mesh `TRIA3` (B) and 9.9% for mesh `QUAD4` (C) ,
- `COQUE_3D` : 16% for mesh `TRIA7` (D) and 10.8% for mesh `QUAD9` (E),
- `POUT_D_TGM` : 9% (F).

But it is noted that with a finer mesh at the ends and the center of the plate, place or plasticization appears, it is possible to minimize the error compared to the reference solution.