SSNL130 – Indeformable plate on a carpet of springs

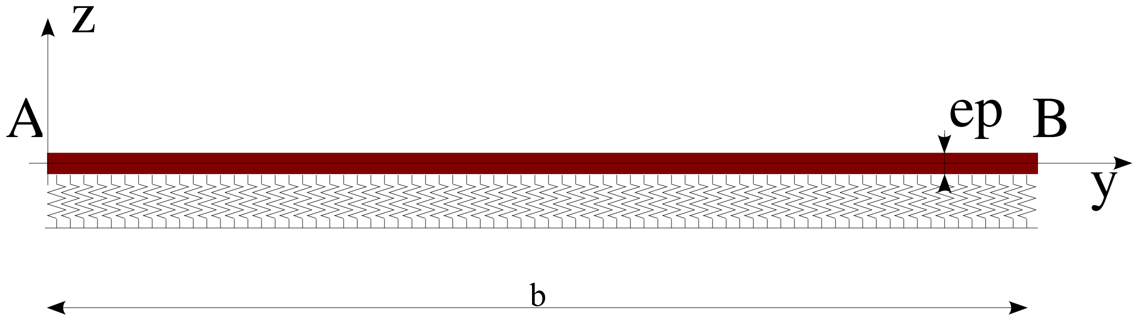
Summarized:
The purpose is to test and validate of the command the possibilities AFFE_CARA_ELEM, option RIGI_PARASOL in $2D$ and $3D$ affected of the behavior DIS_CHOC.
This case test models a plate, considered as indeformable, posed on a carpet of springs.

- Springs are modelled by DIS_T (K_T_D_L), that makes it possible to impose boundary conditions at the ends of springs which are not related to solid.
- Behavior DIS_CHOC allows a unilateral behavior of springs, which leaves a possibility of separation of the plate with respect to the carpet of spring.

1 Problem of reference

1.1 Geometry

a rectangular plate of width a and length b, leaned on a carpet of springs.



Appear 1.1-a : Diagram of the plate and springs in the plane (y, z).

Dimensions:

$a = 1.00\,m$
$b = 2.00\,m$
$ep = 0.30\,m$

1.2 Properties of the material

Modulus Young: $2.0\mathrm{E}+11\,Pa$
Poisson's ratio: 0.3
Total stiffness of the carpet of springs: $K = 10000.0\,N/m$

1.3 Boundary conditions and loadings

the loading is a loading of pressure of the form $P = p.(y-b)^2$, with $p = 5\mathrm{N}/m^4$

Displacements imposed at the ends of springs off-line to the plate:

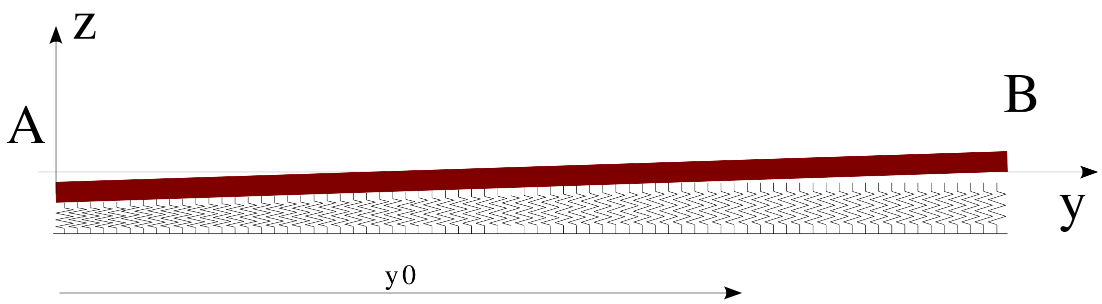
- in the time interval $[0,1]$ displacement is imposed on 0.0 following DX, DY and DZ,
- in the time interval $[1,2]$ displacement is imposed on 0.0 following DX and DY. According to DZ it is imposed by the function $Dz = (t-1.0)*0.5\mathrm{E}\text{-}02$.

1.4 Initial conditions

Without object for a static analysis.

2 Reference solution

2.1 Method of calculating of the continuous problem



Appears 2.1-a : Diagram of the plate and springs after loading.

The resolution of the problem consists in calculating vertical displacements of the corners of the plate and the position of the point of separation with respect to the carpet of springs.
The balance equations are the following ones:
Force resulting due to the loading:

$$Fp=\iint_s P.ds=a.p.\frac{b^3}{3} \quad [2.1\text{-}1]$$

Moment resulting at the point A due to the loading:

$$Mp_A=\iint_s P.y.ds=a.p.\frac{b^4}{12} \quad [2.1\text{-}2]$$

the plate is regarded as rigid, its displacement is form $z(y)=U_a\left(1-\frac{y}{y_0}\right)$. With U_a the vertical displacement of the point A and y_0 the position of separation.
Force of reaction of springs:

$$Fr=\iint_s \frac{K}{a.b}U_a\left(1-\frac{y}{y_0}\right)ds=K\,U_a\frac{y_0}{2b} \quad [2.1\text{-}3]$$

Moment of reaction of springs to point: A

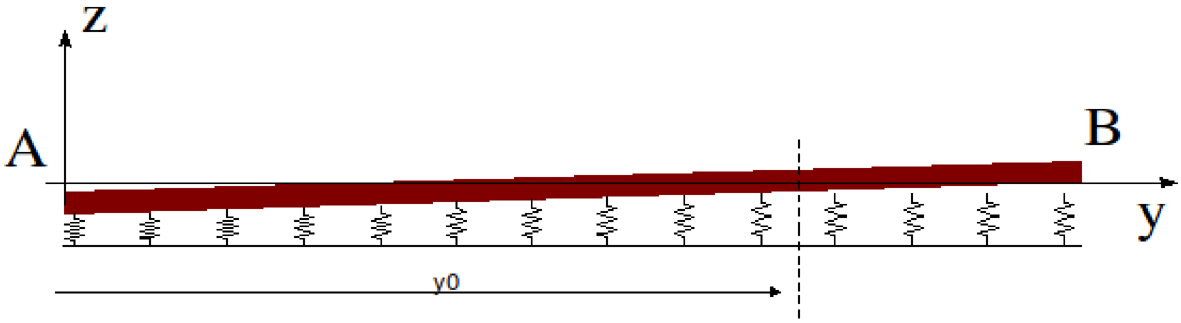
$$Mr_A=\iint_s \frac{K}{a\,b}U_a\left(1-\frac{y}{y_0}\right)y\,ds=K\,U_a\frac{y_0^2}{6b} \quad [2.1\text{-}4]$$

the resolution of equations 2.1-1, 2.1-2, 2.1-3, 2.1-4 (equilibrium of the forces and the moments) gives result according to:

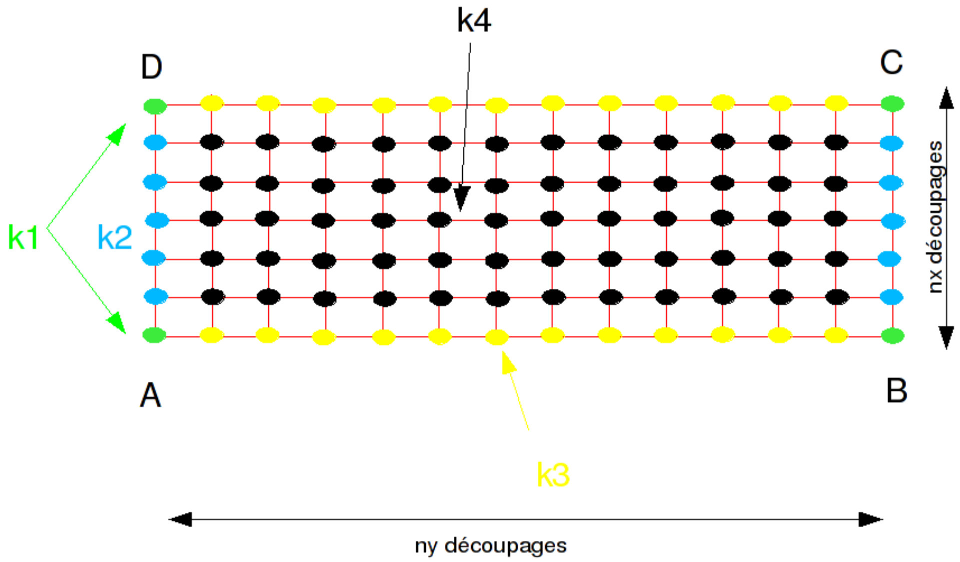
$$y_0=\frac{3b}{4} \quad U_a=-\frac{8\,p\,a\,b^3}{9K} \text{ one of deduced } U_b=-\frac{U_a}{3}$$

2.2 Method of calculating of the problem discretized

In this analysis the carpet from springs is not regarded any more as continuous. Springs are regularly distributed. As previously vertical displacements of the corners of the plate and the position of line of separation with respect to the carpet of springs will be calculated.



Appear 2.2-a : Diagram of the plate and springs after loading.



Appear 2.2-b : Discretization of the plate in the plane (x, y) **.**

The figure above locates springs according to their stiffness. This stiffness is calculated by option RIGI_PARA_SOL of the command AFFE_CARA_ELEM. The assignment of the values is made according to the surface of the zone that they affect. If K is the total stiffness of the carpet of spring, one thus has:

$$\mathrm{k4} = \frac{\mathrm{K}}{nx\,ny} \qquad \mathrm{k2} = \mathrm{k3} = \frac{\mathrm{k4}}{2} = \frac{\mathrm{K}}{2\,nx\,ny} \qquad \mathrm{k1} = \frac{\mathrm{k4}}{4} = \frac{\mathrm{K}}{4\,nx\,ny} \tag{2.2-1}$$

the balance equations are the following ones:
Force of reaction of springs:

$$Fr_{(j)} = U_a \cdot \left[K'_x + K''_x \cdot \sum_{j=1}^{n} \left(1 - j \frac{b}{ny\, y_0} \right) \right] \tag{2.2-2}$$

Moment of reaction of springs along line AB :

$$Mr_{(j)} = U_a \cdot K''_x \cdot \sum_{j=1}^{n} \left(1 - j \frac{b}{ny\, y_0} \right) \cdot j \frac{b}{ny} \tag{2.2-3}$$

with $K'_x = (2\,\mathrm{k1} + \mathrm{k2}(nx-1))$ $\quad K''_x = (2\,\mathrm{k3} + \mathrm{k4}(nx-1))$

$$n\frac{b}{ny}\leq y_0\leq(n+1)\frac{b}{ny}$$

the resolution of the equations (equilibrium of the forces and the moments) the solution of the equilibrium gives:

$$U_a=\frac{p\,a\,b^3 ny(3ny-8n-4)}{6K(1+n+n^2)} \qquad y_0=\frac{b\,n(1+n)(3ny-8n-4)}{3ny(ny+2n(ny-2)-4n^2)} \qquad []$$

where n and y_0 must observe the following conditions:

$$n.\frac{b}{ny}\leq y_0\leq(n+1)\frac{b}{ny} \qquad 0\leq y_0\leq b \qquad n \text{ integer}$$

2.3 Quantities and results of reference

the quantities tested will be vertical displacements with the 4 corners of the plate.

2.4 Uncertainties on the solution

No, the solution is analytical.

3 Modelization A

3.1 Characteristic of the modelization

the plate is modelled by elements DKT. Springs are modelled by AFFECTED seg2 of a modelization DIS_T whose characteristics are K_T_D_L. They are the discrete ones in translation having a diagonal matrix, to see the documentation of AFFE_CARA_ELEM.

3.2 Characteristics of the mesh

the plate is cut out with $ny=16$ and $nx=4$. Dimensions of the plate are $a=1\,m$ and $b=2\,m$

3.3 Quantities tested and results

For time step the n°1, displacements of the ends of the springs, off-line to the plate, are imposed on zero. The results of *Code_Aster* are compared with the discrete solution, which corresponds to the solution of the modelled problem. This solution is obtained for $n=12$, equation 2.2-4.

Nature of the Solution	$U_A=U_D$	$U_B=U_C$
results continues	$\frac{-4}{1125}$	$\frac{4}{3375}$
discrete Solution ($n=12$)	$\frac{-208}{58875}$	$\frac{176}{153075}$
Tolerance	4.0E-04	7.0E-03

For time step the n°2, displacements of the ends of the springs, off-line to the plate, are moved of $+5.0\text{E-}03\,m$. The results of *Code_Aster* are compared with the discrete solution, which corresponds to the solution of the modelled problem.

Nature of the Solution	$U_A=U_D$	$U_B=U_C$
results continues	$\frac{-4}{1125}+\frac{5}{1000}$	$\frac{4}{3375}+\frac{5}{1000}$
discrete Solution B $n=12$ (	$\frac{-208}{58875}+\frac{5}{1000}$	$\frac{176}{153075}+\frac{5}{1000}$
)	4.0E-04	7.0E-03

4 Tolerance Modelization

4.1 Characteristic of the modelization

the plate is modelled in $2D$ of plane strains, by elements QUAD4. Springs are modelled by AFFECTED seg2 of a modelization 2D_dis_t whose characteristics are K_T_D_L. They are the discrete ones in translation having a diagonal matrix, to see the documentation of AFFE_CARA_ELEM.

4.2 Characteristics of the mesh

the plate is cut out with $ny=16$. The length of the plate is $b=2\,m$.

4.3 Quantities tested and results

For time step the n°1, displacements of the ends of the springs, off-line to the plate, are imposed on zero. The results of *Code_Aster* are compared with the discrete solution, which corresponds to the solution of the modelled problem. The solution of equilibrium is obtained for $n=12$, equation 2.2-4.

Nature of the Solution	U_A	U_B
results continues	$\frac{-4}{1125}$	$\frac{4}{3375}$
discrete Solution ($n=12$)	$\frac{-208}{58875}$	$\frac{176}{153075}$
Tolerance	2.0E-07	2.0E-07

For time step the n°2, displacements of the ends of the springs, off-line to the plate, are moved of $+5.0\text{E-}03\,m$. The results of *Code_Aster* are compared with the discrete solution, which corresponds to the solution of the modelled problem.

Nature of the Solution	U_A	U_B
results continues	$\frac{-4}{1125}$	$\frac{4}{3375}$
discrete Solution ($n=12$)	$\frac{-208}{58875}+\frac{5}{1000}$	$\frac{176}{153075}+\frac{5}{1000}$
Tolerance	2.0E-07	2.0E-07

5 Summary of the results

the use of the discrete elements, affected on nodes or segments, with a material of the dis_contact TYPE and used with STAT_NON_LINE (behavior COMP_INCR and relation DIS_CHOC) makes it possible to model a unilateral behavior of springs.

In $2D$ as in $3D$, the use of the key word RIGI_PARASOL of the command AFFE_CARA_ELEM makes it possible to assign to springs stiffness proportional to the length or the surface of the elements to which they are connected.

The behavior being unilateral, it is necessary that *Code_Aster* makes several iterations to find the equilibrium position. It is also possible to encounter problems of convergence related to a loss of accuracy, due to a bad conditioning of the stiffness matrix during iterations. Stiffness of springs being able to cancel itself from one iteration to another.