SSNL126 - Elastoplastic buckling of a straight beam

Summarized:

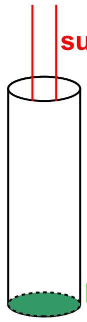
A slender straight beam of circular section is subjected to a compression force at an end, and is embedded at the other end. The behavior of the material is elastoplastic, with a linear isotropic hardening. During the rise in load, one calculates the critical loads of elastic buckling, then plastic.

Two modelizations make it possible to test the criterion of buckling in elastoplasticity:

Voluminal modelization a: mesh, small strains and small displacements.
Modelization b: voluminal mesh, small strains and large displacements (GREEN).

1 Problem of reference

1.1



Straight beam, circular $L=1\mathrm{m}$
length Section of radius $R=0.01\mathrm{m}$.

1.2 Material properties

elastoplastic Material with isotropic linear hardening:

Young modulus: $E=210000\,MPa$
Poisson's ratio: $\nu_{.}=0$. (assumption of beam of Eulerian-Bernoulli)
Elastic limit: $\sigma_Y=4\,MPa$
Tangent modulus: $E_T=70000\,MPa$

1.3 Boundary conditions and loadings

- Boundary conditions: fixed support on all basic surface
- Forces surface on the upper face: to $t=1\mathrm{s}$, $F=6.5\,MPa$

This load is applied into 10 time step équi-distribute.

2 Reference solution

2.1 Method of calculating used for the analytical reference solution

Solution:

In small displacements:

- in elastic mode (for $F<\sigma_Y$) the theoretical breaking value corresponds to the load of Eulerian. In the frame of a kinematics of beam, the critical load is worth:

  $F_{cr}=\frac{\pi^2 EI}{4L^2}$, therefore critical pressure: $P_{cr}=\frac{\pi^2 EI}{4SL^2}$

  with $I=\frac{\pi R^4}{4}$ and $S=\pi R^2$ is $P_{cr}=\frac{\pi^2 ER^2}{16L^2}$

- in elastoplastic mode, as one considers a uniform compression without elastic discharge and because of constitutive law, the critical load of buckling is worth:

  $F_{cr}=\frac{\pi^2 Et.I}{4L^2}$ that is to say a pressure criticizes: $P_{cr}=\frac{\pi^2 EtR^2}{16L^2}$

2.2 Results of reference

Values of the critical load for the two loading cases.

In elastic mode, for $F<4\,MPa$, that is to say $t<0.61538462$, one must obtain: $P_{cr}=12.95\,MPa$.

In plastic mode the critical value of pressure of buckling is: $4{,}32\,MPa$.

The critical coefficients according to the loading are:

Time step	**surface Force (in MPa)**	**Coefficient criticizes**	**Critical load (in MPa)**
1	0.65	19.9290	12.9539
2.1.3		9.9645	12.9539
3	1.95	6.6430	12.9539
4.2.6		4.9823	12.9539
5	3.25	3.9858	12.9539
6.3.9		3.3215	12.9539
7	4.55	0.9490	4.3180
8.5.2		0.8304	4.3180
9	5.85	0.7381	4.3180
10.6.5		0.6643	4.3180

3 Modelization A

3.1 Characteristic of the modelization

Mesh 3D voluminal.

3.2 Characteristics of the mesh

Many nodes: 600
Number of meshes and types: 90 HEXA20

3.3 Values tested

Urgent	Reference
0.2	– 9.9645
1	– 0.6643

4 Modelization B

4.1 Characteristic of the modelization

Mesh 3D voluminal. Large displacements and strains (but small rotations)

the surface force applied is worth here $-20\,MPa$ with $t=1\mathrm{s}$, in order to pass, during the evolution of the loading, by the critical point.

This load is applied into 10 time step équirépartis.

Two complete computations are carried out: one with a purely elastic behavior, in order to be able to compare result with the elastic solution of reference, and the other with an elastoplastic behavior.

4.2 Characteristics of the mesh

Many nodes: 600
Number of meshes and types: 90 HEXA20

4.3 Values tested

In elastic behavior

One test the end value of the critical coefficient: (test of non regression)

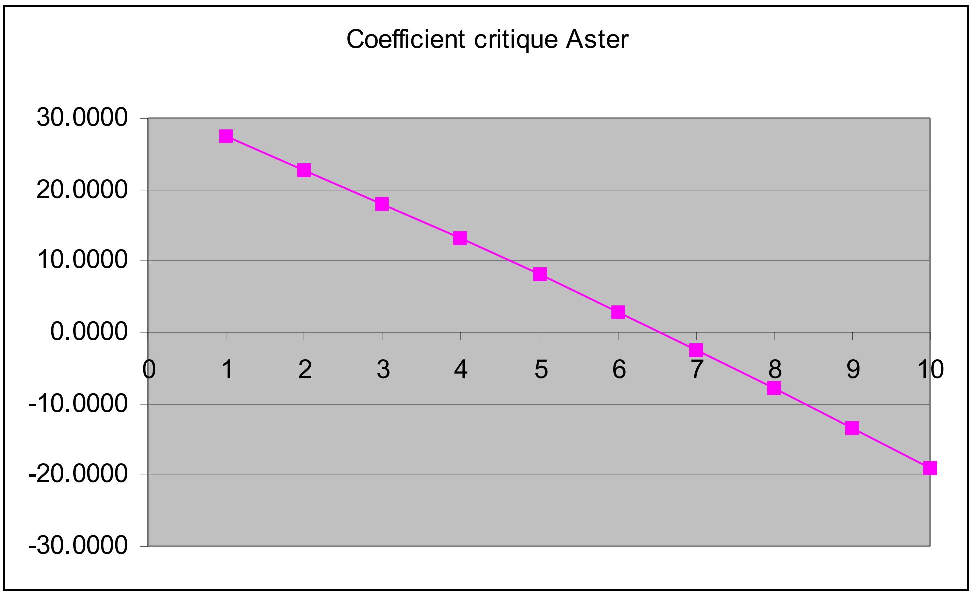
Urgent	Reference
1	– 19.0657

In the case of large displacements or large deformations, the value of the critical coefficient must be interpreted differently case small displacements: the structure becomes unstable when the "critical load" is cancelled.

The evolution of this coefficient in the course of time is the following one:

Time step	surface Force (in MPa)	Coefficient criticizes *Aster*	Critical load Eulerians
1	2	27.5797	12.9539
2	4	22.8250	12.9539
3	6	17.9808	12.9539
4	8	13.0407	12.9539
5	10	7.9975	12.9539
6	12	2.8434	12.9539
7	14	– 2.4301	12.9539
8	16	– 7.8324	12.9539
9	18	– 13.3738	12.9539
10	20	– 19.0657	12.9539

the critical coefficient thus passes well by 0 between times 6 and 7, and more precisely (cf curves following) in the neighborhoods of time 6.5, which corresponds well to the critical load in elasticity.



In elastoplasticity, one tests times when the critical coefficient changes sign. The tests are of non regression since one does not have analytical solution in this case.

Time	Reference
0.4	4.9917
0.5	– 1.3186

5 Summary of the results

the results as of the modelization in small displacements is in conformity with the analytical reference (less than 2% of variation in plasticity). The results in large displacements cannot be compared with a reference solution, but the change of sign of the critical coefficient is in conformity with the expected solution.