SSNL119 - Static response of a reinforced concrete beam (rectangular section) with nonlinear behavior

Abstract:

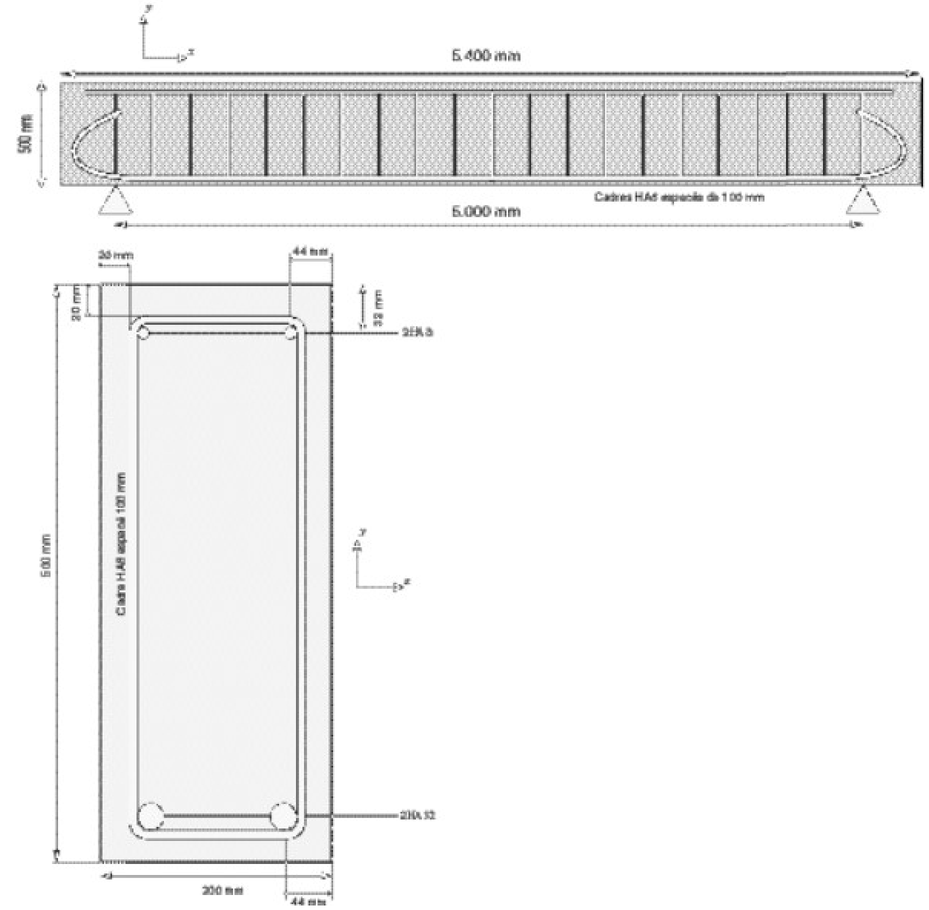
The problem consists in a beam modelization analyzing the response of a reinforced concrete beam in bending 3 points until failure via multifibre [R3.08.08]. This test corresponds to a static analysis of a beam having a linear behavior not -. The concrete is modelled:

- for modelization a: with the lenitive model endommageable of behavior of Borderie in its version $1D$ [R7.01.07], and elastoplastic steel.
- for modelization b: with the lenitive model endommageable of behavior of Mazars in its version $1D$ [R7.01.08], and elastoplastic steel.

The validation of the case test is obtained with comparison with the got results of reference using code FEAP-LMT. Computations are carried out until the failure of structure.

1 General characteristics

1.1 Geometry



Appears 1.1-a : Plan of the beam.

1.2 Material properties

- Concrete:
  Young's modulus: $E=37272\,MPa$
  Poisson's ratio: $\nu=0.2$
  threshold of elasticity in tension: $\sigma_{ft}=3.9\,MPa$
  threshold of elasticity in compression: $\sigma_{fc}=38.3\,MPa$
  threshold of elastic strain in compression: $\varepsilon_{fc}=2.0.10^{-3}$
  energy of cracking $G_f^1=110\,J/m^2$

- steel :
  Young's modulus: $E=200\,000\,MPa$
  Poisson's ratio: $\nu=0.33$
  yield stress: $\sigma_e=400\,MPa$
  Tangent modulus (plastic slope) $E_T=3280\,MPa$

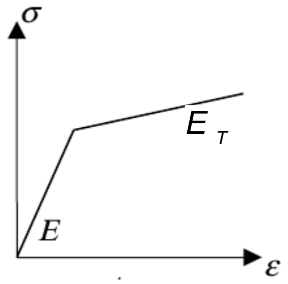


Figure 1.2-a : Curve of stress-strain of steel

1.3 Boundary conditions and loadings

simple Bearing in B : $dy=0$
Bearing doubles in A : $dx=dy=dz=0$ just as $drx=dry=0$.

Quasi-static loading: monotonous displacement dy to the bottom applied to mid--span in C (deflection test 3 points), according to a linear function of time:

t	dy
0,0	0,0 cm
3,0	$-3{,}0$ cm

NB: transverse reinforcements are not taken into account in computations.

2 Reference solution

the reference solution is a computation carried out using the computer code FEAP [bib1] and the version beam multifibers developed with the LMT Cachan from the library of elements FEDEAS [bib2], [bib3]. It is about a computation multifibers with the same finite elements (beam of Eulerian Bernoulli with enrichment) and the same models of behavior of the materials. The discretization is the same one for Code_Aster and the reference.

2.1 Bibliographical references

1 Taylor R.L., FEAP: A finite element analysis program. University of California, Berkeley, 2000.

2 Filippou F.C., static and dynamic analysis for evaluating of structures. 3rd European Conference one Structural Dynamics Eurodyn 96, Florence Italy, 395-402, 1996.

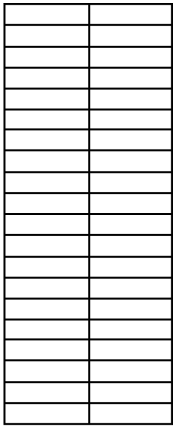
3 Neuenhofer A, Filippou FC. Evaluating of nonlinear frame finite-element models. Newspaper of Structural Engineering (ASCE) 1997; 123(7): 958-966.

3 Modelization A

3.1 Characteristic of the modelization

longitudinal Mesh of beam:
It is composed of 17 nodes and 16 elements `POU_D_EM`.

Cross section of beam: The concrete is modelled by a mesh (`DEFI_GEOM_FIBRE/SECTION`) composed of $2x20$ quadrilaterals (40 fibers):



Steel is modelled by 4 specific fibers (`DEFI_GEOM_FIBRE/FIBER`).

For the modelization A, the concrete is modelled with the model damage of Christian Borderie in version $1D$ (`LABORD_1D`) [R7.01.07]. The material parameters used are the following:

$$Y_{01}=310\,Pa \quad Y_{02}=7000\,Pa \quad A_1=9{,}0.10^{-3}\,Pa^{-1} \quad A_2=5{,}2.10^{-6}\,Pa^{-1}$$

$$B_1=1{,}2 \quad B_2=2{,}0 \quad \beta_1=10^6\,Pa \quad \beta_2=-40.10^6\,Pa \quad \sigma_f=-3{,}5.10^6\,Pa$$

Steel is modelled by the model `VMIS_CINE_LINE`. The material parameters used are the following:

$$D\_SIGM\_EPSI=3.28\,10^9\,Pa$$

$$SY=4.0\,10^8\,Pa$$

the list of time chosen to discretize the monotonous way is:

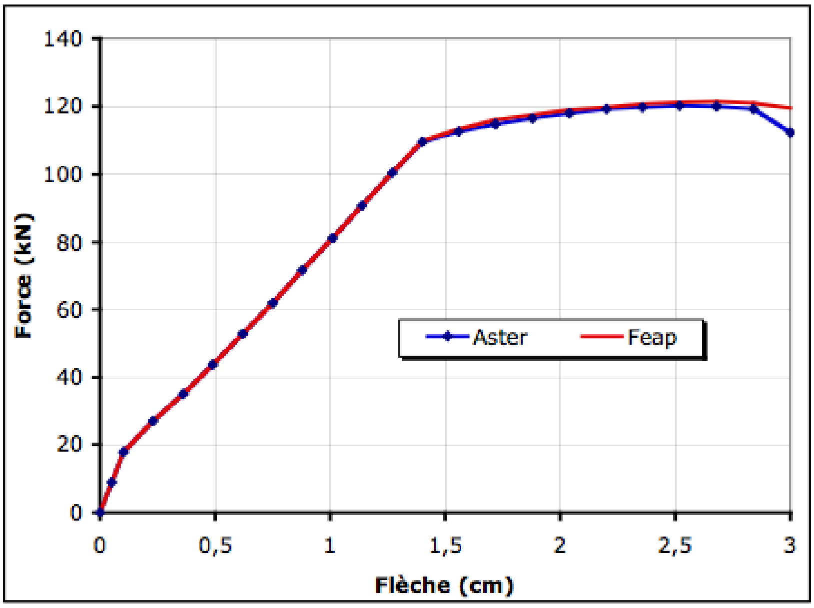
t	dy	Many steps
$0{,}0$	$0{,}0\,cm$	
$0{,}1$	$-0{,}1\,cm$	2
$1{,}4$	$-1{,}4\,cm$	10
$3{,}0$	$-3{,}0\,cm$	10

3.2 Quantities tested and results

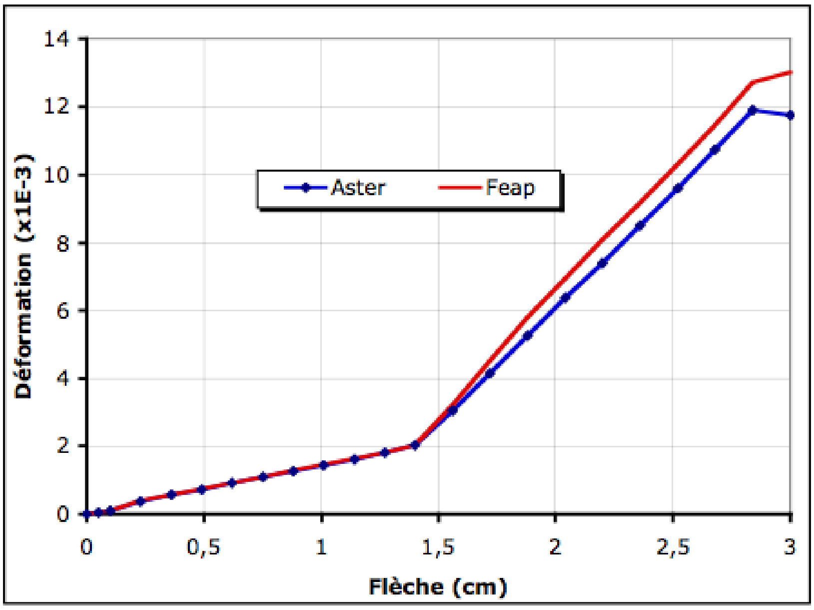
the following curves compare the Code_Aster *results* with those of reference. The discretization and the materials parameters are identical in the Aster computation and the computation of reference. The deflection is that of the center of the beam, the local results (forced, strains) are those of the first Gauss point of the 9th element (nearest to the medium of the beam).

The greatest differences are observed on the strains in tended steel and compressed concrete stresses, beyond the plasticization of tended steels (deflection of $1{,}4\,cm$). The implementation of the model of plasticity of steels is probably slightly different in the two codes.

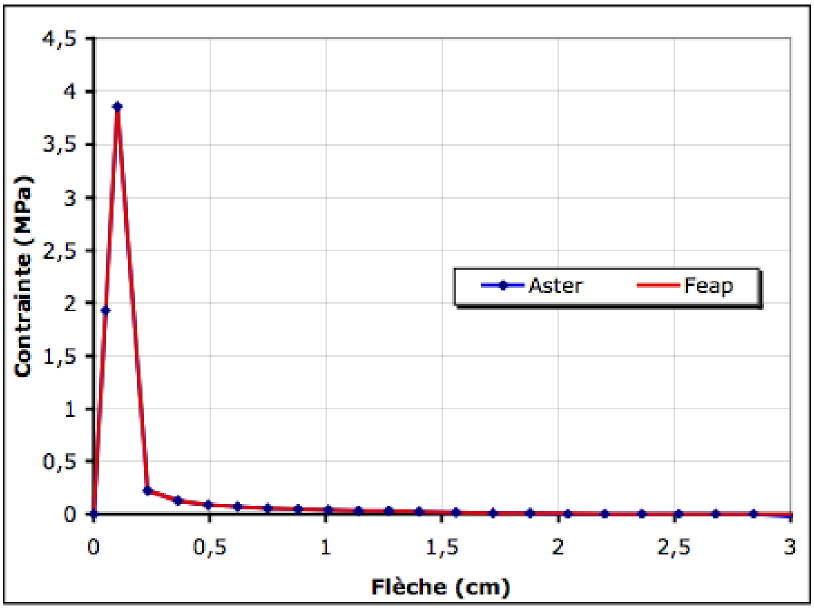
From $2{,}84\,cm$ deflection, the failure of the beam is reached: tended and compressed steels are plasticized, the compressed concrete exceeded the peak of strength on major the part of the section, it does not have there more reserve of capacity. The total force decreases and one observes an elastic return in steels. For these the last two points, the solvers have difficulty converging, the residue do not decrease in a monotonous way. The solution obtained is very sensitive numerically and varies between the codes.



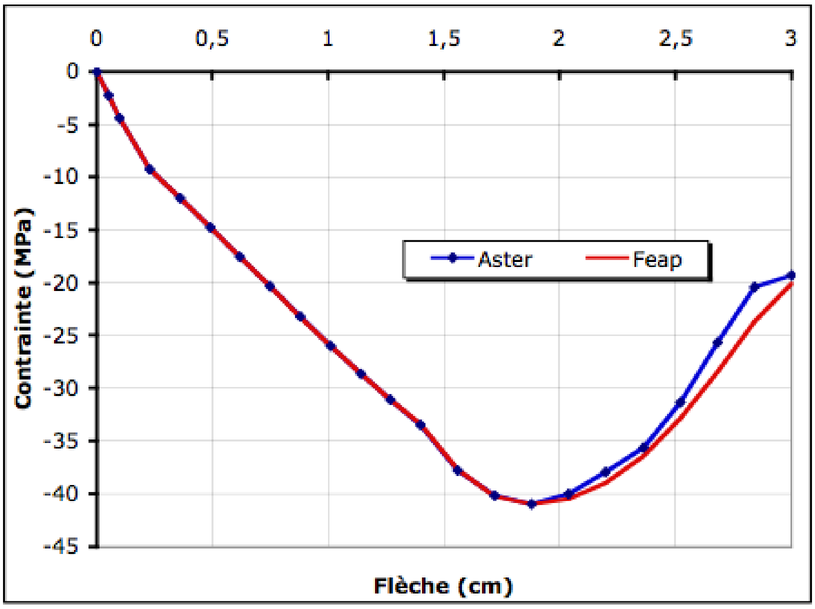
Appear 3.2-aReaction on a bearing, according to the deflection in the center.



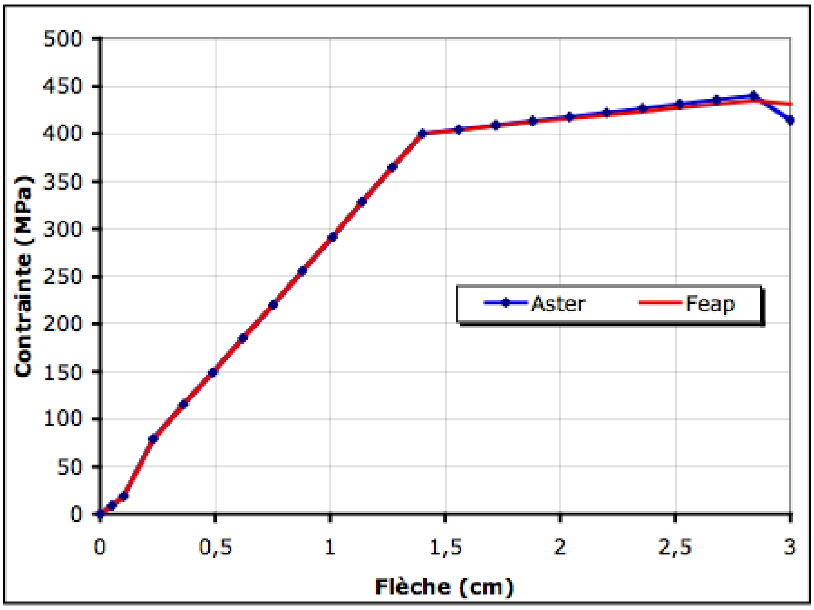
Appear : Strain of tended steels, according to the deflection in the center.



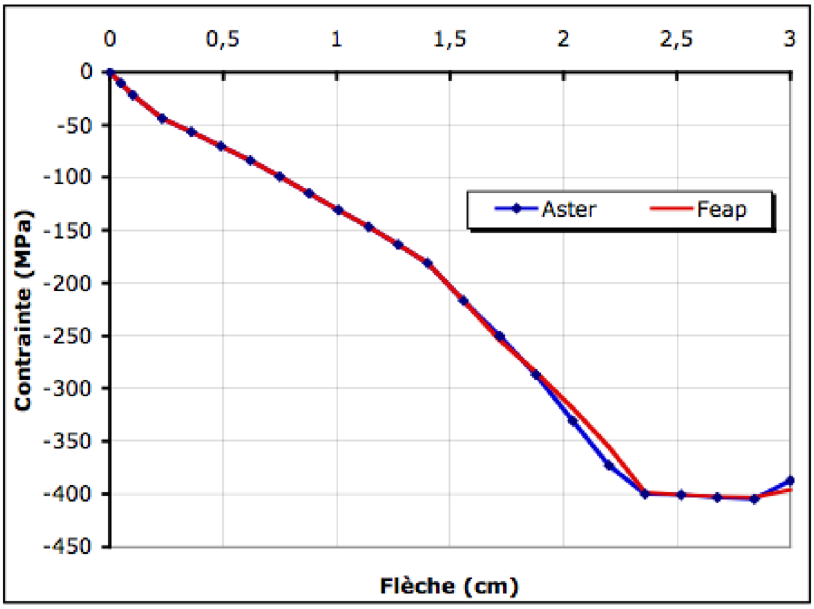
Appear 3.2-c : Stress tended concrete, according to the deflection in the center.



Appear Stress compressed concrete, according to the deflection in the center.



Appear 3.2-e : Stress tended steel, according to the deflection in the center.



Appear 3.2-f : Stress compressed steel, according to the deflection in the center.

Tests of results (`TEST_RESU`) are carried out with each inflection on the curved deflection-reaction [Figure 3-a], i.e. for values of deflection: $0.1\,cm$, $1.4\,cm$ and $2.68\,cm$ (a little before failure).

One evaluates successively two methods of management of time step: the first with the list defined in the § 3.1, then the second with this same list but with in the event of failure the intervention of additional iterations, a minimal subdivision of step of $0.10\text{E-}10$, 50 levels maximum of subdivision.

Deflection $0.1\,cm$	**TYPE OF REFERENCE**	**REFERENCE**	**Tolerance**
Reaction in A	`SOURCE_EXTERNE`	1.77860E+04	3.00E-04
Forced M9, PG1, SP41 Steel tightened	`SOURCE_EXTERNE`	1.88600E+07	3.00E-04
Forced M9, PG1, SP44 compressed Steel	`SOURCE_EXTERNE`	-2.18900E+07	3.00E-04
M9 Strain, PG1, SP41 Steel tightened	`SOURCE_EXTERNE`	9.43200E-05	3.50E-03
Forced M9, PG1, SP1 compressed Concrete	`SOURCE_EXTERNE`	-4.41800E+06	6.00E-05
Forced M9, PG1, SP40 Concrete tightened	`SOURCE_EXTERNE`	3.85500E+06	1.00E-04

Deflection $1.4\,cm$	**TYPE OF REFERENCE**	**REFERENCE**	**Tolerance**
Reaction in A	`SOURCE_EXTERNE`	1.09700E+05	2.50E-04
Forced M9, PG1, SP41 Steel tightened	`SOURCE_EXTERNE`	4.00100E+08	3.00E-04
Forced M9, PG1, SP44 compressed Steel	`SOURCE_EXTERNE`	-1.81100E+08	1.00E-03
M9 Strain, PG1, SP41 Steel tightened	`SOURCE_EXTERNE`	2.02600E-03	5.00E-04
Forced M9, PG1, SP1 compressed Concrete	`SOURCE_EXTERNE`	-3.33800E+07	3.00 E -03
M9 Stress, PG1, SP40 Concrete tightened	`SOURCE_EXTERNE`	2.64100E+04	2.00E-02

Deflection $2.68\,cm$	**TYPE OF REFERENCE**	**REFERENCE**	**Tolerance**
Reaction in A	`SOURCE_EXTERNE`	1.20030E+05	1.50E-03
Forced M9, PG1, SP41 Steel tightened	`SOURCE_EXTERNE`	4.31000E+08	2.00E-02
Forced M9, PG1, SP44 compressed Steel	`SOURCE_EXTERNE`	-4.02100E+08	3.50E-03
M9 Strain, PG1, SP41 Steel tightened	`SOURCE_EXTERNE`	1.14600E-02	9.00E-02
Forced M9, PG1, SP1 compressed Concrete	`SOURCE_EXTERNE`	-2.84800E+07	2.00E-01
Forced M9, PG1, SP40 Concrete tightened	`SOURCE_EXTERNE`	3.36600E+03	3.00E-01

Deflection $2.68\,cm$	**TYPE OF REFERENCE**	**REFERENCE**	**Tolerance**
M9 Strain, PG1, SP41 Steel tightened	`NON_REGRESSION`	1.07550E-02	2.50E-02
Forced M9, PG1, SP1 compressed Concrete	`NON_REGRESSION`	-2.56200E+07	5.50E-02
Forced M9, PG1, SP40 Concrete tightened	`NON_REGRESSION`	2.80130E+03	8.00E-02

The graphs below present:

- evolution of the "Residue" during iterations,

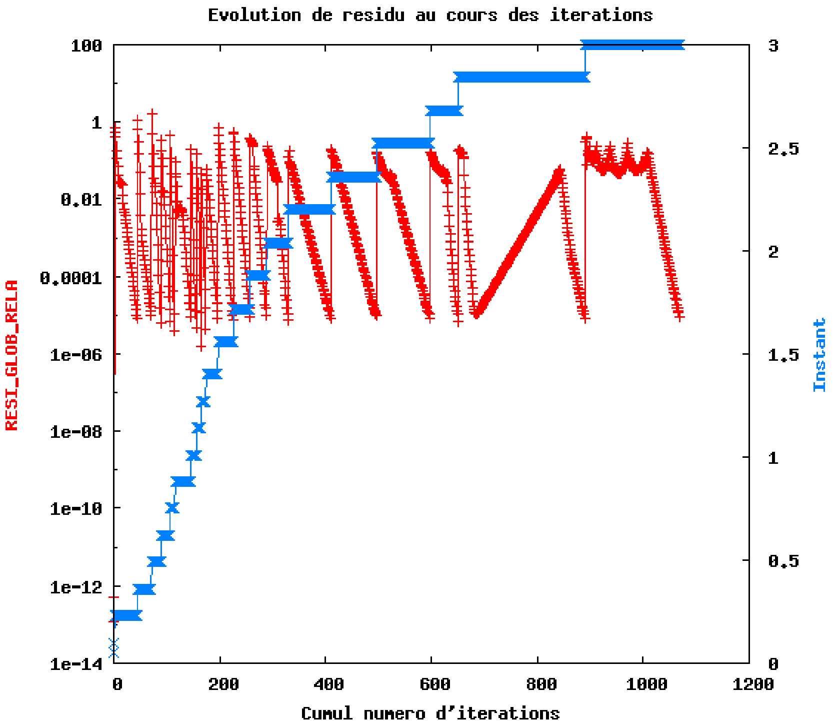


Figure : Evolution of the "Residue" during iterations

- evolution of the "Nombre of iterations" per increment of loading,
- evolution of "Time" by increment of loading,
- evolution of the "TEMPS CPU (by time)" by increment of loading,

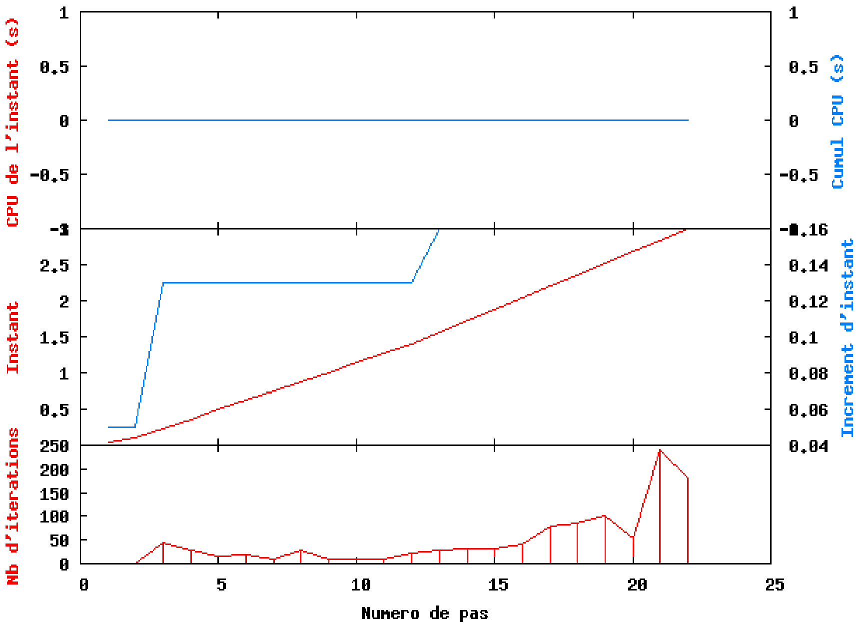


Figure Evolution of the iterations, times and TEMPS CPU per time during increments

- evolution of the “`RESI_GLOB_RELA` ” by number of step of loading,

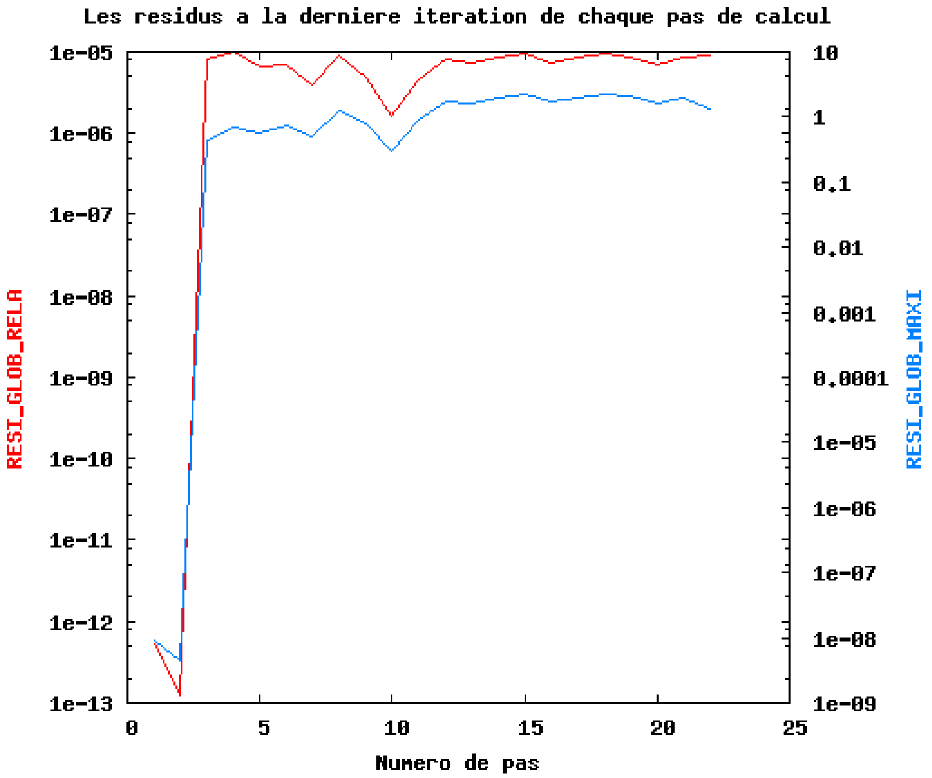


Figure 3.2-iEvolution of the total residue during increments

4 Modelization B

4.1 Characteristic of the modelization

the only difference compared to the modelization A is the concrete which is modelled with the damage model of Mazars in version $1D$ (`MAZARS`) [R7.01.08]. The material parameters used are the following:

$$AC=1.71202987 \quad BC=2.01163780E+03 \quad BT=1.21892353E+04 \quad BETA=1.10E+00$$
$$AT=1.0E+00 \quad EPSD0=8.20396008E-05$$

the parameters of the model of Mazars are given as well as possible to stick to the curve of behavior of the concrete exit of modelization A.

4.2 Grandeurs tested and results

the following tables give the results of *Code_Aster* . The discretization is identical to that of modelization A. the tests are only in NON-regression, because the constitutive law used E is that of Mazars . The deflection is that of the center of the beam, the local results (forced, strains) are those of the second Gauss point of the 9th element (nearest to the medium of the beam).

Reaction of bearing to point: A

Deflection (m)	Standard	Quantity	Place reference	Tolerance
1.00E-03	REAC_NODA DY	group: A	NON_REGRESSION	3.0E-04
1.40E-02	REAC_NODA DY	group: A	NON_REGRESSION	3.0E-04
2.68E-02	REAC_NODA DY	group: A	NON_REGRESSION	3.0E-04

Forced in tended steels:

Deflection (m)	Standard	Quantity	Place reference	Tolerance
1.00E-03	SIXX	nets: M9, point: 2 sous_point: 41	NON_REGRESSION	3.0E-04
1.40E-02	SIXX	net: M9, point: 2 sous_point: 41	NON_REGRESSION	3.0E-04
2.68E-02	SIXX	net: M9, point: 2 sous_point: 41	NON_REGRESSION	3.0E-04

Forced in compressed steels:

Deflection (m)	Standard	Quantity	Place reference	Tolerance
1.00E-03	SIXX	nets: M9, point: 2 sous_point: 44	NON_REGRESSION	3.0E-04
1.40E-02	SIXX	net: M9, point: 2 sous_point: 44	NON_REGRESSION	3.0E-04
2.68E-02	SIXX	net: M9, point: 2 sous_point: 44	NON_REGRESSION	3.0E-04

Strain in tended steels:

Deflection (m)	Standard	Quantity	Place reference	Tolerance
1.00E-03	EPXX	nets: M9, point: 2 sous_point: 41	NON_REGRESSION	3.0E-04
1.40E-02	EPXX	net: M9, point: 2 sous_point: 41	NON_REGRESSION	3.0E-04
2.68E-02	EPXX	net: M9, point: 2 sous_point: 41	NON_REGRESSION	3.0E-04

Forced in the compressed concrete:

Deflection (m)	Standard	Quantity	Place reference	Tolerance
1.00E-03	SIXX	nets: M9, point: 2 sous_point: 1	NON_REGRESSION	3.0E-04
1.40E-02	SIXX	nets: M9, point: 2 sous_point: 1	NON_REGRESSION	3.0E-04
2.68E-02	SIXX	nets: M9, point: 2 sous_point: 1	NON_REGRESSION	3.0E-04

Forced in the tended concrete:

Deflection (m)	Standard	Quantity	Place reference	Tolerance
1.00E-03	SIXX	nets: M9, point: 2 sous_point: 40	NON_REGRESSION	3.0E-04
1.40E-02	SIXX	net: M9, point: 2 sous_point: 40	NON_REGRESSION	3.0E-04
2.68E-02	SIXX	net: M9, point: 2 sous_point: 40	NON_REGRESSION	3.0E-04

the curves are obtained with *Code_Aster*. They are to be compared with those obtained with modelization A.

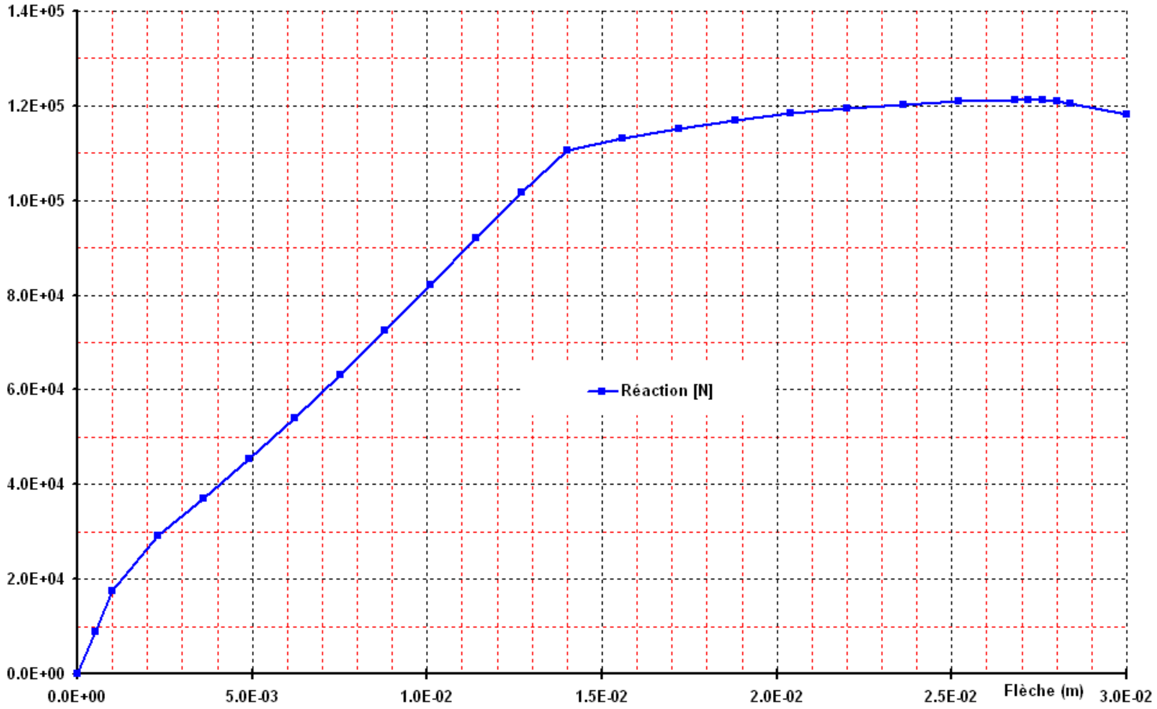
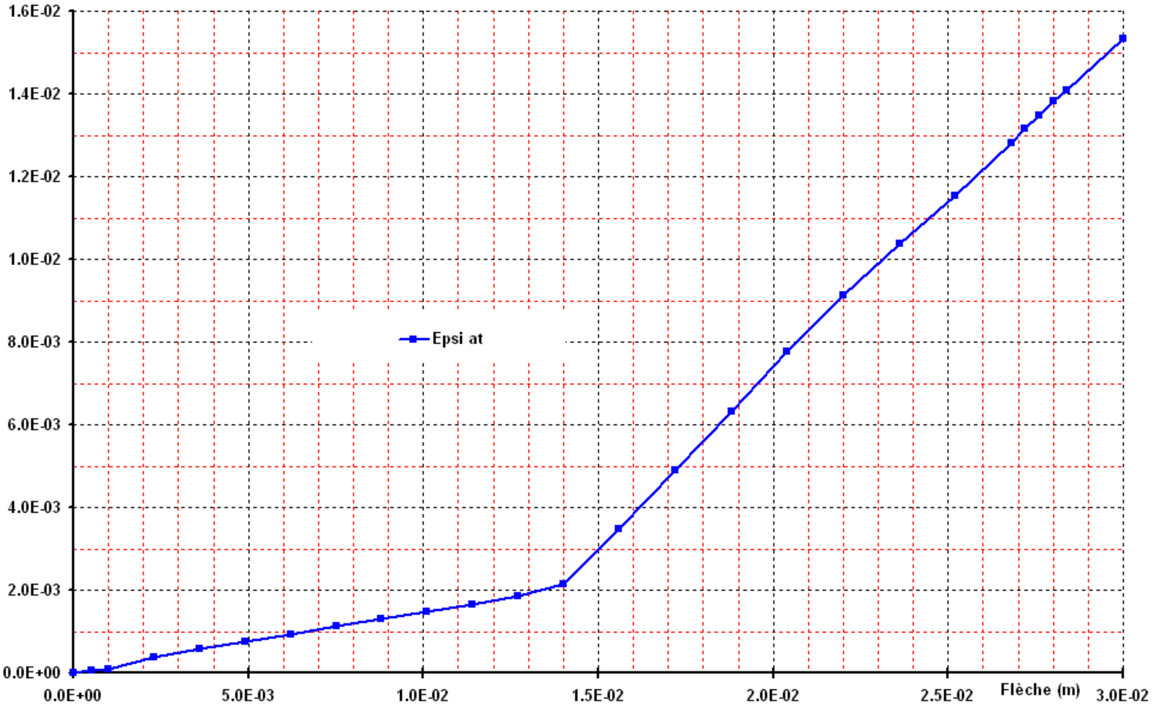
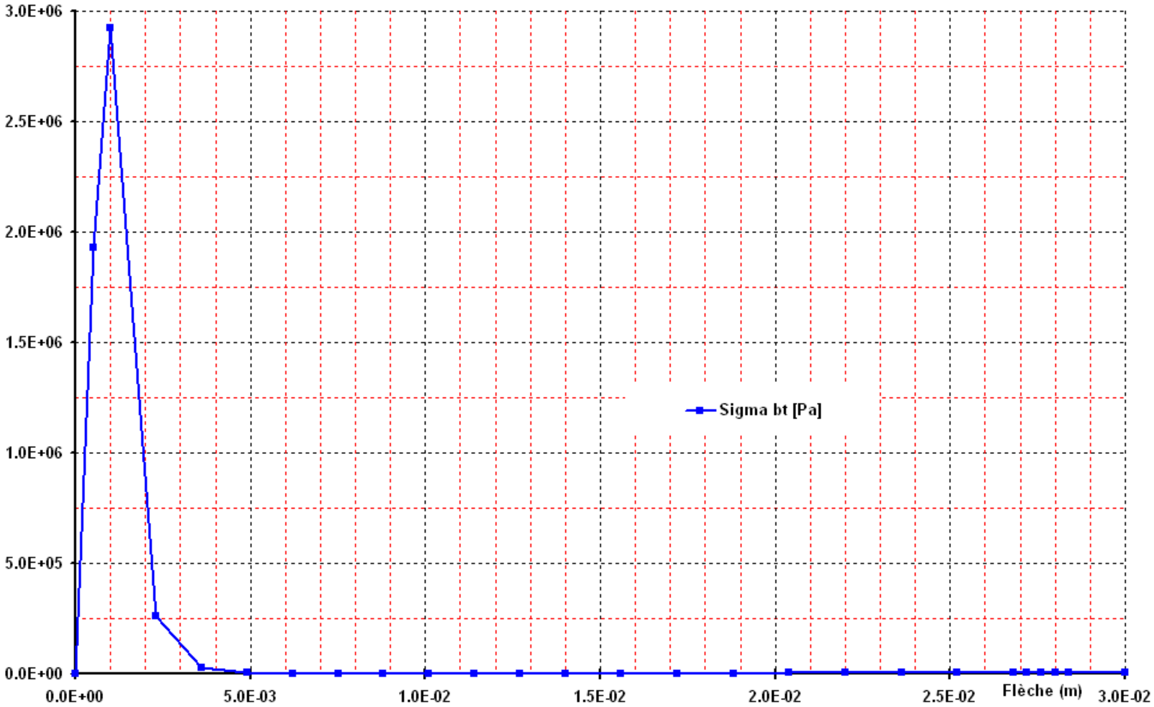


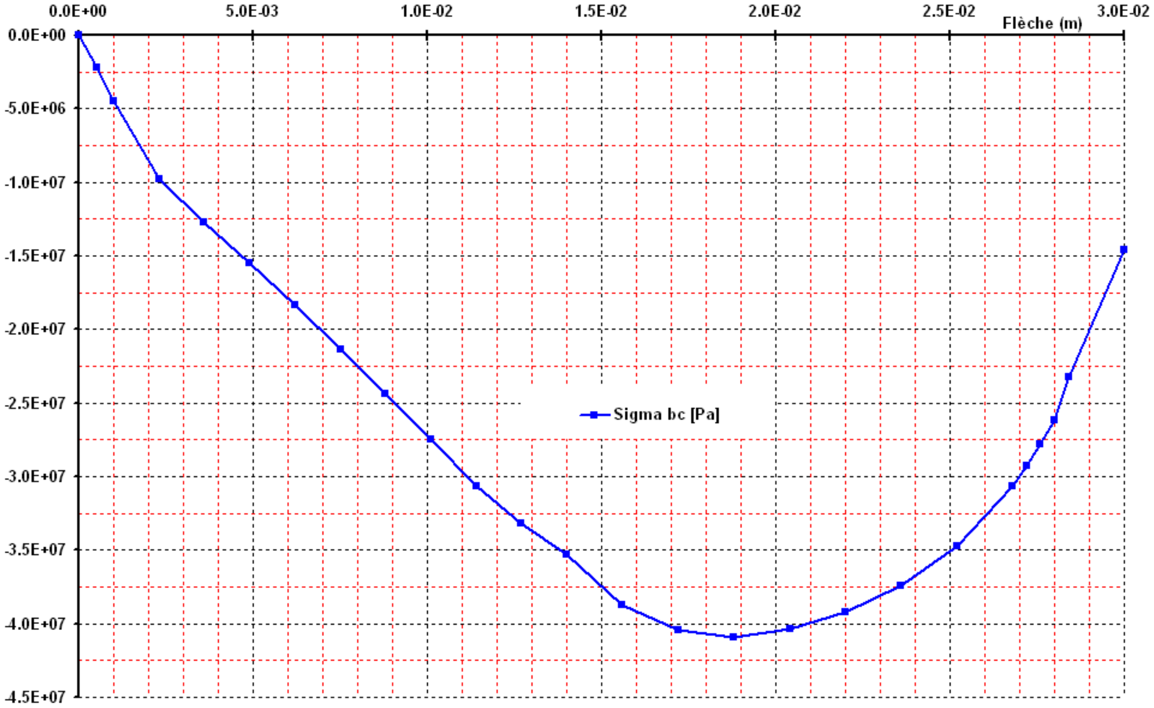
Figure 4.2-a : Reaction on a bearing, according to the deflection in the center.



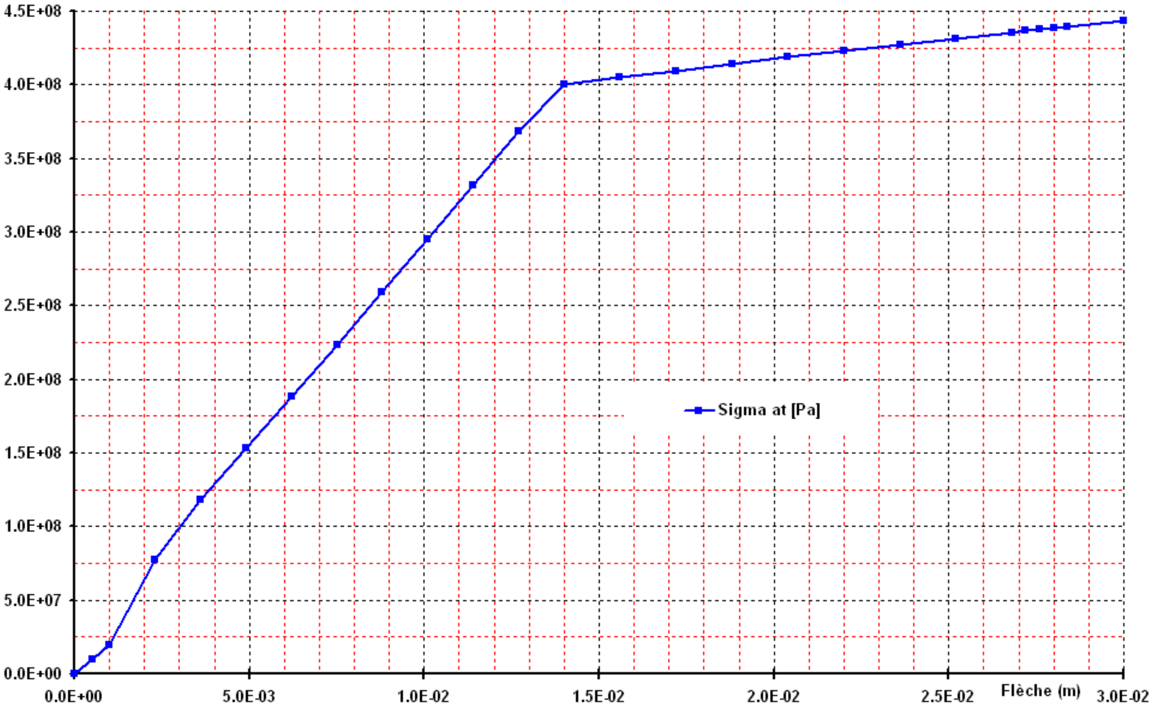
Appear Strain of tended steels, according to the deflection in the center.



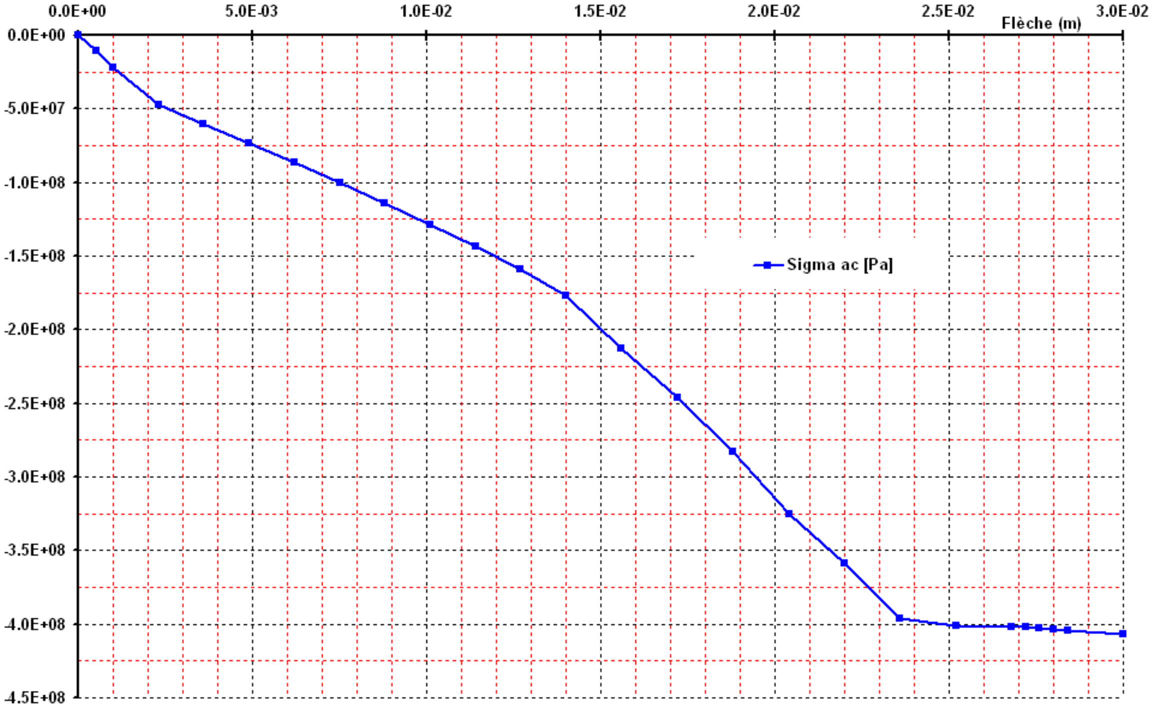
Appear 4.2-c : Stress tended concrete, according to the deflection in the center.



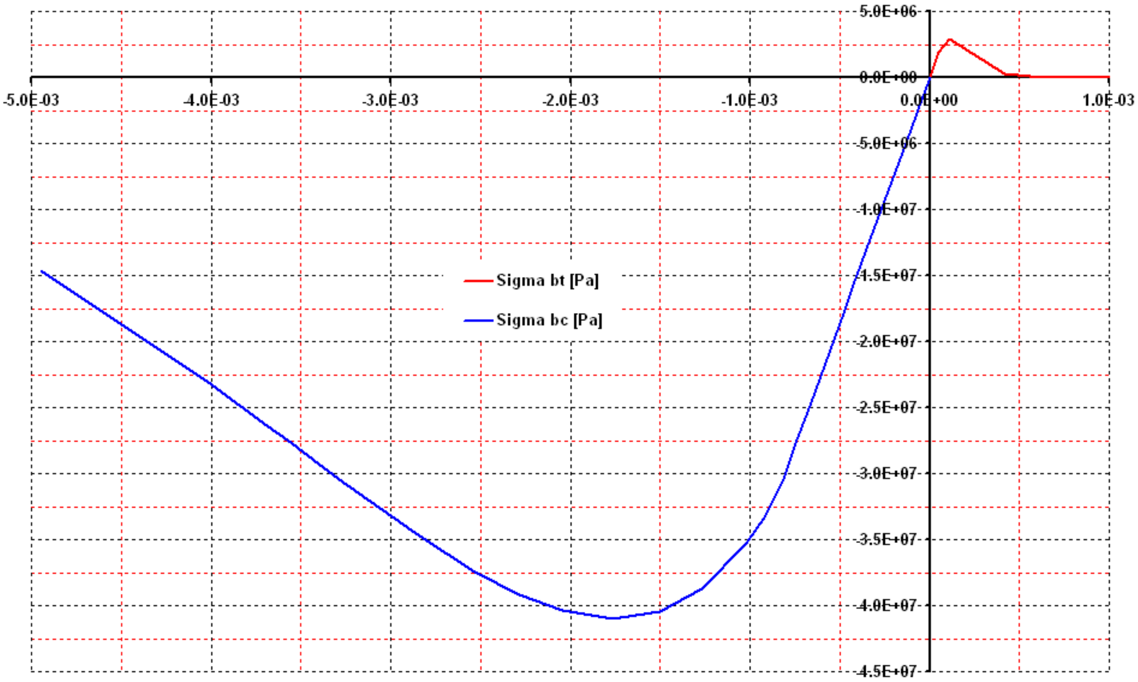
Appear : Stress compressed concrete, according to the deflection in the center.



Appear Stress tended steel, according to the deflection in the center.



Appear4.2-f : Stress compressed steel, according to the deflection in the center.



Appear 4.2-g: Co mportement of the concrete .

5 Summary of the results

For modelization a:
the results got with *Code_Aster* are in concord with those of reference (solution of computation with code FEAP-LMT).
As expected, this unidimensional constitutive law of the concrete of introduced Borderie of softening, therefore a loss of unicity of the solutions. That explains the increasing variations with the reference obtained with software FEAP.
The method of subdivision tested produces results very close to those obtained in absence of subdivision.

Comments:
It was tempted to refine time step but that does not improve anything, because there is always one time step (a little before $3\,cm$) which puts as much, even more, of iterations to converge, and even which does not converge if one refines too much. That corresponds to the moment when the compressed concrete is crushed on an important zone and it occurs a kind of snap-back, all the more violent one that time step are small.

With large a enough step one ends up a little further finding more easily an equilibrium, with $3\,cm$. The search of the new state of equilibrium of the element (position of the neutral axis for each Gauss point and balancing of the axial load between the two Gauss points with the degree of freedom interns) is rather difficult and the tangent matrix is not great help with these local discharges on a large number of fibers.

Cutting suggested functions better than the various tried attempts at refining.

The finer cutting is, the less that converges, phenomenon which one often observes with lenitive models, it is much more rarely true in the multifibre beams, but here one has a rather severe case test which activates all failures (tension of the concrete, plasticization of tended reinforcements, crushing of the compressed concrete, perhaps even plasticization of compressed steels). It is an example of optimized design.

For modelization b:
the tests are carried out in NON-regression owing to the fact that it does not exist of reference resulting from another code with the constitutive law of Mazars. The coefficients of the model of Mazars are fixed in order to stick as well as possible to the curve of behavior of the concrete obtained with the model of Borderie.
The comparison of the curves between the modelizations A and B show that there are not great differences.

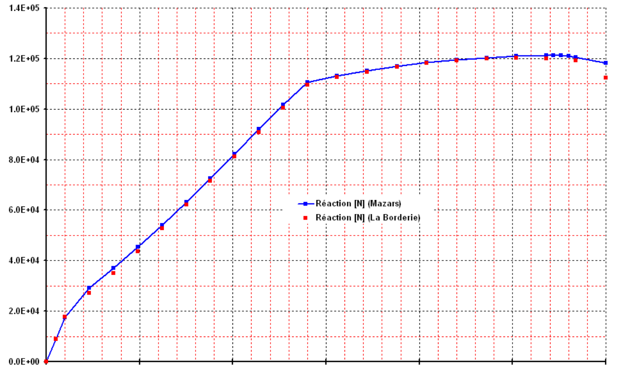
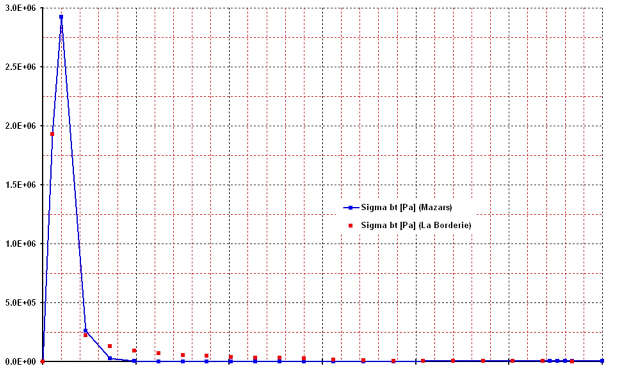
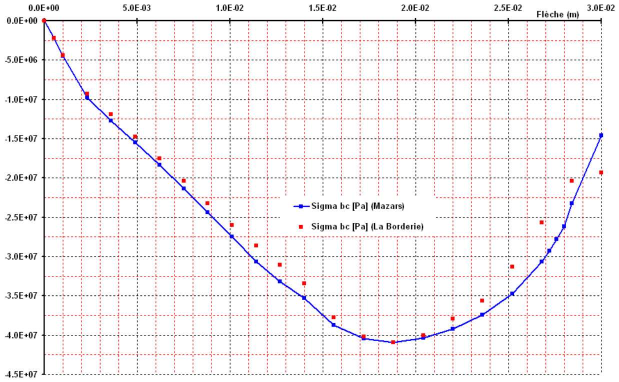


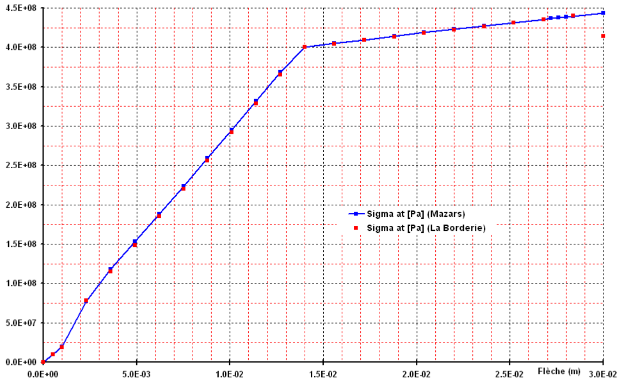
Figure 5-a : 5-a Reaction on a bearing, according to the deflection in the center.



Appear 5-b : Stress tended concrete, according to the deflection in the center.



Appear : Stress compressed concrete, according to the deflection in the center.



Appear 5-d : Stress tended steel, according to the deflection in the center.

Contrary to the modelization "has", it does not have there particular refinement of time step to realize.
The computation is held without problem. For the modelization there "has" is 22 time step, which corresponds to 800 iterations of Newton for complete computation. For the modelization "B" there are 26 time step (release of a subdivision for nonconvergence in 10 iterations), which corresponds to 110 iterations of Newton for complete computation. The TEMPS CPU is connected to the total nombre of iterations of Newton. The modelization "B" convergence 10 times more quickly than the modelization "has", for comparable results.