

SSNL112 - Bar subjected has a Summarized cyclic thermal

loading:

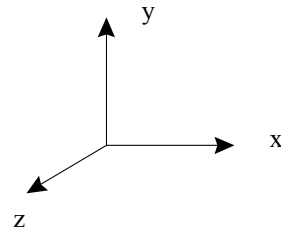
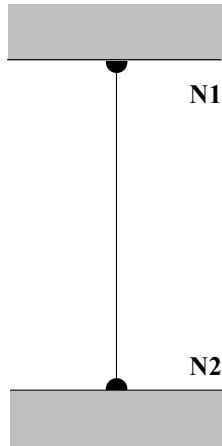
This case test enters the frame of the validation of the behavior models in elastoplasticity of the elements bars for the quasi-static mechanics of structures.

A clamped bar has these two ends undergoes a cyclic thermal loading inducing forces of traction and compression.

Each modelization makes it possible to validate one of the behavior models nonlinear introduced: Linear isotropic hardening with criterion of Von-Settings (modelization A), linear kinematic hardening with criterion of Von-Settings (modelization B), as well as a model known as of Pinto-Menegotto, representing the cyclic behavior of steel reinforcements in the reinforced concrete (modelizations C and D).

1 Problem of reference

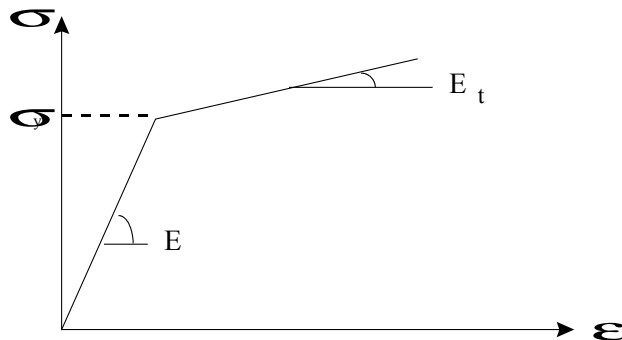
1.1 Geometry



Length of the bar : 1 m
Section of the bar : 5 cm²

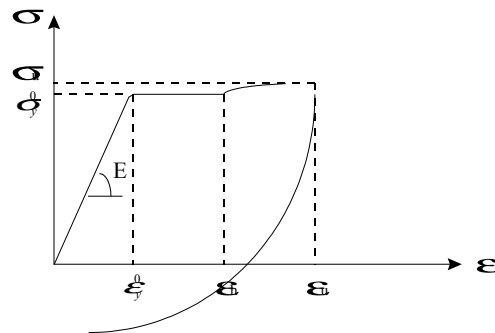
1.2 Properties of the materials

1.2.1 linear Hardenings isotropic and kinematics



Young Modulus: $E = 2 \cdot 10^{11} \text{ Pa}$
 Slope D `hardening: $E_t = 2 \cdot 10^9 \text{ Pa}$
 Elastic limit: $\sigma = 2 \cdot 10^8 \text{ Pa}$
 Poisson's ratio: $\nu = 0,3$
 Thermal coefficient of thermal expansion: $\alpha = 1 \cdot 10^{-5} \text{ K}^{-1}$

1.2.2 Model of Pinto-Menegotto



Modulus Young:	E	=	$2 \cdot 10^{11} Pa$
Elastic limit:	σ_v^0	=	$2 \cdot 10^8 Pa$
Poisson's ratio:	ν	=	0,3
thermal Coefficient of thermal	α	=	$1 \cdot 10^{-5} K^{-1}$
Strain of hardening:	ϵ_h	=	$2 \cdot 3 \cdot 10^{-3}$
ultimate Stress:	σ_u	=	$2 \cdot 58 \cdot 10^8 Pa$
ultimate Strain:	ϵ_u	=	$3 \cdot 10^{-2}$
Coefficient defining the curve ζ :	R_0	=	20
Coefficient defining the curve ζ	A_1	=	18.5
Coefficient defining the curve ζ	A_2	=	0.15
Coefficient of buckling:	C	=	0.5
Coefficient of buckling:	A	=	0.008

1.3 Boundary conditions and loading

Boundary conditions:

The bar is clamped. Displacements are thus blocked in the three directions.

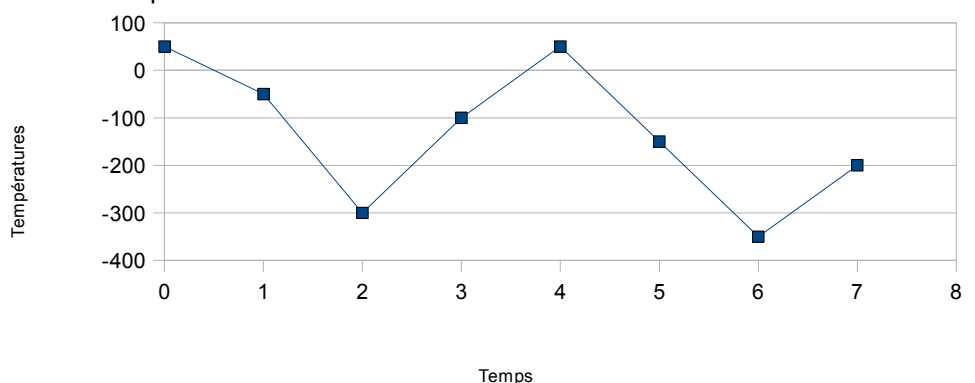
In $N1$ and $N2$: $DX = DY = DZ = 0$

Loading:

The way of loading is described by the change of the temperature, uniform in the bar:

t	0	1	2	3	4	5	6	7
$T(^{\circ}C)$	50	-50	-300	-100	50	-150	-350	-200

the reference temperature is $0^{\circ}C$.



2 Reference solutions

2.1 Method of calculating used for the reference solutions

2.1.1 linear Hardenings

isotropic Hardening

For a uniaxial tension, the plasticity criterion is written:

$$|\sigma_L| - R(p) \leq 0$$

where p is the cumulated plastic strain

$$R(p) = R' p + \sigma^y \quad \text{and} \quad R' = \frac{E E_1}{E - E_1}$$

the criterion is written then:

$$|\sigma_L| - R' p - \sigma^y \leq 0$$

The tensor of the stresses is obtained by:

$$\sigma = \mathbf{A} \cdot (\varepsilon(\mathbf{u}) - \varepsilon^p) - 3K\alpha(T - T^{ref}) \mathbf{Id}$$

One from of thus deduced the statement from σ_L

$$\sigma_L = E(\varepsilon - \alpha T) - E \varepsilon^p \quad (T^{ref} = 0)$$

In our case, $\varepsilon = 0$ therefore:

$$\sigma_L = E \varepsilon_L - E \varepsilon^p \quad \text{with} \quad \varepsilon_L = -\alpha T$$

Thus:

- If $|\sigma_L| - R(p) < 0$:
 $p=0$ and $\sigma_L = E \varepsilon_L$

- If $|\sigma_L| - R(p) = 0$:

$$p = \left(\frac{|\sigma_L| - \sigma^y}{R'} \right)$$

$$\sigma_L = E \varepsilon_L - E \varepsilon^p$$

Application to the way of Urgent

loading 1:

$\sigma = E \varepsilon = 200\text{MPa}$ and $R(p) = R' p + \sigma^y = 100\text{MPa}$ because $p=0$.

One has well $\sigma_L - R(p) \leq 0$.

The criterion is not crossed, the evolution is elastic: $\sigma_L = 100\text{MPa}$ and $N = 100\text{kN}$

Time 2:

The criterion is reached:

$$\sigma_L = \frac{E}{E + R'} (R' \varepsilon_L + \sigma^y) = \frac{2 \cdot 10^{11}}{2 \cdot 10^{11} + 2.02 \cdot 10} (2.02 \cdot 10 \times 3.5 \cdot 10^{-3} + 2 \cdot 10) = 205\text{MPa}$$

$N = 102.5\text{kN}$

and $p = 2.475 \cdot 10^{-3}$

Time 3:

One discharges elastically:

$$\sigma_L = E \varepsilon_L - E \varepsilon^p = 2 \cdot 10^{11} (1.5 \cdot 10^{-3} - 2.475 \cdot 10^{-3}) = -195 \text{ MPa}$$
$$N = -97.5 \text{ kN}$$

Time 4:

One plasticizes again:

The criterion is written: $|\sigma| - R' p - \sigma^y = 0$ with $p = p_1 + p_2$ where $p_1 = 2.475 \cdot 10^{-3}$

One thus obtains:

$$p_2 = \frac{|\sigma| - \sigma^y}{R'} - p_1$$
$$\sigma = -E \varepsilon^p = -E (p_1 - p_2)$$
$$\sigma = \left(\frac{R'}{R' + E} \right) \left(-2 E p_1 - \frac{E \sigma^y}{R'} \right) = -207.9 \text{ MPa}$$

And thus $N = -103.95 \text{ kN}$

Urgent 5:

One discharges elastically:

$$\sigma_L = E \varepsilon_L - E \varepsilon^p = 2 \cdot 10^{11} (2 \cdot 10^{-3} - 1.0395 \cdot 10^{-3}) = 192.1 \text{ MPa}$$
$$N = 96.05 \text{ kN}$$

Times 6 and 7:

The reasoning is identical

One finds:

$$N_{(inst.6)} = 105.87 \text{ kN}$$
$$N_{(inst.7)} = -44.13 \text{ kN}$$

Kinematic hardening

the method of calculating is identical, but in this case, the plasticity criterion is written:

$$\sigma - X_{eq} - \sigma^y \leq 0 \text{ with } X_{eq} = C (\varepsilon^p)_{eq} = \frac{3}{2} C \varepsilon^p = \frac{E E_t}{E - E_t} \varepsilon^p$$

With the preceding notations, the criterion is written:

$$|\sigma_L - R' \varepsilon^p| - \sigma^y \leq 0 \text{ from where } \sigma_L = R' \varepsilon^p \pm \sigma^y \text{ (according to the meaning of flow).}$$

Application to the way of Urgent loading 1:

The criterion is not crossed, the evolution is elastic: $\sigma_L = 100 \text{ MPa}$ and $N = 100 \text{ kN}$

Time 2:

The criterion is reached: $|\sigma_L - R' \varepsilon^p| - \sigma^y = 0$

$$\sigma_L = R' \varepsilon^p + \sigma^y = 2.02 \cdot 10^8 \times 2.475 \cdot 10^{-3} + 2 \cdot 10^8 = 205 \text{ MPa}$$

Time 3:

One discharges elastically:

$$\sigma_L = E \varepsilon_L - E \varepsilon^p = 2 \cdot 10^{11} (1.5 \cdot 10^{-3} - 2.475 \cdot 10^{-3}) = -195 \text{ MPa}$$
$$N = -97.5 \text{ kN}$$

Time 4:

$$|\sigma - R' \varepsilon^p| - \sigma^y = 0 \quad \text{with } p_1 = 2.475 \cdot 10^{-3}$$

$$\varepsilon^p = p_1 - p_2$$

$$p_2 = p_1 - \frac{|\sigma + \sigma^y|}{R'}$$

$$\sigma = -E \varepsilon^p = -E(p_1 - p_2)$$

$$\sigma = -E \left(\frac{|\sigma + \sigma^y|}{R'} \right) = -198 \text{MPa}$$

$$N = -99 \text{kN}$$

Time 5:

One discharges elastically:

$$\sigma_L = E \varepsilon_L - E \varepsilon^p = 2 \cdot 10^{11} (2 \cdot 10^{-3} - 9.9 \cdot 10^{-4}) = 202 \text{MPa}$$

$$N = 101 \text{kN}$$

Times 6 and 7:

The reasoning is identical. One finds:

$$N_{(inst.6)} = 103 \text{kN}$$

$$N_{(inst.7)} = -47 \text{kN}$$

2.1.2 Model of Pinto-Menegotto

This model is described in the Handbook of Reference of *Code_Aster* [R5.03.09] [bib1]. The constitutive law of steels is made up of two distinct parts: the monotonic loading made up of three successive zones (linear elasticity, plastic bearing and hardening) and the cyclic loading where the way between two points of inversion (semi-cycle) is described by a curve of analytical statement of the type $\sigma = f(\varepsilon)$.

As previously the imposed strains are thermal strains: $\varepsilon = -\alpha T$

2.1.2.1 Case without buckling

First loading

- Linear elasticity: $\sigma = E \varepsilon$

Time 1:

$$N = E \varepsilon S = 2 \cdot 10^{11} \times 1 \cdot 10^{-3} \times 5 \cdot 10^{-4} = 100 \text{kN}$$

- Plastic bearing: $\sigma = \sigma^y$
- Polynomial of degree 4: $\sigma = \sigma_u - (\sigma_{su} - \sigma_y^0) \left(\frac{\varepsilon_u - \varepsilon}{\varepsilon_u - \varepsilon_h} \right)^4$

Time 2:

$$\varepsilon = 3.5 \cdot 10^{-3} > \varepsilon_h = 2.3 \cdot 10^{-3}, \text{ one uses the polynomial of degree 4:}$$

$$\sigma = 209.416 \text{MPa and } N = 104.708 \text{kN}$$

Cycles

Semi-cycle 1:

One determines ζ_p^0 :

$$\zeta_p^0 = \varepsilon_r^0 - \varepsilon_y^0 = 3.5 \cdot 10^{-3} - 1.10 \cdot 10^{-3} = 2.5 \cdot 10^{-3} \text{ because } \varepsilon_r^0 = \varepsilon_{(inst.2)}$$

Then $\Delta \sigma^0$:

$$\Delta \sigma^0 = E_h \zeta_p^0 = 2 \cdot 10^9 \times 2.5 \cdot 10^{-3} = 5 \text{ MPa}$$

From where $\sigma_y^1 = \sigma_y^0 \cdot \text{sign}(-\zeta_p^0) + \Delta \sigma^0 = -200 + 5 = -195 \text{ MPa}$

One calculates then ε_y^1 :

$$\varepsilon_y^1 = \varepsilon_r^0 + \frac{\sigma_y^1 - \sigma_r^0}{E} = 3.5 \cdot 10^{-3} + \frac{(-195 - 209.416) \cdot 10}{2.0 \cdot 10^{11}} = 1.477 \cdot 10^{-3}$$

One determines thus $\sigma^* = f(\varepsilon^*)$, definite by:

$$\sigma^* = b \varepsilon^* + \left(\frac{1-b}{(1+(\varepsilon^*)^R)^{(1/R)}} \right) \varepsilon^* \quad , \text{ with } b = \frac{E_h}{E}$$

$$\varepsilon^* = \frac{\varepsilon - \varepsilon_r^0}{\varepsilon_y^1 - \varepsilon_r^0}$$

$$\sigma^* = \frac{\sigma - \sigma_r^0}{\sigma_y^1 - \sigma_r^0}$$

$$\xi_p^0 = \frac{\zeta_p^0}{|\varepsilon_y^1 - \varepsilon_r^0|} \quad \text{and} \quad R^l = R_0 - \frac{A_1 \cdot \xi_p^0}{A_2 + \xi_p^0}$$

One obtains $\xi_p^0 = -1.23$ and $R^l = 3.51$

One can then calculate the value of σ at times 3 and 4:

Time 3:

$$\varepsilon^* = \frac{\varepsilon_{(inst.3)} - \varepsilon_r^0}{\varepsilon_y^1 - \varepsilon_r^0} = \frac{1.5 \cdot 10^{-3} - 3.5 \cdot 10^{-3}}{1.477 \cdot 10^{-3} - 3.5 \cdot 10^{-3}} = 0.988$$

$$\sigma^* = b \varepsilon^* + \left(\frac{1-b}{(1+(\varepsilon^*)^R)^{(1/R)}} \right) \varepsilon^* = 0.01 \times 0.988 + \left(\frac{1-0.01}{(1+(0.988)^{3.51})^{(1/3.51)}} \right) = 0.82$$

and $\sigma = \sigma^* (\sigma_y^1 - \sigma_r^0) + \sigma_r^0 = 0.82 \times (-195 - 209.416) + 209.416 = -122 \text{ MPa}$

from where $N = -61 \text{ kN}$

Time 4:

One uses the same method, with $\varepsilon = 0$.

$$\varepsilon^* = 1.73$$

$$\sigma^* = 0.56$$

$$\sigma = -20 \text{ MPa}$$

$$N = -10 \text{ kN}$$

Semi-cycle 2: Time 5 and 6:

The method of calculating is identical, one determines:

$$\zeta_p^1 \quad \sigma_y^2 \quad \varepsilon_y^2 \quad \zeta_p^1 \quad R^2, \text{ then } \sigma_{(inst.5)}^* = f(\varepsilon_{(inst.5)}^*) \text{ and } \sigma_{(inst.6)}^* = f(\varepsilon_{(inst.6)}^*)$$

finally $\sigma_{(inst.5)}$ and $\sigma_{(inst.6)}$.

Semi-cycle 3: Time 7 : Idem

2.1.2.2 Case with buckling

First loading

Identical to the preceding case.

Cycles

Semi-cycle 1 (compression):

The method of calculating is identical, but the value of the slope of the asymptote is modified:

A new coefficient is calculated b_c :

$$b_c = a(5.0 - L/D) e \left(b \xi' \frac{E}{\sigma^y - \sigma^\infty} \right) = 0.006 \times (5.0 - 5.9) e^{(0.01 \times 1.477 \cdot 10^{-3} \frac{2 \cdot 10^{11}}{2 \cdot 10^8 - 1.36 \cdot 10^8})} = -0.0057$$

It is necessary then, as in the model without buckling, to determine σ_y^n . The reasoning is identical, but one adds a complementary stress σ_s^* in order to correctly position the curve compared to the asymptote.

$$\sigma_s^* = \gamma_s b E \frac{b - b_c}{1 - b_c} = 0.028 \times 0.01 \times 2 \cdot 10^{11} \times \frac{0.01 + 0.0057}{1 + 0.0057} = 0.87 \text{ MPa}$$

where γ_s is given by:
$$\gamma_s = \frac{11.0 - L/D}{10(e^{c(L/D)} - 1.0)} = 0.028$$

Semi-cycle 2 (tension):

- In tension, a reduced Young modulus is adopted:

$$E_r = E \left(a_5 + (1.0 - a_5) e^{-a_6 \xi^{n_2}} \right) = 2 \cdot 10^{11} \times (0.88 + (1 - 0.88) e^{(-620 \times 1.473 \cdot 10^{-6})}) = 1.99 \cdot 10^{11} \text{ MPa}$$

with $a_5 = 1.0 + (5.0 - L/D)/7.5 = 0.88$

the rest of the method is identical.

2.2 Results of reference

constant normal N Force on the bar

2.3 Uncertainty on the solution

No, the solution is analytical

2.4 bibliographical References

[Manual of reference of *the Code_Aster* [R5.03.09].

[2] S. ANDRIEUX: Thermoelastoplastic TD 1 Three bars perfect Von Mises. In "Initiation with thermoplasticity in *the Code_Aster*", HI-74/96/November 13th, 1996 (manual of reference of the course).

3 Modelization A

3.1 Characteristic of the modelization

The model is made up D `an element of bar (BAR).

Constitutive law: elastoplasticity with linear isotropic hardening - Criterion of Von Mises

3.2 Characteristics of the mesh

2 nodes.

1 mesh SEG2

3.3 Quantities tested and results

Identification	Times	Reference	Variation (%)
normal force N1		1.0000E+05	0
normal force N2		1.0250E+05	0
normal force N	3	- 9.7500E+04	0
normal force N4		- 1.0395E+05	0
normal force N	5	9.6050E+04	0
normal force N	6	1.0587E+05	0
normal force N	7	- 4.4129E+04	0

4 Modelization B

4.1 Characteristic of the modelization

The model is made up D `an element of bar (BAR).

Constitutive law: elastoplasticity with linear kinematic hardening - Criterion of Von Mises

4.2 Characteristics of the mesh

2 nodes.

1 mesh SEG2

4.3 Quantities tested and results

Identification	Times	Reference	Variation (%)
normal force N1		1.0000E+05	0
normal force N2		1.0250E+05	0
normal force N	3	- 9.7500E+04	0
normal force N4		- 9.9000E+04	0
normal force N	5	1.0100E+05	0
normal force N	6	1.0300E+05	0
normal force N	7	- 4.7000E+04	0

5 Modelization C

5.1 Characteristic of the modelization

The model is made up D `an element of bar (BAR).
Constitutive law: model of Pinto-Menegotto without buckling (value of ELAN lower than 5).

5.2 Characteristics of the mesh

2 nodes.
1 mesh SEG2

5.3 Quantities tested and results

Identification	Times	Reference	Variation (%)
normal force N1		1.0000E+05	0
normal force N2		1.0470E+05	0
normal force N	3	- 6.0777E+04	0
normal force N4		- 9.1430E+04	0
normal force N	5	7.6082E+04	0
normal force N	6	1.0125E+05	0
normal force N	7	- 3.7965E+04	0

6 Modelization D

6.1 Characteristic of the modelization

The model is made up of 1 element of bar (BAR).

Constitutive law: model of Pinto-Menegotto with buckling (value of ELAN higher than 5).

6.2 Characteristics of the mesh

2 nodes.

1 mesh SEG2

6.3 Quantities tested and results

Identification	Times	Reference	Variation (%)
normal force N1		1.0000E+05	0
normal force N2		1.0470E+05	0
normal force N	3	- 6.0556E+04	0
normal force N4		- 8.9078E+05	0
normal force N	5	7.6905E+05	0
normal force N	6	1.0125E+05	0
normal force N	7	- 3.8119E+04	0

7 Summary of the results

the results calculated by *Code_Aster* are in excellent agreement with the analytical solutions.