SSNL106 - Beam elastoplastic in tension and pure bending

Summarized:

This test validates elastoplasticity in a straight beam in tension and bending, for a perfect elastoplastic behavior or with linear hardening as well as options for the postprocessing of the beams.

- Static analysis,
- Behavior elastic,
- Behavior elastoplastic,
- 3 sections: rectangular, circular full, hollow circular,
- 1 types of hardening: linear (VMIS_ISOT_LINE)

7 modelizations make it possible to test elements POU_D_TGM, POU_D_EM, TUYAU_3M (3 and 4 nodes) COQUE_3D.

The test makes it possible to validate the operation of the integration of these constitutive laws and the algorithm of resolution until complete plasticization of the beam.

An eighth modelization makes it possible to test the operation of DYNA_NON_LINE on a static computation quasi - of tension of a beam modelled in POU_D_TGM.

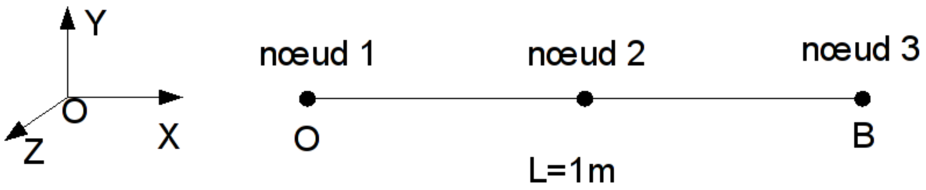
Moreover, the modelization G is used to factor key word test the operation of the ETAT_INIT of STAT_NON_LINE with the structure STRX_ELGA field .

1 Problem of reference

1.1 Geometry

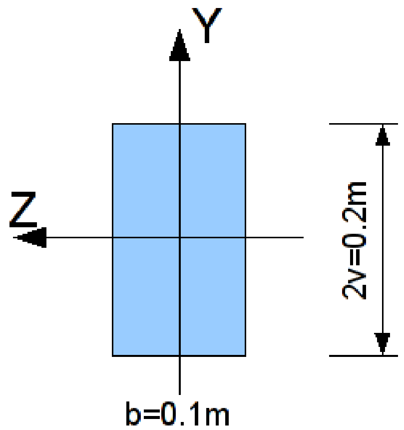
the geometrical values are expressed in meters.

Straight beam length $L=1$, direction x .

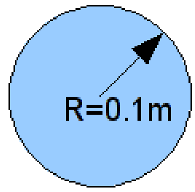


One calculates simultaneously 2 types of section:

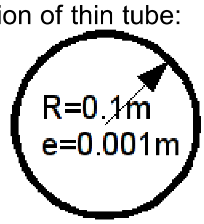
Rectangular section



circular Section



For the modelization D, one calculates 1 section of thin tube:



1.2 Material properties

$E=2.10^{11}\,Pa$

$\nu=0.3$

ECRO_LINE :

$SY=\sigma_y=150\,10^6\,Pa$

H = D_SIGM_EPSI = $2\,10^9\,Pa$ or 0

1.3 Boundary conditions and loadings

Fixed support in O
imposed Displacement B

$$DX^e = \frac{L.\sigma_y}{E} = 0.7510^{-3}\,m$$

DX varies DX^e the rotating one $3\mathrm{DX}^e$

imposed B

$$DRZ^e = 0.7510^{-2}\,m$$

DRZ varies some from DRZ^e with $20 \times DRZ^e$ then decrease until $-2 \times DRZ^e$

Note::

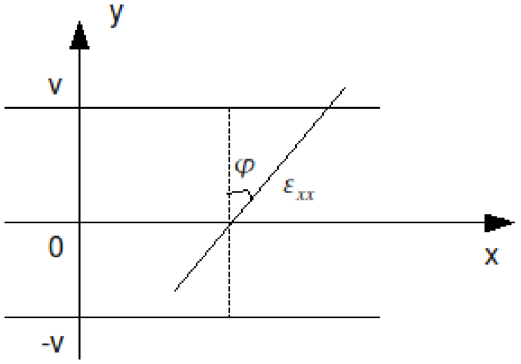
In pure bending, MZ and DRZ do not depend on x . The curvature $\varphi = \frac{d(DRZ)}{dx} = DRZ(B)$

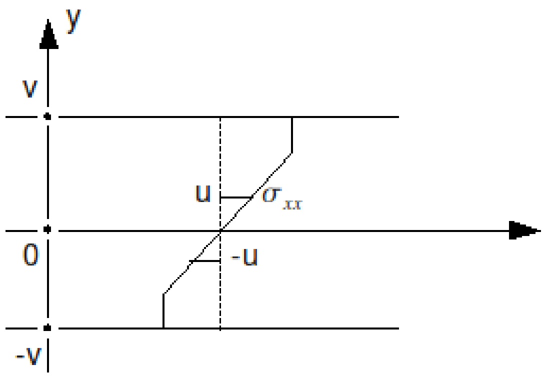
2 Reference solution

2.1 Method of calculating used for the reference solution

2.1.1 Pure bending - Hardening linear

analytical Solution:





$\varepsilon_{xx}=\varphi \quad y \qquad \varphi$: curvature

Computation of the moment by:

$$M(u)=\int_s \sigma_{xx}(y).\,y\ ds$$

$\sigma_{xx}=E\,\varepsilon_{xx}$ for $0\leq|y|\leq u$

$$\sigma_{xx}=\sigma_y+H\left(\varepsilon_{xx}-\frac{\sigma_y}{E}\right)$$

$u<|y|\leq v$

One obtains:

for the rectangular section:

$$\frac{M}{M_e}=\left(1-\frac{H}{E}\right)\left(\frac{3}{2}-\frac{1}{2}\left(\frac{\varphi_e}{\varphi}\right)^2\right)+\frac{H\,\varphi}{E}/\varphi \quad \text{with} \quad \varphi_e=\frac{M_e}{EI}\ M_e=\frac{I_z\cdot\sigma^y}{v}$$

for the circular section:

$$M(\mu)=\frac{R^3\sigma^y}{E}\left[\frac{\pi}{4}\frac{H}{\mu}+\frac{4}{3}(E-H)\left(1-\mu^2\right)^{3/2}+\frac{E-H}{2\mu}\left(Arc\sin\mu-\mu\left(1-2\mu^2\right)\sqrt{1-\mu^2}\right)\right]$$

with $\mu=\frac{u}{R}=\frac{\sigma_y}{ER\,\varphi}=\frac{\varphi_e}{\varphi}$

In discharge, after having reached the Yield-point load in load, one obtains a contrary Yield-point load of sign.

for the tubular section:
(assumption of beam of Navier-Bernoulli)
the Yield-point load ($H=0$) is worth:

$$\frac{M}{M_e}=\frac{4}{\pi}$$

The total solution for a thin tube is [bib1]:

$$\frac{M(\mu)}{M_e}=\frac{\lambda}{\mu}+\frac{2(1-\lambda)}{\pi\mu}\left(arc\sin\mu+\mu\sqrt{1-\mu^2}\right) \text{ with } \lambda=\frac{E_T}{E}=\frac{H}{E+H}$$

2.1.2 Tension - Hardening linear

analytical Solution: one has immediately $N=S\sigma_y\left(1-\frac{H}{E}\right)+\frac{HS}{L}.DX$.

2.1.3 Pure tension - analytical

Elasticity Solution:

$$N=\frac{E.S.\delta u}{L} \quad \sigma=\frac{E.\delta u}{L} \text{, with } \delta u=7.5\text{E}-03$$

2.1.4 Pure bending - analytical

Elasticity Solution:

$$Mfz=\frac{3.0\,\delta u\,E\,I}{L^2} \quad \sigma=\frac{M_{fz}.h}{I} \text{, with } \delta u=7.5\text{E}-03$$

$h=R$ for the circular section, $h=v$ the rectangular section.

2.2 Bibliographical references

1.J.H. LAU and T.T. LAU: Newspaper of Pressure Vessel Technology vol. 106 p188-195 - May 1984.

3 Modelization A

3.1 Characteristic of the modelization

2 elements POU_D_TGM by type of section. There are thus 2 groups of elements comprising each one 2 elements.

Group $GR1$:	rectangular section
$GC1$:	circular section
simple Tension with hardening	on $GR1$ and $GC1$ (ECRO_LINE)
Pure bending without hardening	on $GR1$ and $GC1$ (ECRO_LINE)
Elasticity	on $GR1$ and $GC1$

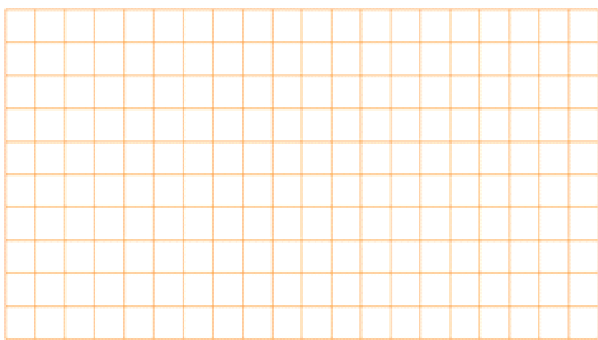
3.2 Characteristics of the mesh

- Mesh of the beam

  2×2 elements POU_D_TGM

- Mesh of the sections

(111 nodes, 188 TRIA3)



231 nodes, 200 QUAD4

3.3 Quantities tested and simple

- Tension results (with $DX^e = 0.75\text{E}-03$)

$DX_{(B)}$	Standard	N°ordre	GROUP_MA	NOEUD		Identification of Reference	Reference	Tolerance
$2\,DX^e$	N	11	GR1	R3	EFGE_ELNO	ANALYTIQUE	$3.\text{E}+06$	$0.10\,\%$
$3\,DX^e$	N	21	GR1	R3	EFGE_ELNO	ANALYTIQUE	$3.\text{E}+06$	$0.10\,\%$
$2\,DX^e$	N	11	GC1	C3	EFGE_ELNO	ANALYTIQUE	$4.82\text{E}+06$	$2.5\,\%$
$3\,DX^e$	N	21	GC1	C3	EFGE_ELNO	ANALYTIQUE	$4.87\text{E}+06$	$2.5\,\%$
$3\,DX^e$	21	GC1	C3	DEGE_ELNO	EPXX	ANALYTIQUE	$2.25\text{E}-03$	$0.10\,\%$

$DX_{(B)}$	N°ordre	DO NOT NET		Stand ard	Subpoint	Identification of Reference	Reference	Tolerance
$3\,DX^e$	21	SR1	1	1	VARI_ELGA V1	ANALYTIQUE	$1.5\mathrm{E}{-03}$	0.1%
$3\,DX^e$	21	SC1	1	1	VARI_ELGA V1	ANALYTIQUE	$1.5\mathrm{E}{-03}$	1.5%

$DX_{(B)}$	Standard	N°ordre	GROUP_MA	NOEUD		Identification of Reference	Reference	Tolerance
$2\,DX^e$	11	GR1	R3	REAC_NODA	DX	ANALYTIQUE	$3.\mathrm{E}{+06}$	0.10%
$3\,DX^e$	21	GR1	R3	REAC_NODA	DX	ANALYTIQUE	$3.\mathrm{E}{+06}$	0.10%
$2\,DX^e$	11	GC1	C3	REAC_NODA	DX	ANALYTIQUE	$4.82\mathrm{E}{+06}$	2.5%
$3\,DX^e$	21	GC1	C3	REAC_NODA	DX	ANALYTIQUE	$4.87\mathrm{E}{+06}$	2.5%

- Pure bending (with $DRZ^e = 0.75\mathrm{E}{-02}$)

$DRZ_{(B)}$	Standard	N°ordre	GROUP_MA	NOEUD	Identification of Reference	Reference	Tolerance
DRZ^e	1	GR1	R3	MFZ (Nm)	ANALYTIQUE	$1.0\mathrm{E}{+05}$	0.5%
$5\,DRZ^e$	21	GR1	R3	MFZ	ANALYTIQUE	$1.48\mathrm{E}{+05}$	1.0%
$10\,DRZ^e$	31	GR1	R3	MFZ	ANALYTIQUE	$1.495\mathrm{E}{+05}$	1.0%
$20\,DRZ^e$	41	GR1	R3	MFZ	ANALYTIQUE	$1.499\mathrm{E}{+05}$	1.0%
$-2.\,DRZ^e$	71	GR1	R3	MFZ	ANALYTIQUE	$-1.5\mathrm{E}{+05}$	1.0%
$10\,DRZ^e$	31	GR1	R3	KZ	ANALYTIQUE	0.075	0.1%
DRZ^e	1	GC1	C3	MFZ	ANALYTIQUE	$1.178\mathrm{E}{+05}$	2.5%
$5\,DRZ^e$	21	GC1	C3	MFZ	ANALYTIQUE	$1.96\mathrm{E}{+05}$	2.0%
$10\,DRZ^e$	31	GC1	C3	MFZ	ANALYTIQUE	$1.99\mathrm{E}{+05}$	1.5%
$20\,DRZ^e$	41	GC1	C3	MFZ	ANALYTIQUE	$1.998\mathrm{E}{+05}$	1.5%
$-2.\,DRZ^e$	71	GC1	C3	MFZ	ANALYTIQUE	$-2.0\mathrm{E}{+05}$	2.0%

$DX_{(B)}$	Standard	N°ordre	GROUP_MA	NOEUD		Identification of Reference	Reference	Tolerance
DRZ^e	1	GR1	R3	REAC_NODA	DRZ	ANALYTIQUE	$1.0\text{E}+05$	0.2%
$5\,DRZ^e$	21	GR1	R3	REAC_NODA	DRZ	ANALYTIQUE	$1.48\text{E}+05$	1.0%
$10\,DRZ^e$	31	GR1	R3	REAC_NODA	DRZ	ANALYTIQUE	$1.495\text{E}+05$	1.0%
$20\,DRZ^e$	41	GR1	R3	REAC_NODA	DRZ	ANALYTIQUE	$1.499\text{E}+06$	1.0%
DRZ^e	1	GC1	C3	REAC_NODA	DRZ	ANALYTIQUE	$1.178\text{E}+05$	2.5%
$5\,DRZ^e$	21	GC1	C3	REAC_NODA	DRZ	ANALYTIQUE	$1.96\text{E}+05$	2.0%
$10\,DRZ^e$	31	GC1	C3	REAC_NODA	DRZ	ANALYTIQUE	$1.99\text{E}+05$	1.5%
$20\,DRZ^e$	41	GC1	C3	REAC_NODA	DRZ	ANALYTIQUE	$1.998\text{E}+06$	1.5%

4 Modelization C

4.1 Characteristic of the modelization

2 elements POU_D_EM by type of section. There are thus 2 groups of elements comprising each one 2 elements.

Group	$GR1$:	rectangular section
	$GC1$:	circular section
simple Tension with hardening		on $GR1$ and $GC1$ (ECRO_LINE)
Pure bending without hardening		on $GR1$ and $GC1$ (ECRO_LINE)
Elasticity		on $GR1$ and $GC1$

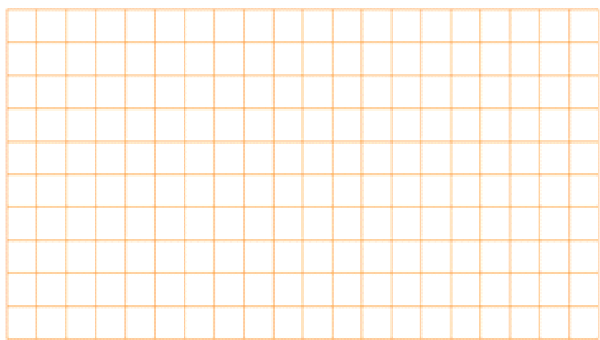
4.2 Characteristics of the mesh

- Mesh of the beam

  2×2 elements POU_D_EM

- Mesh of the sections

(111 nodes, 188 TRIA3)



231 nodes, 200 QUAD4

4.3 Quantities tested and simple

- Tension results (with $DX^e=0.75\mathrm{E}-03$)

$DX_{(B)}$	Standard	N°ordre	GROUP_MA	NOEUD		Identification of Reference	Reference	Tolerance
$2\,DX^e$	N	11	GR1	R3	EFGE_ELNO	ANALYTIQUE	$3.\mathrm{E}+06$	$0.10\,\%$
$3\,DX^e$	N	21	GR1	R3	EFGE_ELNO	ANALYTIQUE	$3.\mathrm{E}+06$	$0.5\,\%$
$2\,DX^e$	N	11	GC1	C3	EFGE_ELNO	ANALYTIQUE	$4.82\mathrm{E}+06$	$2.5\,\%$
$3\,DX^e$	N	21	GC1	C3	EFGE_ELNO	ANALYTIQUE	$4.87\mathrm{E}+06$	$2.5\,\%$
$3\,DX^e$	21	GC1	C3	DEGE_ELNO	EPXX	ANALYTIQUE	$2.25\mathrm{E}-03$	$0.10\,\%$

$DX_{(B)}$	N°ordre	Standard	GROUP_MA	NOEUD		Identification of Reference	Reference	Tolerance
$2\,DX^{e}$	11	GR1	R3	REAC_NODA	DX	ANALYTIQUE	$3.E+06$	$0.10\,\%$
$3\,DX^{e}$	21	GR1	R3	REAC_NODA	DX	ANALYTIQUE	$3.E+06$	$0.50\,\%$
$2\,DX^{e}$	11	GC1	C3	REAC_NODA	DX	ANALYTIQUE	$4.82E+06$	$2.5\,\%$
$3\,DX^{e}$	21	GC1	C3	REAC_NODA	DX	ANALYTIQUE	$4.87E+06$	$2.5\,\%$

- Pure bending (with $DRZ^{e}=0.75E-02$)

$DRZ_{(B)}$	Standard	N°ordre	GROUP_MA	NOEUD	Identification of Reference	Reference	Tolerance
DRZ^{e}	1	GR1	R3	MFZ (Nm)	ANALYTIQUE	$1.0E+05$	0.5%
$5\,DRZ^{e}$	21	GR1	R3	MFZ	ANALYTIQUE	$1.48E+05$	1.0%
$10\,DRZ^{e}$	31	GR1	R3	MFZ	ANALYTIQUE	$1.495E+05$	1.0%
$20\,DRZ^{e}$	41	GR1	R3	MFZ	ANALYTIQUE	$1.499E+05$	1.5%
$-2.\,DRZ^{e}$	71	GR1	R3	MFZ	ANALYTIQUE	$-1.5E+05$	$2.0\,\%$
$10\,DRZ^{e}$	31	GR1	R3	KZ	ANALYTIQUE	0.075	$0.1\,\%$
DRZ^{e}	1	GC1	C3	MFZ	ANALYTIQUE	$1.178E+05$	$2.5\,\%$
$5\,DRZ^{e}$	21	GC1	C3	MFZ	ANALYTIQUE	$1.96E+05$	1.5%
$10\,DRZ^{e}$	31	GC1	C3	MFZ	ANALYTIQUE	$1.99E+05$	1.0%
$20\,DRZ^{e}$	41	GC1	C3	MFZ	ANALYTIQUE	$1.998E+05$	$2.0\,\%$
$-2.\,DRZ^{e}$	71	GC1	C3	MFZ	ANALYTIQUE	$-2.0E+05$	$2.0\,\%$

$DX_{(B)}$	Standard	N°ordre	GROUP_MA	NOEUD		Identification of Reference	Reference	Tolerance
DRZ^{e}	1	GR1	R3	REAC_NODA	DRZ	ANALYTIQUE	$1.0E+05$	0.5%
$5\,DRZ^{e}$	21	GR1	R3	REAC_NODA	DRZ	ANALYTIQUE	$1.48E+05$	1.0%
$10\,DRZ^{e}$	31	GR1	R3	REAC_NODA	DRZ	ANALYTIQUE	$1.495E+05$	1.0%
$20\,DRZ^{\epsilon}$	41	GR1	R3	REAC_NODA	DRZ	ANALYTIQUE	$1.499E+06$	1.5%
DRZ^{e}	1	GC1	C3	REAC_NODA	DRZ	ANALYTIQUE	$1.178E+05$	$2.5\,\%$
$5\,DRZ^{e}$	21	GC1	C3	REAC_NODA	DRZ	ANALYTIQUE	$1.96E+05$	1.5%
$10\,DRZ^{e}$	31	GC1	C3	REAC_NODA	DRZ	ANALYTIQUE	$1.99E+05$	1.0%
$20\,DRZ^{\epsilon}$	41	GC1	C3	REAC_NODA	DRZ	ANALYTIQUE	$1.998E+06$	1.0%

5 Modelization D

5.1 Characteristic of the modelization

2 elements PIPE for the tubular section.
Simple tension: (ECRO_LINE)
Pure bending: without hardening

Moreover, one blocks the degrees of freedom which correspond to the mode 3d' ovalization: $U03$ $V03$ $W03$.

5.2 Characteristics of the mesh

2 elements PIPE (METUSEG3)

5.3 Quantities tested and results

One tests the nodal reactions:

Identification	Standard	Node	Sequence number of reference	Value of reference	Tolerance
DX	B	11	ANALYTIQUE	9.47152E+4	0.10 %
DX	B	21	ANALYTIQUE	9.5653E+4	0.10 %
DRZ	B	1	ANALYTIQUE	4.64217E+3	0.10 %
DRZ	B	21	ANALYTIQUE	5.9106E+3	0.50 %

One tests respectively the strains, the equivalent strains and the equivalent stresses :

Quantity	Nets	Standard	Node	Sequence number of reference	Value of reference	Tolerance
$EPYY$	$SC1$	$C1$	1	NON_REGRESSION	−2.25E−4	0.01 %
$INVA_2$	$SC1$	$C1$	1	NON_REGRESSION	5.89491E−4	0.01 %
$VMIS$	$SC1$	$C1$	1	NON_REGRESSION	0.0	0.10 %

6 Modelization E

6.1 Characteristic of the modelization

2 elements PIPE to 4 nodes for the tubular section.
Simple tension: (ECRO_LINE)
Pure bending: without hardening
Moreover, one blocks the degrees of freedom which correspond to the mode 3d' ovalization: $U03$ $V03$, $W03$

6.2 Characteristic of the mesh

2 elements PIPE (meshes SEG4)

6.3 Quantities tested and results

One tests the nodal reactions:

Quantity	Standard	Sequence number	Node of reference	Value of reference	Tolerance
DX	11	B	ANALYTIQUE	$9.47152E+4$	0.10%
DX	21	B	ANALYTIQUE	$9.5653E+4$	0.10%
DRZ	1	B	ANALYTIQUE	$4.64217E+3$	0.10%
DRZ	21	B	ANALYTIQUE	$5.9106E+3$	0.50%

One tests respectively the strains, the equivalent strains and the equivalent stresses :

Quantity	Sequence number	Nets	Standard	Node of reference	Value of reference	Tolerance
$EPYY$	1	$M1$	$N1$	NON_REGRESSION	$-2.25E-4$	0.01%
$INVA\_2$	1	$M1$	$N1$	NON_REGRESSION	$5.89491E-4$	0.01%
$VMIS$	1	$M1$	$N1$	NON_REGRESSION	0.0	0.10%
KY	1	$M1$	$N1$	ANALYTIQUE	$-7.5E-3$	0.10%

Quantity	Sequence number	Does not net		Standard a reference	Value of reference	Tolerance
KY	1	$M1$	3	ANALYTIQUE	$-7.5E-3$	0.1%

7 Modelization F

7.1 Characteristic of the modelization

112 elements COQUE_3D for the tubular section, and 2 elements pipe to apply the boundary conditions. The length of the mesh shells is of $0.98\mathrm{m}$. The length of each element pipe is of $0.01\mathrm{m}$.

A connection COQUE_TUYAU is applied at each end of the mesh shells, with an element pipe. Moreover, one blocks the degrees of freedom of the pipes which correspond to the mode 3d' ovalization: $U03$ $V03$, $W03$

simple Tension: (ECRO_LINE)
Pure bending: without hardening

7.2 Characteristics of the mesh

112 meshes QUAD9 and 2 meshes SEG3.

7.3 Quantities tested and Tension

results:

$DX_{(B)}$	Standard	N°ordre	Identification of Reference	Reference	Tolerance
$2\,DX^e$	11	N	ANALYTIQUE	$9.47E+04$	1.10%
$3\,DX^e$	21	N	ANALYTIQUE	$9.565E+04$	1.10%

Bending:

$DRZ_{(B)}$	Standard	N°ordre	Identification of Reference	Reference	Tolerance
DRZ^e	1	MFZ	ANALYTIQUE	$4.642E+03$	0.10%
$2.8\,DRZ^e$	19	MFZ	ANALYTIQUE	$5.7824E+03$	0.50%

8 Modelization G

8.1 Characteristic of the modelization

2 elements POU_D_EM for the tubular section.
The section is with a grid in QUAD4: it is discretized by a mesh in the thickness, and 90 meshes on the circumference.

Simple tension: (ECRO_LINE)
Pure bending: without hardening

8.2 Characteristics of the mesh

2 meshes SEG2 for the beam. 90 meshes QUAD4 for the section.

8.3 Quantities tested and Tension

results:

$DX_{(B)}$	Standard	N°ordre	Identification of Reference	Reference	Tolerance
$2\,DX^e$	11	N	ANALYTIQUE	$9.47E+04$	$0.10\,\%$
$3\,DX^e$	21	N	ANALYTIQUE	$9.565E+04$	$0.10\,\%$

Bending:

$DRZ_{(B)}$	Standard	N°ordre	Identification of Reference	Reference	Tolerance
DRZ^e	1	MFZ	ANALYTIQUE	$4.642E+03$	$0.10\,\%$
$5\,DRZ^e$	21	MFZ	ANALYTIQUE	$5.9106E+03$	$0.50\,\%$

9 Modelization H

9.1 Characteristic of the modelization

2 elements POU_D_TGM for the tubular section.
The section is with a grid in QUAD4: it is discretized by a mesh in the thickness, and 90 meshes on the circumference.

Simple tension: (ECRO_LINE)
Pure bending: without hardening

9.2 Characteristics of the mesh

2 meshes SEG2 for the beam. 90 meshes QUAD4 for the section.

9.3 Quantities tested and Tension

results:

$DX_{(B)}$	Standard	N°ordre	Identification of Reference	Reference	Tolerance
$2\,DX^e$	11	N	ANALYTIQUE	$9.47E+04$	$0.10\,\%$
$3\,DX^e$	21	N	ANALYTIQUE	$9.565E+04$	$0.10\,\%$

Bending:

$DRZ_{(B)}$	N°ordre	Standard	Identification of Reference	Reference	Difference
DRZ^e	1	MFZ	ANALYTIQUE	$4.642E+03$	$0.10\,\%$
$5\,DRZ^e$	21	MFZ	ANALYTIQUE	$5.9106E+03$	$0.50\,\%$

10 Modelization I

10.1 Characteristic of the modelization

2 elements POU_D_TGM by type in section. There are thus 2 groups of elements comprising each one 2 elements.

Group	$GR1$:	rectangular section
	$GC1$:	circular section
simple Tension with hardening		on $GR1$ and $GC1$ (ECRO_LINE)
Pure bending without hardening		on $GR1$ and $GC1$ without hardening

10.2 Characteristics of the mesh

- Mesh of the beam

  2×2 elements POU_D_TGM

- Mesh of the sections

(111 nodes, 188 TRIA3)

231 nodes, 200 QUAD4

10.3 Notices

the characteristic of the modelization L is to test the operation of DYNA_NON_LINE in the quasi-static computation of tension of a beam modelled in POU_D_TGM. This kind of modelization has as a characteristic to reveal null pivots on the lines of the mass matrix corresponding to the degrees of freedom of warping. In this case, the initialization of the diagram of NEWMARK is not done any more by inversion of the mass matrix, which is singular, but by zero setting of initial acceleration.

10.4 Quantities tested and Values

10.4.1 results tested

- simple Tension (with $DX^e=0.75\mathrm{E}-03$)

$DX_{(B)}$	Standard	N°ordre	GROUP_MA	NOEUD		Identification of Reference	Reference	Tolerance
$2\,DX^e$	N	11	GR1	R3	EFGE_ELNO	ANALYTIQUE	$3.E+06$	0.10%
$3\,DX^e$	N	21	GR1	R3	EFGE_ELNO	ANALYTIQUE	$3.E+06$	0.10%
$2\,DX^e$	N	11	GC1	C3	EFGE_ELNO	ANALYTIQUE	$4.82E+06$	2.5%
$3\,DX^e$	N	21	GC1	C3	EFGE_ELNO	ANALYTIQUE	$4.87E+06$	2.5%
$3\,DX^e$	21	GC1	C3	DEGE_ELNO	EPXX	ANALYTIQUE	$2.25E-03$	0.10%

$DX_{(B)}$	N°ordre	DOES NOT NET		Standard	Subpoint	Identification of Reference	Reference	Tolerance
$3\,DX^e$	21	GR1	1	1	VARI_ELGA V1	ANALYTIQUE	$1.5E-03$	20.0%
$3\,DX^e$	21	GC1	1	1	VARI_ELGA V1	ANALYTIQUE	$1.5E-03$	1.5%

10.4.2 Observations

One notices that the results in tension resulting from DYNA_NON_LINE are identical to those given by STAT_NON_LINE.

11 Summary of the results

the modelizations out of multifibre beams provide a solution to less of 2.5% of the analytical solution, for a time very weak computation, in comparison with modelizations pipe and shell. The only approximation comes from the mesh of the section.

With regard to the modelizations pipe and shell, the conclusions are the same ones, but this time the difference with the analytical solution comes from this solution which is valid for a very thin beam of tubular section, without effect of ovalization. This ovalization is blocked at the ends, which makes it possible to obtain a solution with less 0.4% of the analytical solution.