# SSNA302 - Plate circular simply supported subjected to pressure

**Summarized:**

This test consists in applying a transverse pressure to a circular plate, simply supported and made up of an elastic material. It is intended particularly to study the taking into account of the geometric nonlinearities in the absence of initial curvature when the transverse stiffness is only due to the effect of plate.

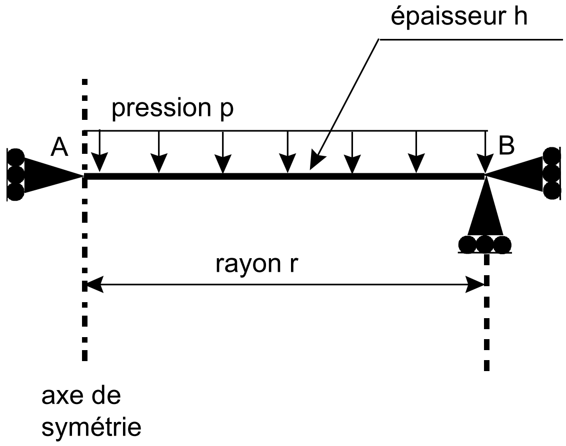
The modelization is made with voluminal elements of type HEXA20 and PENTA15 and surface elements of type QUAD8 and TRIA6 for the application of the pressure.

The reference is software the SAMCEF software. One has for information the results resulting from the theory of the thin shells.

Command `COMP_ELAS` with option `GREEN` is compared with `COMP_INCR` with the option `PETIT_REAC`.

# 1 Problem of reference

## 1.1 Geometry




## 1.2 Material properties

isotropic elastic Material:

$E = 200000\,Mpa$
$\nu = 0.3$

## 1.3 Boundary conditions and loadings

Point: $B$ $u_x = 0$
$u_y = 0.$

One applies a transverse pressure $p$ to the plate: $p = 222.72\,N/mm^2$ . This pressure corresponds to a deflection $w_0$ of $1.5\,mm$ .

Radius $r = 10\,mm$
Thickness $h = 1\,mm$

the problem is axisymmetric.

# 2 Reference solution

## 2.1 Reference solution

the reference is software the SAMCEF software. One has, for information, in the paragraph [§2.2], the theoretical results related to an assumption of type thin shell. Then, the results got with the SAMCEF software are presented according to whether one chooses an assumption of type thick shell or standard volume. It is the latter which is taken into account for the evaluating of *Code_Aster*.

## 2.2 Analytical solution and results of reference

the following formula gives the deflection $w_0$ to the center of the plate:

$$\frac{w_0}{h}+A\left(\frac{w_0}{h}\right)^3=\frac{Bp}{E}\left(\frac{r}{h}\right)^4$$ with $A=1.852$ and the $B=0.696$

forced with mid thickness are worth:

$$\sigma_{rr} = \alpha_r E\frac{w_0}{r^2}$$

$$\sigma_{\theta\theta} = \alpha_t E\frac{w_0}{r^2}$$

The stresses in lower skin are worth

$$\sigma_{rr}' = \beta_r E\frac{w_0 h}{r^2}$$

$$\sigma_{\theta\theta}' = \beta_t E\frac{w_0 h}{r^2}$$

the coefficients are worth:

In the center of the plate:

$$\alpha_r=\alpha_t=0.905$$
$$\beta_r=\beta_t=1.778$$

At the edge of the plate:

$$\alpha_r=0.610$$
$$\alpha_t=0.183$$
$$\beta_r=0$$
$$\beta_t=0.755$$

For a pressure of $222.72\,MPa$, the deflection $w_0$ is worth $1.5\,mm$ and the following stresses are obtained:

| Position | $\sigma_{rr}$ $(MPa)$ | $\sigma_{\theta\theta}$ $(MPa)$ | $\sigma_{rr}'$ $(MPa)$ | $\sigma_{\theta\theta}'$ $(MPa)$ |
|---|---|---|---|---|

| | | | | |
|---|---|---|---|---|
| Centers | 4072.5 | 4072.5 | 5334.0 | 5334.0 |

These results correspond to an assumption of thin shell.

The following table shows the results got by the SAMCEF software for a modelization of type thick shell and standard volume.

| Identification | thick Shell | Deflection |
|---|---|---|
| Volume $w0$ $(mm)$ | – 1.43041 E-3 | – 1.441838 E-3 |
| SIXX $(MPa)$ centers, mid thickness | 3899.88 | 3850.88 |
| SIYY $(MPa)$ centers, mid thickness | 3899.53 | 3850.91 |
| SIXX $(MPa)$ centers, skin inf | 8085.81 | 8133.60 |
| SIYY $(MPa)$ centers, skin inf | 8083.32 | 8133.65 |
| SIXX $(MPa)$ $r=R/2$ , mid thickness | 3596.91 | 3512.79 |
| SIYY $(MPa)$ $r=R/2$ , mid thickness | 3056.37 | 2947.55 |
| SIXX $(MPa)$ $r=R/2$ , skin inf | 7798.18 | 7815.73 |
| SIYY $(MPa)$ $r=R/2$ , skin inf | the 7264.69 | 7307.04 |

values of the stresses are values by element extrapolated with the nodes.

The results got with the SAMCEF software are close one to the other.

The differences of results between the mean theory shell and computation finite element using the assumption thick shell are important.

One chooses to take as reference the voluminal computation obtained by the SAMCEF software.

## 2.3 Bibliographical references

1) Theory of Plates and Shells, Timoshenko S.P., 2nd edition, p 412

# 3 Modelization A

## 3.1 Characteristic of the modelization A

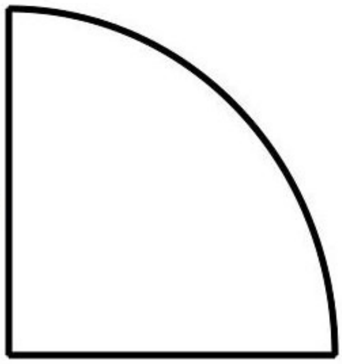



**Figure 3.1-3.1-a3.1-a**

One models only one quarter of plate. One introduces conditions of symmetry on the two sides shown above.

Moreover one fixes according to $x, y, z$ all the nodes of edge located at semi thickness.

In order to most accurately represent the assumptions of thin shell, one introduces linear stresses on the degrees of freedom of the nodes of edge. Those Ci are written:

for two nodes $i$ and $j$ located on both sides of the average average:

$$u_i+u_j=0$$
$$v_i+v_j=0$$

where U and v indicate displacements along the axes $x$ and $y$.

The syntax used in the data file of *Code_Aster* is the following one:

```
LIAISON_DDL (THE NODE IS OUTSIDE THE FIELD OF DEFINITION WITH A RIGHT PROFILE OF THE EXCLU TYPE NODE: Ni Nj ), DDL: ("DX", "DX") , COEF_MULT: (1, 1), COEF_IMPO: 0
```

One applies the pressure linearly by means of 6 increments.

## 3.2 Characteristics of the mesh

| | |
|---|---|
| Many nodes: | 2091 |
| Number of meshes and types: | 368 HEXA20, 28 PENTA15 |
| | 92 QUAD8, 7 TRIA6 |

## 3.3 Values tested

| Identification | Reference |
|---|---|

| | |
|---|---|
| Deflection $w0$ $(mm)$ | – 1.441838E-3 |
| SIXX $(MPa)$ centers, mid thickness | 3850.9 |
| SIYY $(MPa)$ centers, mid thickness | 3850.9 |
| SIXX $(MPa)$ centers, skin inf | 8133.6 |
| SIYY $(MPa)$ centers, skin inf | 8133.6 |
| SIXX $(MPa)$ $r=R/2$, mid thickness | 3512.8 |
| SIYY $(MPa)$ $r=R/2$, mid thickness | 2947.5 |
| SIXX $(MPa)$ $r=R/2$, skin inf | 7815.7 |
| SIYY $(MPa)$ $r=R/2$, skin inf | 7307.0 |

## 3.4 Remarks

One by means of obtains the same accuracy on the results only one increment of loading.

# 4 Modelization B

## 4.1 Characteristic of the modelization B

Idem modelization A, but by treating geometric nonlinearities by the command COMP_INCR option PETIT_REAC.

## 4.2 Characteristics of the mesh

| | |
|---|---|
| Many nodes: | 2091 |
| Number of meshes and types: | 368 HEXA20, 28 PENTA15 |
| | 92 QUAD8, 7 TRIA6 |

## 4.3 Values tested

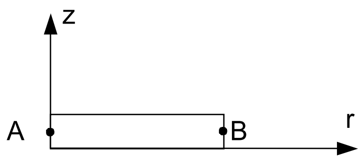
| Identification | Reference | Aster | % difference |
|---|---|---|---|
| | | | |
| Deflection $w0$ $(mm)$ | – 1.441838E-3 | – 1.3692E-3 | – 5.036 |
| SIXX $(MPa)$ center, mid thickness | 3850.9 | 4219.3 | 9.568 |
| SIYY $(MPa)$ centers, mid thickness | 3850.9 | 4219.4 | 9.569 |
| SIXX $(MPa)$ centers, skin inf | 8133.6 | 7950.5 | – 2.251 |
| SIYY $(MPa)$ center, skin inf | 8133.6 | 7950.5 | – 2.252 |
| SIXX $(MPa)$ $r=R/2$, mid thickness | 3512.8 | 3838.4 | 9.268 |
| SIYY $(MPa)$ $r=R/2$, mid thickness | 2947.5 | 3314.2 | 12.440 |
| SIXX $(MPa)$ $r=R/2$, skin inf | 7815.7 | 7593.9 | – 2.839 |
| SIYY $(MPa)$ $r=R/2$, skin inf | 7307.0 | 7123.7 | – 2.509 |

## 4.4 Remarks

One notes an important difference between the reference solution and the solution provided by *Code_Aster*.
It is checked that this variation tends towards 0 when one increases the number of increments.

# 5 Modelization C

## 5.1 Characteristic of the axisymmetric modelization




- C modelization
- boundary conditions:

$B$ : $DX=0$ $DY=0$

As for the modelization A (3D), of the linear stresses are introduced for better representing a kinematics of shell. They relate to the nodes of edge external of the disc. If $i$ and $j$ 2 nodes on both sides of the average average indicate, they are written:

$u_i+u_j=0$ where $u$ indicates Characteristics

## 5.2 radial displacement of the mesh

| | |
|---|---|
| Many nodes: | 149 |
| Number of meshes and types: | 40 QUAD8, 10 SEG3 |

## 5.3 Values tested

| Identification | Reference |
|---|---|
| Deflection $w0$ $(mm)$ | – 1.430 |
| SIXX $(MPa)$ center, mid thickness | 3900 |
| SIZZ $(MPa)$ centers, mid thickness | 3900 |

## 5.4 Remarks

In the absence of an axisymmetric computation of reference the SAMCEF software, one is based on computation in thick shell (always the SAMCEF software) and one compares only the deflection and the forced with the center with mid thickness.

# 6 Summary of the results

the performances in time computation and accuracy of the results are satisfactory by means of command COMP_ELAS option GREEN. On the other hand, the processing of geometric nonlinearities by the command COMP_INCR option PETIT_REAC time step provides results rather far away from the reference solution by adopting a discretization into 6 which leads to costs in already important times computation.