SSNA116 - Triaxial compression test with the model of Hoek-Brown modified in axisymmetric

Abstract

This test makes it possible to validate the elastoplastic constitutive law of Hoek-Brown modified in rock mechanics. It is about a triaxial compression test for which computations are carried out only on the solid part of the soil in pure mechanics.

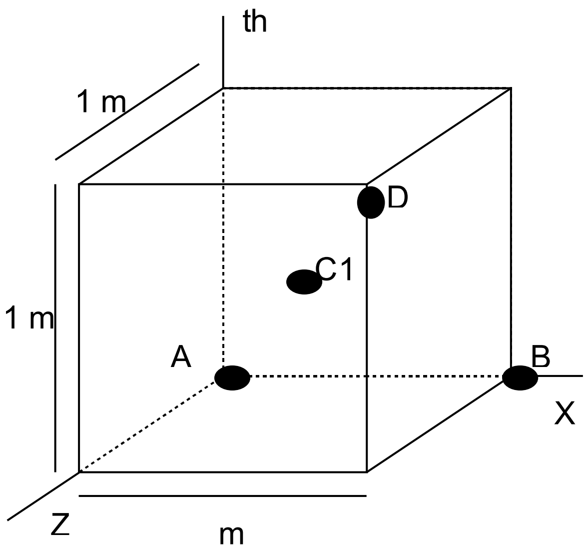
Two levels of containment are applied: $5\,MPa$ and $12\,MPa$. The parameters ϕ^{end}, ϕ^{rup} and ϕ^{res} are taken equal (what returns to a constant voluminal plastic strain): one can in this case compute an analytical solution with the problem and thus to compare the results got with *Code_Aster* with this reference solution.

For reasons of symmetry, one is interested only in the eighth of a sample subjected to a triaxial compression test. The modelization is axisymmetric.

1 Problem of reference

1.1 Geometry

One considers a cube of dimension here $1\text{m}\times 1\text{m}\times 1\text{m}$.



Coordinated of the points (in m):

	A	B	C	D
x	0	1.0.5		1
y	0	0.0.5		1
z	0	0.0.5		1

1.2 Properties of the material

Parameters of the elastic constitutive law:

$E=4500\,MPa$

$\nu=0.3$

Parameters of the model of Hoek-Brown modified:

$\gamma^{rup}=0.005$

$\gamma^{res}=0.017$

$(S\,\sigma_c^2)^{end}=225\,MPa^2$

$(S\,\sigma_c^2)^{rup}=482.5675\,MPa^2$

$(m\,\sigma_c)^{end}=13.5\,MPa$

$(m\,\sigma_c)^{rup}=83.75\,MPa$

$\beta=3\,MPa$

$\phi^{end}=15°$

$\phi^{rup}=15°$

$\phi^{res}=15°$

$\alpha=3.3$

1.3 Initial conditions, in extreme cases and loading

the test breaks up into two phases:

1) Initially, one brings the sample in a homogeneous state $\sigma_{xx}^0=\sigma_{yy}^0=\sigma_{zz}^0$. For that, the corresponding confining pressure is imposed on the front sides ($z=1$), side right ($x=1$) and higher ($y=1$), while displacements are taken null on the sides postpones ($u_z\big|_{z=0}=0$), side left ($u_x\big|_{x=0}=0$) and lower ($u_y\big|_{y=0}=0$).
2) Once the homogeneous state obtained, displacements are maintained blocked on the sides postpones, side left and lower and the confining pressure is always imposed on the front sides and side right. A displacement is forced on the upper face ($u_y(t)$) in order to obtain a strain ε_{yy} equal to – 25% starting from the beginning of the second phase, by constant increments of strain $\Delta\varepsilon_{yy}=-2.5E-4$.

2 Reference solution

2.1 Computation of the reference solution

One places here in the case of a triaxial compression test for which the stresses of containment are applied in the directions x and z for which the direction of imposed strain is the direction y. One supposes moreover than the parameter η is independent of the hardening parameter γ , it is - with - to say $\phi^{end}=\phi^{rup}=\phi^{res}$: it is then possible to calculate an analytical solution with the problem. The plasticity criterion and of flow are written:

$$(\sigma_3-\sigma_1)-\sqrt{S\sigma_c^2(\gamma)-m\sigma_c(\gamma)\sigma_3}-b(\gamma)\left(1-\frac{\sigma_3}{\sigma_3^{b-d}}\right)=0$$

$$\dot{\varepsilon}_1^p=\dot{\lambda}(\eta-1)=\frac{\eta-1}{\eta+1}\dot{\gamma}$$

$$\dot{\varepsilon}_3^p=\dot{\varepsilon}_2^p=\dot{\lambda}(\eta+\frac{1}{2})=\frac{2\eta+1}{2(\eta+1)}\dot{\gamma}$$

$$\dot{\varepsilon}_v^p=3\eta\dot{\lambda}=\frac{3\eta}{\eta+1}\dot{\gamma}$$

An increasing situation of loading is considered for which the preceding equations can be written in a nonincremental way:

$$\varepsilon_1^p=\frac{\eta-1}{\eta+1}\gamma,\quad \varepsilon_3^p=\varepsilon_2^p=\frac{2\eta+1}{2(\eta+1)}\gamma,\quad \varepsilon_v^p=\frac{3\eta}{\eta+1}\gamma$$

The relations of elasticity give:

$$\varepsilon_1-\varepsilon_1^p=\frac{1}{E}(\sigma_1-\sigma_1^0)-\frac{2\nu}{E}(\sigma_3-\sigma_3^0)$$

$$\varepsilon_3-\varepsilon_3^p=\frac{1-\nu}{E}(\sigma_3-\sigma_3^0)-\frac{\nu}{E}(\sigma_1-\sigma_1^0)$$

i.e.:

$$\varepsilon_1 - \frac{\eta-1}{\eta+1}\gamma = \frac{1}{E}(\sigma_1 - \sigma_1^0) - \frac{2\nu}{E}(\sigma_3 - \sigma_3^0)$$

$$\varepsilon_3 - \frac{2\eta+1}{2(\eta+1)}\gamma = \frac{1-\nu}{E}(\sigma_3 - \sigma_3^0) - \frac{\nu}{E}(\sigma_1 - \sigma_1^0)$$

with σ_3^0 and σ_1^0 values of σ_1 and σ_3 at the beginning of the loading. It thus remains by means of σ_1 formula γ according to the plasticity criterion to obtain γ, σ_1 and ε_3.

1st case: $\gamma \le \gamma^{rup}$

While noting $S\sigma_c^2(\gamma) = A_1 + \gamma A_2$ and $m\sigma_c(\gamma) = B_1 + \gamma B_2$ where A_1 A_2, B_1 and B_2 are given in the documentation of reference of the constitutive law, γ is solution of the polynomial of degree 2:

$$\left(\frac{\eta-1}{\eta+1}\right)^2 \gamma^2 - \left[2\varepsilon_1\left(\frac{\eta-1}{\eta+1}\right) + \frac{A_2 - \sigma_3 B_2}{E^2}\right]\gamma + \varepsilon_1^2 - \frac{A_1 - \sigma_3 B_1}{E^2} = 0,$$

with γ in the interval $[0, \gamma^{rup}]$.

2nd case: $\gamma^{rup} \le \gamma \le \gamma^{res}$

By taking again the notations of the documentation of reference of the model of Hoek-Brown modified for has, D, C and σ_3^{b-d}, γ is solution of the polynomial of degree 2:

$$\frac{a}{E}\left(1 - \frac{\sigma_3}{\sigma_3^{b-d}}\right)\gamma^2 + \left[-\left(\frac{\eta-1}{\eta+1}\right) + \frac{d}{E}\left(1 - \frac{\sigma_3}{\sigma_3^{b-d}}\right)\right]\gamma + \varepsilon_1 + \frac{\sqrt{(S\sigma_c^2)^{rup} - \sigma_3 (m\sigma_c)^{rup}}}{E} + \frac{c}{E}\left(1 - \frac{\sigma_3}{\sigma_3^{b-d}}\right) = 0$$

avec γ dans l'intervalle $\left[\gamma^{rup}, \gamma^{res}\right]$

3rd case: $\gamma^{res} \le \gamma$

In this case, σ_1 is constant:

$$\sigma_1 = \sigma_3 - \sqrt{(S\sigma_c^2)^{res} - \sigma_3 (m\sigma_c)^{res}} - b^{res}\left(1 - \frac{\sigma_3}{\sigma_3^{b-d}}\right)$$

and $\gamma = \dfrac{\sigma_1 - \sigma_1^0}{E} - \varepsilon_1$.

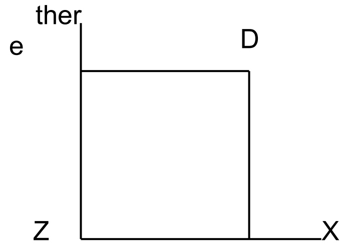
2.2 Forced results of

reference $\sigma_{xx}(\sigma_3)$, $\sigma_{yy}(\sigma_1)$ and $\sigma_{zz}(\sigma_3)$ at the point D.
Displacements $\varepsilon_{xx}(\varepsilon_3)$ and $\varepsilon_{yy}(\varepsilon_1)$ at the point D.

3 Modelization A

3.1 Characteristic of the axisymmetric

modelization $2D$ Modelization



Cutting: $1m$ in height, $1m$ width
Loading of phase 1: $\sigma_{xx}^0=\sigma_{yy}^0=\sigma_{zz}^0=-5\ MPa$ (confining pressure)
Boundary conditions: $u_x\big|_{x=0}=u_y\big|_{y=0}=u_z\big|_{z=0}=0$

3.2 Characteristics of the mesh

Many nodes: 4
Number of meshes and types: 1 `QUAD4` and 4 `SEG2`

3.3 Quantities tested and results

Localization	**Sequenc e number**	**Forced** (MPa)	**Reference solution**
Point D	12	σ_{xx}	-5
	70	σ_{xx}	-5
	12	σ_{zz}	-5
	70	σ_{zz}	-5
	12	σ_{yy}	-18.50
	16	σ_{yy}	-22.5675778
	32	σ_{yy}	-30.8797526
	41	σ_{yy}	-34.9342281
	42	σ_{yy}	-32.9136722
	46	σ_{yy}	-26.8215156
	52	σ_{yy}	-22.7560224
	70	σ_{yy}	-20.721512

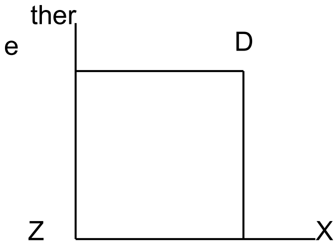
Localization	**Sequence number**	**Strain**	**Reference solution**
Point D	12.0.9	ε_{xx}	E-3
	16	ε_{xx}	1.24644 E-3

32	ε_{xx}	3.48682 E-3
41	ε_{xx}	4.81373 E-3
42	ε_{xx}	5.22653 E-3
46	ε_{xx}	6.66403 E-3
52	ε_{xx}	8.27551 E-3
70	ε_{xx}	12.01865 E-3
12	ε_{yy}	-0.003
70	ε_{yy}	-0.0175

4 Modelization B

4.1 Characteristic of the axisymmetric

modelization $2D$ Modelization



Cutting: $1m$ in height, $1m$ width
Loading of phase 1: $\sigma_{xx}^{0}=\sigma_{yy}^{0}=\sigma_{zz}^{0}=-12\,\mathrm{MPa}$ (confining pressure)
Boundary conditions: $u_x|_{x=0}=u_y|_{y=0}=u_z|_{z=0}=0$

4.2 Characteristics of the mesh

Many nodes: 4
Number of meshes and types: 1 `QUAD4` and 4 `SEG2`

4.3 Quantities tested and results

Localization	Sequence number	Forced (MPa)	Reference solution
Point D	16.-12	σ_{xx}	
	80.-12	σ_{xx}	
	16.-12	σ_{zz}	
	80.-12	σ_{zz}	
	16.-30	σ_{yy}	
	20	σ_{yy}	-33.4287301
	36	σ_{yy}	-43.5095082
	49	σ_{yy}	-50.4230084
	52	σ_{yy}	-48.4775526
	56	σ_{yy}	-46.4935733
	60	σ_{yy}	-45.0479008
	70	σ_{yy}	-43.1174944
	80	σ_{yy}	-42.8023313

Localization	Sequence number	Strain	Reference solution

Point D	16.1.2	ε_{xx}	E-3
	20	ε_{xx}	1.61504 E-3
	36	ε_{xx}	3.66549 E-3
	49	ε_{xx}	5.46863 E-3
	52	ε_{xx}	6.265 E-3
	56	ε_{xx}	7.26131 E-3
	60	ε_{xx}	8.19982 E-3
	70	ε_{xx}	10.36527 E-3
	80	ε_{xx}	12.35726E-3
	16	ε_{yy}	-0.004
	80	ε_{yy}	-0.02

5 Summary of the results

the got results make it possible to validate the model of Hoek-Brown modified integrated in *Code_Aster* in the typical case of a constant voluminal plastic strain.