SSNA115 – Wrenching of a rigid reinforcement with cohesive elements

Summarized:

This case test has as an aim the numerical study of the wrenching of a rigid reinforcement embedded in a hollow roll. Decoherence is described with three cohesive models.:

Modelization a: from elements with discontinuity by means of interns with a cohesive `model` CZM_EXP (confer to the documentation [R7.02.14]) modelization `AXIS_ELDI`.

Modelization b: from elements of joint with cohesive model `CZM_LIN_REG` (confer to the documentation [R7.02.11]) by means of modelization `AXIS_JOINT`.

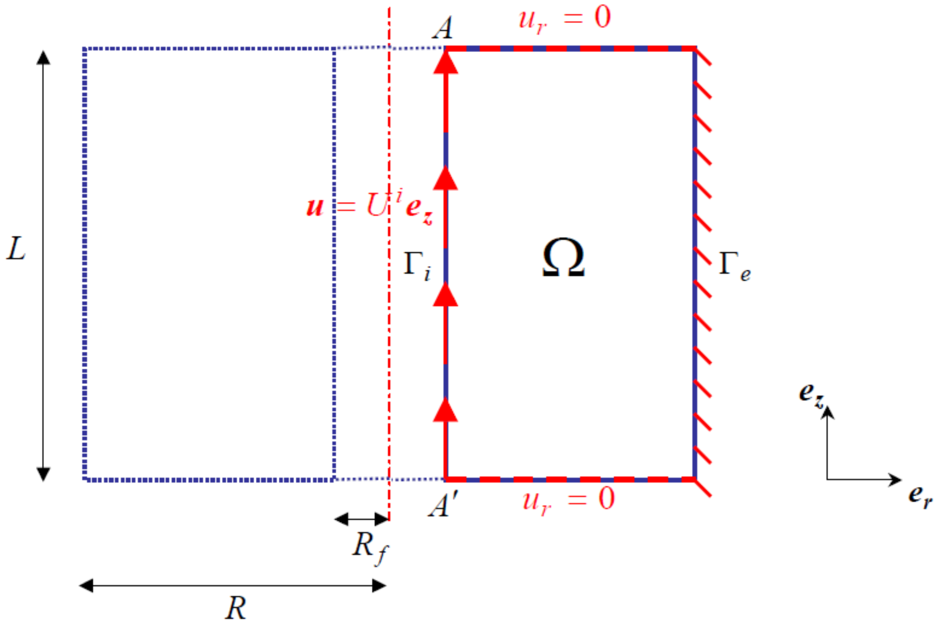
Modelization C : from elements of interface with cohesive model `CZM_TAC_MIX` (confer to the documentation [R7.02.11]) by means of modelization `AXIS_INTERFACE_S`.

To validate the results we will lean on the analytical solution developed in [bib3]. The interested reader will be able to also refer to it for a thorough study of this case test.

1 Problem of reference

1.1 Geometry and loading

Is a hollow roll length L, interior radius R_f and external radius R. That is to say a rigid reinforcement of circular section of radius R_f clamped in its center. One notes Γ_i and Γ_e surfaces interior and external of the hollow roll (see [Figure 1.1-a]). The loading consists in applying, to rigid reinforcement, a displacement $U^i e_z$ ($U^i > 0$) as well as a null displacement to external edge Γ_e.



Appear 1.1-a: Diagram of the field and loading

One makes the assumption of an axisymmetric solution what enables us to restrict our study has a field 2D rectangular Ω. Dimensions of the field are the following ones: $R_f = 0.5\,\text{mm}$, $R = 5.5\,\text{mm}$, $L = 10\,\text{mm}$. The loading on rigid reinforcement will be taken into account by applying the displacement imposed $U^i \boldsymbol{e}_z$ to all the side Γ_i of the field $2D$ as well as a null displacement on the side Γ_e to take into account the fixed support of the cylinder. Finally one forces a null radial displacement on the sides lower and higher of the field in order to avoid a singularity related to a change of boundary condition than the points A and A' (see [Figure 1.1-a]). These boundary conditions will lead to a anti-plane solution (independent of z) what makes it possible to obtain an analytical solution more simply.

2 Reference solution

the reference solution is an analytical solution drawn from [bib3], itself inspired by a unidimensional study suggested in [bib1] and in a more general way leaning on the energy approach of the fracture suggested by G.A. Francfort and J.J. Marigo [bib2]. We will not return in the details of the computation of this solution, we will present just the analytical value of the total response of structure. The displacement imposed U according to the corresponding force F is worth:

$$U(F)=\frac{Fl}{2\pi R_f L\mu}+sign(F)\psi'^{-1}\left(\frac{|F|}{2\pi R_f L}\right) \qquad \text{éq 2-1}$$

where μ the coefficient of Lamé indicates ($\mu = E/2$ here), ψ the density of energy of cracking and where $l=R_f\ln(R/R_f)$ is a length structural feature decisive for the brutal or progressive evolution of decoherence.

The reverse of derivative of the density of energy surface takes the following values according to whether one adopts the cohesive model CZM_EXP (i.e elements with discontinuity), model CZM_LIN_REG (i.e elements of joint) or law CZM_TAC_MIX (i.e elements of interface). (see documentations [R7.02.14] and [R7.02.11]). CZM_EXP

: CZM_LIN_REG $\psi'^{-1}(x)=-\frac{G_c}{\sigma_c}\ln\left(\frac{x}{\sigma_c}\right)$

or CZM_TAC_MIX: Bibliographical $\psi'^{-1}(x)=2\frac{G_c}{\sigma_c}\left(1-\frac{x}{\sigma_c}\right)$

2.1 references CHARLOTTE

1) Mr., FRANKFURT G.A., MARIGO J.J. and TRUSKINOVSKY L.: Revisiting brittle fracture ace year energy minimization problem: comparison of Griffith and Barenblatt surfaces energy models. Proceedings of the Symposium one "Continuous Ramming and Fracture" The dated science library, Elsevier, edited by A. BENALLAL, Paris, pp. 7-18, (2000). FRANKFURT

2) G.A. and MARIGO J.J.: Revisiting brittle fracture ace year energy minimization problem. J. Mech. Phys. Solids, 46 (8), pp. 1319-1342 (1998). laverne

3) J.: ENERGY formulation of the fracture by models of cohesive forces: numerical considerations theoretical and establishments, Doctorate of the University Paris 13, November 2004. Modelization

3 A Characteristic

3.1 of the modelization simulation

is carried out into axisymmetric. The elements with internal discontinuity make it possible to represent crack along formula. Γ_i have as a modelization AXIS_ELDI and a cohesive behavior CZM_EXP. The other elements of the mesh are QUAD4 with an elastic behavior ELAS in modelization AXIS. Material parameters

3.2 the values

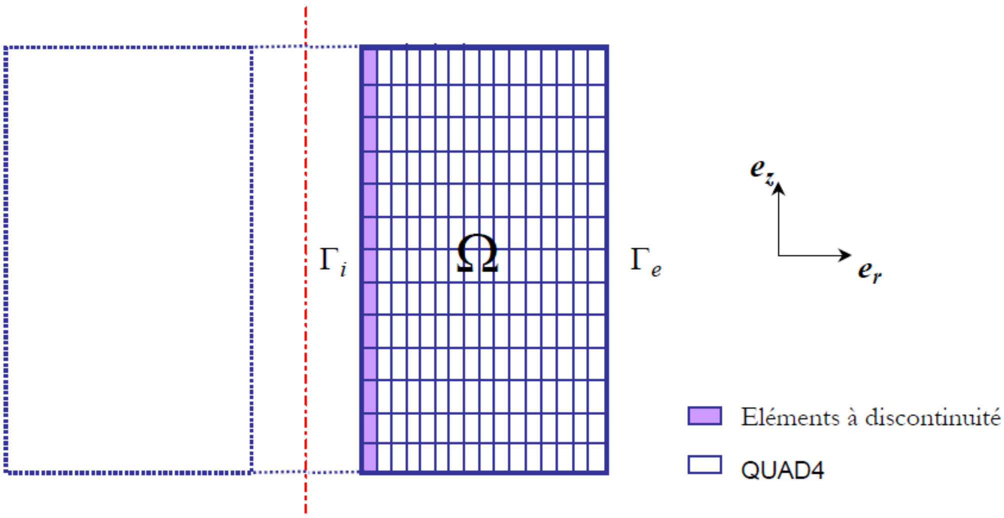
of the Young modulus, the Poisson's ratio, the critical stress and the tenacity of the material are taken in the following way: (NB: they

$$E=1.5\text{ MPa},\ \nu=0,\ \sigma_c=1.1\text{ Mpa},\ G_c=0.9\text{ N.mm}^{-1}$$

are values "tests" which do not correspond to any material in particular.) Characteristics

3.3 of the mesh One meshes

carries out a mesh structured in quadrangles of the field with Ω 76 in the height and 28 meshes in the radial direction. One lays out a layer of elements with internal discontinuity along using Γ_i command CREA_MAILLAGE and of key word CREA_FISS (confer to the documentation [U4.23.02]). The directional sense of the elements to discontinuity is carried out so that the normal direction is directed according to (the tangential $-\boldsymbol{e}_r$ direction is thus according to). The rest $-\boldsymbol{e}_z$ of the field is divided into meshes linear (see [Figure $QUAD4$ 3.2-a]). Figure 3.2



- has: Mesh of the Quantities field

3.4 tested and results the shear stress

along σ_t the crack (i.e in the elements with discontinuity) corresponds contrary to the force divided by F the surface of decoherence: . Moreover $2\pi R_f L$ by leaning on the form of the density of energy of surface defined in ψ [R7.02.12] and according to [éq 2-1] one deduces the following relation: éq 3.4-1 The

$$U(\sigma_t) = -\frac{l\,\sigma_t}{\mu} + sign(\sigma_t)\frac{G_c}{\sigma_c}\ln\left(\frac{|\sigma_t|}{\sigma_c}\right)$$ latter

will enable us to carry out tests summarized in the table below. Quantity tested

Theory Code_Aster		*Difference*	() Shear stress %
: VI7 of the mesh $PG1$ Urgent: $MJ38$ 6.00070E+00 7.69747E-01	7.69747262777 84	E-01 3.41E-05 Shear stress	
: VI7 of the mesh $PG1$ Urgent: $MJ38$ 1.20004E+01 4.34935E-01	4.34934909870 33	E-01 -2.07E-05 Shear stress	
: VI7 of the mesh $PG1$ Urgent: $MJ38$ 1.93334E+01 1.28483E-01	1.28483194462 10	E-01 1.51E-04 Displacement	
DY Urgent Node : $N5$ 1.20004E+01 1.57674E+00	1.57674153065 66	E+00 9.71E-05 Modelization	

4 B Characteristic

4.1 of the modelization simulation

is carried out into axisymmetric. The elements of interface make it possible to represent crack along formula. The Γ_i have as a modelization AXIS_JOINT and a cohesive behavior CZM_LIN_REG . The other elements of the mesh are QUAD4 with an elastic behavior ELAS in modelization AXIS. Material parameters

4.2 the values

of the Young modulus, the Poisson's ratio, the critical stress and the tenacity of the material are taken in the following way: In addition

$$E = 100 \text{ MPa} , \nu = 0 , \sigma_c = 3\text{Mpa} , G_c = 0.9 \text{ N.mm}^{-1}$$

, the parameter of regularization of the cohesive model is taken equal to PENA_ADHERENCE = 0.00001. (NB: they are values "tests" which do not correspond to any material in particular.) Characteristics

4.3 of the mesh The mesh

is identical to the precedent with the difference which the layer of cohesive elements is made up of elements with one thickness low, directed with command ORIE_FISSURE . Quantities tested

4.4 and results to test

the numerical solution, one uses the equation [éq 2-1]. One notes the resultant F^R of the force along multiplied formula Γ_i par. Quantity 2π tested

Theory Code_Aster		*Difference*	() at time %
U^i : 3 2.298338E-01	2.2983379490657	E-01 -2.22E-06 at	time
F^R : 3 1.049985E+01	1.0499850000001	E+01 1.05E-11 at	time
U^i : 6 3.884798E-01	3.8847977822122	E-01 -5.61E-06 at	time
F^R : 6 5.99982E+00	5.9998200000014	E+00 2.27E-11 at	time
U^i : 8 5.2809E-01	5.2809010497189	E-01 1.99E-05 at	time
F^R : 8 2.03974E+00	2.0397408000020	E+00 3.92E-05 Modelization	

5 C Characteristic

5.1 of the modelization simulation

is carried out into axisymmetric. The elements of interface make it possible to represent crack along formula. The Γ_i have as a modelization AXIS_INTERFACE _S and a cohesive behavior CZM_TAC_MIX . The other elements of the mesh are QUAD8 with an elastic behavior ELAS in modelization AXIS. Material parameters

5.2 the values

of the Young modulus, the Poisson's ratio, the critical stress and the tenacity of the material are taken in the following way: In addition

$$E = 100 \text{ MPa} \;,\; \nu = 0 \;,\; \sigma_c = 3\text{Mpa} \;,\; G_c = 0.9 \text{ N.mm}^{-1}$$

, the parameter of penalization of Lagrangian is taken equal to PENA_LAGR = 1000. (NB : they are values "tests" which do not correspond to any material in particular.) Characteristics

5.3 of the mesh The mesh

is identical to the modelization A with two differences near: all meshes are quadratic (QUAD8) and lay down it cohesive elements is made up of elements with one thickness low, directed with command ORIE_FISSURE . Quantities tested

5.4 and results to test

the numerical solution, one uses the equation [éq 2-1]. One notes the resultant F^R of the force along multiplied Γ_i par. Quantity 2π tested

Theory Code_Aster		***Difference***	**() at time** $\%$
U^i : 3 1.576931E-01	1.5769307873679	E-01 -1.35E-05 at	time
F^R : 3 1.25499E+01	1.2549900398014	E+01 3.17E-06 at	time
U^i : 6 2.656474E-01	2.6564739519277	E-01 -1.81E-06 at	time
F^R : 6 9.486833E+00	9.4868329805100	E+00 -2.05E-07 at	time
U^i : 8 3.63577E-01	3.6357700583331	D-01 1.60E-06 at	time
F^R : 8 6.7082E+00	6.7082039325061	D+00 5.86E-05 Summary	of

6 the results One notes

that the three element types allow a good prediction of decoherence. Indeed the latter develops in an identical way on all the height of the cylinder. Moreover, the numerical results are very close to the analytical solution. In addition, the models suggested make it possible to reproduce correctly the evolution brutal (case of the modelization A) or progressive (case of the modelizations B and C) of cracking according to the lengths structural feature and of the material. The interested reader will be able to refer to [bib3] for more details.