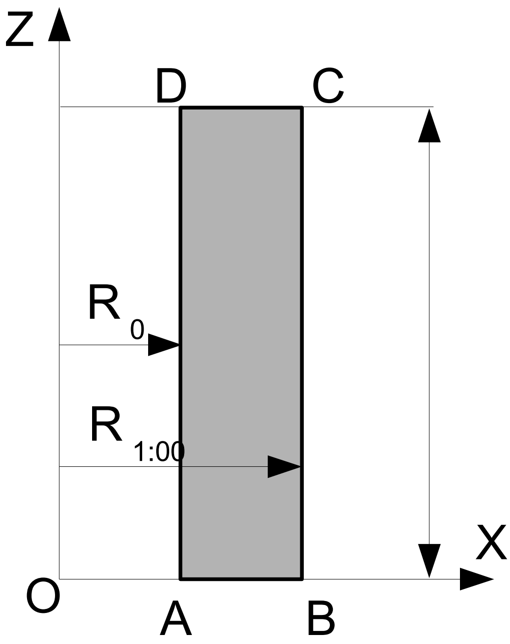
SSNA107 – Hollow roll in nonlinear viscoelasticity

Summarized:

This benchmark makes it possible to validate the model of LEMAITRE established in Code_Aster in the case of viscoelastic behavior nonlinear. The found results are compared with an analytical solution.

1 Problem of reference

1.1 Geometry



Geometry of the cylinder (m) :

$R_0=1$
$R_1=1.02$
$H=1$

1.2 Properties of the Elastic

material

- Young Modulus: $E=1.0\times 10^6\,Pa$
- Poisson's ratio: $\nu=0.3$

LEMAITRE

$$g(\sigma,\lambda,T)=\left(\frac{1}{K}\frac{\sigma}{\left(\frac{1}{\lambda^m}\right)}\right)^n \text{ with } n=2\ ;\ \frac{1}{K}=1\ ;\ \frac{1}{m}=0$$

CIN1_CHAB

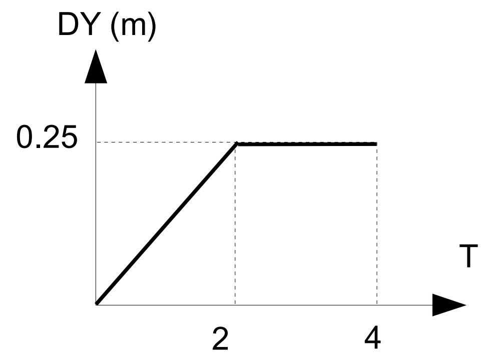
$R\_0=0.$
$R\_I=0.$
$B=0.$
$C\_I=0.$
$K=0.$
$W=0.$
$G\_0=0.$

$A\_I=0.$

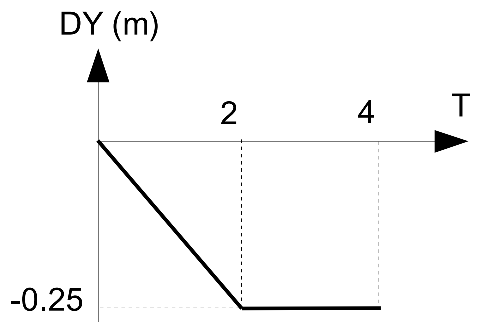
1.3 Boundary conditions and loadings

imposed Displacement (m):

Dimensioned CD



With dimensions AB



2 Reference solution

2.1 Méthode de calcul used for the reference solutions

the group of this demonstration can be read with more details in the document $[1]$

the stress tensor is written:

$$\sigma=\begin{pmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & \sigma_z\end{pmatrix}$$

Because of the loading, one a:

$$\begin{cases}\varepsilon_z-\varepsilon_{vz}=\dfrac{\sigma_z}{E}\\ \varepsilon_\theta-\varepsilon_{v\theta}=-\dfrac{\nu}{E}\sigma_z\\ \varepsilon_r-\varepsilon_{vr}=-\dfrac{\nu}{E}\sigma_z\end{cases} \text{ and } \dot{\varepsilon}_v=\frac{3}{2}g(\sigma^*)\frac{\sigma^D}{\sigma^*}$$

thus

$$\begin{cases}\dot{\varepsilon}_{vz}=g(\sigma_z)\\ \dot{\varepsilon}_{v\theta}=-\dfrac{1}{2}g(\sigma_z)\\ \dot{\varepsilon}_{vr}=-\dfrac{1}{2}g(\sigma_z)\end{cases}$$

If $t\le t_0$, one has $\varepsilon_z=\dfrac{\varepsilon_0}{t_0}t$,

That is to say $a=\sqrt{\dfrac{\varepsilon_0}{t_0}}$

One obtains $\varepsilon_z=a^2t$

While replacing, one finds:

$$\dot{\varepsilon}_{vz}=g((a^2t-\varepsilon_{vz})E)$$

One poses $E=1$ and $z=a^2t-\varepsilon_{vz}$, one obtains: $\dot{z}=a^2-z^2$

While integrating with $z(0)=0$ one obtains:

$z = a \tanh(at)$

For $t \leq t_0$

$$\begin{cases} \sigma_r = \sigma_\theta = 0 \\ \sigma_z = a \tanh(at) \\ \varepsilon_r = \varepsilon_\theta = a[(\frac{1}{2} - \nu)\tanh(at) - \frac{1}{2}at] \\ \varepsilon_z = a^2 t \\ w = ar[(\frac{1}{2} - \nu)\tanh(at) - \frac{1}{2}at] \end{cases}$$

If $t \geq t_0$

$$\varepsilon_z = a^2 t_0$$
$$\dot{\varepsilon}_{vz} = g(a^2 t_0 - \varepsilon_{vz}) = (a^2 t_0 - \varepsilon_{vz})^2$$

What gives while integrating:

$$\varepsilon_{vz} = a^2 t_0 - \frac{1}{\frac{1}{a \tanh(at_0)} + t - t_0}$$

There is thus with final

$$\begin{cases} \sigma_r = \sigma_\theta = 0 \\ \sigma_z = \frac{1}{\frac{1}{a \tanh(at_0)} + t - t_0} \\ \varepsilon_r = \varepsilon_\theta = (\frac{1}{2} - \nu)\frac{1}{\frac{1}{a \tanh(at_0)} + t - t_0} - \frac{1}{2}a^2 t_0 \\ \varepsilon_z = a^2 t_0 \\ w = r\left[(\frac{1}{2} - \nu)\frac{1}{\frac{1}{a \tanh(at_0)} + t - t_0} - \frac{1}{2}a^2 t_0\right] \end{cases}$$

2.2 the Forced reference Variables

- DX Déplacementau B

•node $SIXX$, $SIYY$ and $SIZZ$ with the node B

2.3 Results of reference

Quantity	Not	Urgent	Reference
DX	B	4	-0.2109
$SIXX$	B	4	$0.$
$SIYY$	B	4	0.2616
$SIZZ$	B	4	$0.$

2.4 Uncertainty on the analytical

solution Solution

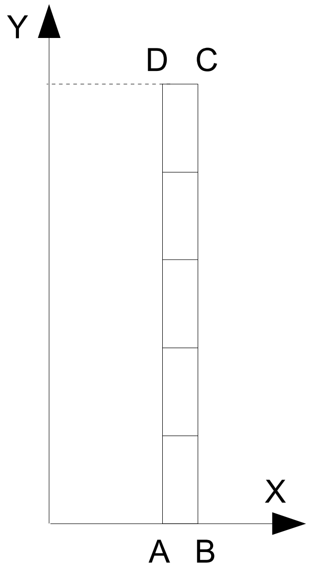
2.5 bibliographical References

[1] pH. BONNIERES, Mr. ZIDI: Introduction of viscoplasticity into the modulus of thermomechanics of Cyrano3 : Principle, description and validation, Note HI-71/8334.

3 Modelization A

3.1 Characteristic of the modelization A

viscoelastic `Modelization`
AXIS Behavior model of `LEMAITRE`



3.2 Characteristic of the mesh

Many nodes	12			
Number of meshes	17	Is:		
			SEG2	12
			QUAD4	5

Nodes groups:
A, B, C, D

Mesh groups :

$MAIL$	: surface	$ABCD$
DAB	: segment	AB
DBC	: segment	BC
DCD	: segment	CD
DDA	: segment	DA

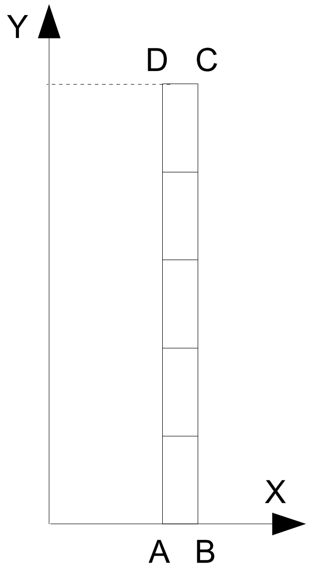
3.3 Quantities tested and Quantity

results	Not	Urgent	Reference	Aster	Variation %
DX	B	4	-0.2109	-0.2109	0,001%
$SIXX$	B	4	$0.$	4,82E-9	-
$SIYY$	B	4	0.2616	0.2616	0,002%
$SIZZ$	B	4	$0.$	4,82E-9	-

4 Modelization B

4.1 Characteristic of the modelization B

viscoelastic `Modelization`
AXIS Behavior model of `VISC_CIN1_CHAB`



4.2 Characteristic of the mesh

Many nodes 12
Number of meshes 17 Is:

`SEG2` 12
`QUAD4` 5

Nodes groups:
A, B, C, D

Mesh groups :

MAIL	: surface	*ABCD*
DAB	: segment	*AB*
DBC	: segment	*BC*
DCD	: segment	*CD*
DDA	: segment	*DA*

4.3 Quantities tested and Quantity

results	Not	Urgent	Reference	Aster	Variation %
DX	*B*	4	-0.2109	-0.2109	0,026%
SIXX	*B*	4	0.	1,55E-10	-
SIYY	*B*	4	0.2616	0.2616	0,130%
SIZZ	*B*	4	0.	1,55E-10	-

5 Summary of the results

the results calculated by Code_Aster are in excellent agreement with the analytical solutions.