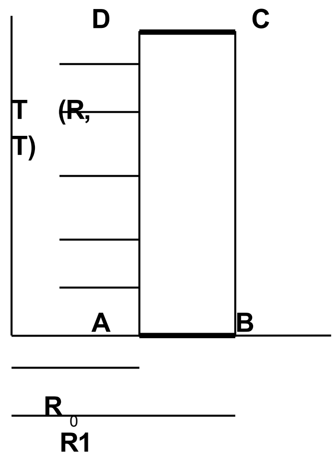
SSNA106 - Hollow roll subjected to a Summarized behavior

thermoviscoelastic:

This benchmark makes it possible to validate the model of LEMAITRE established in *Code_Aster* in the linear case of behavior thermoviscoelastic. The found results are compared with an analytical solution.

1 Problem of reference

1.1 Geometry



R_0 $1\,m$
R_1 $2\,m$

1.2 Properties of the materials

Modulus Young: $E=1\,MPa$
Poisson's ratio: $\nu=0.3$
Coefficient of thermal expansion: $\alpha=0.7$

Model of LEMAITRE:

$$g(\sigma,\lambda,T)=\left(\frac{1}{K}\frac{\sigma}{\lambda^{\frac{1}{m}}}\right)^n \text{ with } \frac{1}{K}=1\,,\ \frac{1}{m}=0\,,\ n=1$$

1.3 Boundary conditions and loading

Boundary conditions:
The cylinder is blocked in DY on the sides $[AB]$ and $[CD]$.

Loading:
The cylinder is subjected to a field of temperature $T(r,t)=t\,r^2$

2 Reference solutions

2.1 Method of calculating used for the reference solutions

the group of this demonstration can be read with more details in the document [bib1].

In the case of a linear viscoelastic isotropic material, one can describe the behavior in the course of time using two functions $I(t)$ and $K(t)$ so that the strains and the forced can be written:

$$\varepsilon(t)=(I+K)*\frac{d\sigma(t)}{d\tau}-K*\frac{d\left(Tr\left(\sigma(t)\right)\right)}{d\tau}\mathbf{I}_3+\alpha T(r,t)\mathbf{I}_3$$

where I_3 indicates the matrix identity of row 3

and $*$ the product of convolution: $(f*g)(t)=\int_0^t f(t-\tau)g(\tau)d\tau$

The thermo-elastic problem are equivalent, via the transform of Laplace is:

$$\begin{cases}\varepsilon^+=(I^++K^+)\sigma^+-K^+Tr(\sigma^+)\mathbf{I_3}+\dfrac{\alpha r^2}{p}\mathbf{I_3}\\ \sigma_r^{+}{}'=\dfrac{d\sigma_r^+}{dr}=\dfrac{1}{r}\left(\sigma_\theta^+-\sigma_r^+\right)\\ \varepsilon_z^+=0\\ \left(r\varepsilon_\theta^+\right)=\varepsilon_r^+\end{cases}$$

By eliminating the sign "+":

$$\begin{cases}\sigma_r'+\dfrac{1}{r}\left(\sigma_r-\sigma_\theta\right)=0\\ \left(I+K\right)\sigma_z-K(\sigma_r+\sigma_\theta+\sigma_z)+\dfrac{\alpha r^2}{p}=0\\ \left[r\left((I+K)\sigma_\theta-K\left(\sigma_r+\sigma_\theta+\sigma_z\right)+\dfrac{\alpha r^2}{p}\right)\right]=(I+K)\sigma_r-K\left(\sigma_r+\sigma_\theta+\sigma_z\right)+\dfrac{\alpha r^2}{p}\end{cases}$$

maybe,

$$\begin{cases}\sigma_r'+\dfrac{1}{r}(\sigma_r-\sigma_\theta)=0\\ \sigma_z=\dfrac{K}{I}(\sigma_r+\sigma_\theta)-\dfrac{\alpha r^2}{pI}\\ \left[r\left((I+K)\sigma_\theta-\dfrac{\left(I+K\right)K}{I}\left(\sigma_r+\sigma_\theta\right)+\dfrac{\alpha r^2}{p}\right)\right]=(I+K)\sigma_r-\dfrac{\left(I+K\right)K}{I}\left(\sigma_r+\sigma_\theta\right)+\dfrac{\left(I+K\right)}{I}\dfrac{\alpha r^2}{p}\end{cases}$$

$$(I+K)\sigma_\theta + r\left((I+K)\sigma_\theta - \frac{(I+K)K}{I}(\sigma_r+\sigma_\theta) + \frac{(I+K)}{I}\frac{\alpha r^2}{p}\right) = (I+K)\sigma_r$$

According to the balance equation, one has $\sigma_\theta = r\sigma_r' + \sigma_r$, one obtains:

$$(I+K)\sigma_r' + r\left((I+K)(r\sigma_r'+\sigma_r) - \frac{(I+K)K}{I}(2\sigma_r + r\sigma_r') + \frac{(I+K)}{I}\frac{\alpha r^2}{p}\right) = 0$$

$$\left[(2\sigma_r + r\sigma_r') + \frac{\alpha r^2}{p(I-K)}\right] = 0 ,$$

$2\sigma_r + r\sigma_r' = A + \dfrac{\alpha r^2}{p(K-I)}$ which while integrating compared to R gives:

$$\sigma_r = \frac{A}{2} + \frac{B}{r^2} + \frac{\alpha r^2}{4p(K-I)} ,$$

the boundary conditions $\sigma_r(r_0) = \sigma_r(r_1) = 0$ give:

$$A = -\frac{\alpha}{2p(K-I)}(r_0^2 + r_1^2)$$

$$B = \frac{\alpha r_0^2 r_1^2}{4p(K-I)}$$

One thus has by taking again the initial notations:

$$\begin{cases} \sigma_r^+ = \dfrac{\alpha}{4p(I^+-K^+)}(r_0^2 + r_1^2 - r^2 - \dfrac{r_0^2 r_1^2}{r^2}) \\ \sigma_\theta^+ = \dfrac{\alpha}{4p(I^+-K^+)}(r_0^2 + r_1^2 - 3r^2 + \dfrac{r_0^2 r_1^2}{r^2}) \\ \sigma_z^+ = \dfrac{\alpha}{p(I^+-K^+)}(\dfrac{K^+}{I^+}\dfrac{(r_0^2+r_1^2)}{2} - r^2) \end{cases}$$

Maybe, by taking the opposite transform,

$$\sigma = \begin{pmatrix} \dfrac{\alpha}{2k}(1-e^{-bt})\left(r_0^2 + r_1^2 - r^2 - \dfrac{r_0^2 r_1^2}{r^2}\right) & 0 & 0 \\ 0 & \dfrac{\alpha}{2k}(1-e^{-bt})\left(r_0^2 + r_1^2 - 3r^2 + \dfrac{r_0^2 r_1^2}{r^2}\right) & 0 \\ 0 & 0 & \dfrac{\alpha}{k}\left[(1-e^{-bt})(r_0^2 + r_1^2 - 2r^2) + \dfrac{r_0^2 + r_1^2}{r^2}(1-e^{-Ekt})\right] \end{pmatrix}$$

One from of deduced ε_V and w :

$$w(r,t) = \frac{1-2\nu}{Ek}\alpha r\left[(1-e^{-bt})\left[r_0^2 + r_1^2 - \frac{r_0^2 r_1^2}{r^2}\right] + (1-e^{-Ekt})\left[\frac{-(r_0^2 + r_1^2)}{4}\right] + \frac{3Ekt}{4(1-2\nu)}\left[\frac{r_0^2 r_1^2}{r^2} + r^2\right]\right]$$

2.2 Results of reference

Displacement DX on the node B

2.3 Uncertainty on the solution

0% : analytical solution

2.4 bibliographical References

pH. BONNIERES, two analytical solutions of axisymmetric problems in linear viscoelasticity and with unilateral contact, Note HI-71/8301

3 Modelization A

3.1 Characteristic of the modelization

the problem is modelled in axisymetry

3.2 Characteristics of the mesh

120 meshes QUAD4

3.3 Quantities tested and results

Identification	Times	Reference	Tolerance %
$DX(B)$	0.24	1.110	0.1%

4 Summary of the results

the results calculated by *Code_Aster* are in agreement with the analytical solutions but very strongly depend on the refinement of the mesh.