SSNA105 - Hollow roll subjected to a pressure, linear viscoelasticity, Summarized

contact:

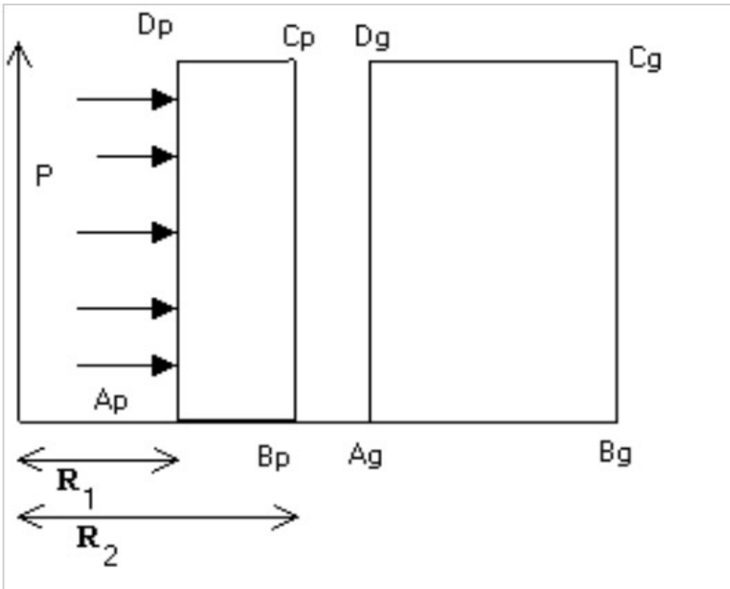
This benchmark makes it possible to validate the model of LEMAITRE established in *Code_Aster* in the case of viscoelastic behavior linear. The found results are compared with an analytical solution.

This test takes again the same modelization as the benchmark SSNA104A to which one adds a cylinder (pastille) and one treats the contact.

1 Problem of reference

1.1 Geometry

the diagram is not on the scale, the difference between the two cylinders was amplified for a better visibility.



R_1	0.82
R_2	0.92
R_3	1.
R_4	2.

1.2 Properties of the materials

the pastille is made up of an elastic material, the sheath consists of a viscoelastic material.
The elastic data coincide for the two materials.
Young modulus: $E=1\,MPa$
Poisson's ratio: $\nu=0.3$

Model of LEMAITRE:

$$g(\sigma,\lambda,T)=\left(\frac{1}{K}\frac{\sigma}{\lambda^{\frac{1}{m}}}\right)^n \text{ with } \frac{1}{K}=1 \quad \frac{1}{m}=0\,,\; n=1$$

1.3 Boundary conditions and loading

Boundary conditions:
The cylinder is blocked in DY on the sides $[AP, BP]$, $[AG, BG]$ and $[CP, PD][CG, PG]$.

Loading:
The cylinder is subjected to a pressure interns on $[DP, AP]$, this pressure is calculated so that at time $t=0$, the sheath has the same behavior as the cylinder modelled in the test ssna104a.

$$p_1(t) = \begin{cases} \left(\frac{r_3}{r_2} - 1\right)\frac{E\left(r_2^2 - r_1^2\right)}{2r_1^2(1-\nu)} & si\ \ -1 \le t \le 0 \\ A\left[B\left(r_3 - r_2 + C\left(D + Ge^{-Ekt} + Ht\right)\right) + K\right] & si\ \ 0 < t \le 5 \end{cases}$$

with

$A = \frac{r_2^2 - r_1^2}{2r_1^2(1-\nu)}$ $B = \frac{E}{r_2(1+\nu)}$, $C = \frac{P_0 r_3^3}{r_4^2 - r_3^2}$ with $P_0 = 1.\text{E}-3\ MPa$, pressure of the test ssna104a.

$$D = \frac{1}{E}\left[(1+\nu)\frac{r_4^2}{r_3^2} + \frac{3}{2}(1-2\nu)\right] \quad G = -\frac{(1-2\nu)^2}{2E} \quad H = \frac{3}{2}k\frac{r_4^2}{r_3^2}, \quad K = \frac{P_0 r_2^2}{r_2^2 - r_1^2}\left(1 - 2\nu + \frac{r_1^2}{r_2^2}\right)$$

One treats the contact between the two cylinders.

2 Reference solutions

2.1 Method of calculating used for the reference solutions

the group of this demonstration can be read with more details in the document [bib1].

Phase without contact

One wants to find the value of $p_1(t)$ to be applied to the internal wall of the pastille for which the contact takes place.
For the pastille, one finds:

$$\sigma=\begin{pmatrix} \gamma\left(1-\frac{r_2^2}{r^2}\right) & 0 & 0 \\ 0 & \gamma\left(1+\frac{r_2^2}{r^2}\right) & 0 \\ 0 & 0 & 2\nu\gamma \end{pmatrix} \text{ where } \gamma=\frac{p_1(t)r_1^2}{r_2^2-r_1^2}$$

$$\varepsilon_\theta=\frac{1+\nu}{E}\gamma\left[1-2\nu+\frac{r_2^2}{r^2}\right]=\frac{w}{r}.$$

The condition of being written contact: $w(r_3)-w(r_2)=0$, one has $r_3-r_2=r_2\frac{2(1+\nu)\gamma}{E}(1-\nu)$

From where $\gamma=\left(\frac{r_3}{r_2}-1\right)\frac{E}{2(1-\nu)^2}$

$$p_1\lim=\left(\frac{r_3}{r_2}-1\right)\frac{E(r_2^2-r_1^2)}{2r_1^2(1-\nu)^2}.$$

Phase with contact

One wants that from time $t=0$, the sheath has same behavior as in the test ssna104a.
When there is contact, one a:

$$w_P(r_2)=W_G(r_3)+r_3-r_2,$$

therefore by recovering the value of displacements in the test ssna104, one must obtain:

$$w_P(r_2)=r_3-r_2+\frac{p_0 r_3^3}{r_4^2-r_3^2}\left\{\frac{1}{E}\left((1+\nu)\frac{r_4^2}{r_3^2}+\frac{1-2\nu}{2}(3-(1-2\nu)e^{-Ekt})\right)+\frac{3}{2}k\frac{r_4^2}{r_3^2}t\right\}.$$

The stress field of the pastille is given by

$$\sigma=\begin{pmatrix} \gamma_1\left(1-\frac{r_2^2}{r^2}\right)-\gamma_0\left(1-\frac{r_2^2}{r^2}\right) & 0 & 0 \\ 0 & \gamma_1\left(1+\frac{r_2^2}{r^2}\right)-\gamma_0\left(1+\frac{r_2^2}{r^2}\right) & 0 \\ 0 & 0 & \sigma_Z \end{pmatrix}$$

with $\gamma_1=\frac{p_1 r_1^2}{r_2^2-r_1^2}$ and $\gamma_0=\frac{p_0 r_1^2}{r_2^2-r_1^2}$.

Like $\varepsilon_Z=\frac{1+\nu}{E}\sigma_Z-\frac{\nu}{E}(2(\gamma_1-\gamma_0)+\sigma_Z)=0$, one finds: $\sigma_Z=2\nu(\gamma_1-\gamma_0)$.

There is thus $\varepsilon_\theta=\frac{1+\nu}{E}\sigma_\theta-\frac{\nu}{E}(\sigma_r+\sigma_\theta+\sigma_Z)=\frac{1+\nu}{E}\left[(1-2\nu)(\gamma_1-\gamma_0)+\gamma_1\frac{r_2^2}{r^2}-\gamma_0\frac{r_1^2}{r^2}\right]=\frac{w}{r}$

$w_P(r_2)=\frac{1+\nu}{E}r_2\left[2(1-\nu)\gamma_1-\gamma_0\left(1-2\nu+\frac{r_1^2}{r_2^2}\right)\right]$, one finds $p_1(t)$ given by the formula a little higher.

2.2 Results of reference

Displacement DX on the node B

2.3 Uncertainty on the solution

0% : analytical solution

2.4 bibliographical References

•pH. BONNIERES, two analytical solutions of axisymmetric problems in linear viscoelasticity and with unilateral contact, Note HI-71/8301

3 Modelization A

3.1 Characteristic of the modelization

the problem is modelled in axisymetry. The contact is treated by the discrete formulation.

3.2 Characteristics of the mesh

600 meshes QUAD4
160 meshes SEG2

3.3 Quantities tested and results

Identification	Times	Reference	Tolerance
$DX(B)$	0.9	2,1400 E-3	RELATIF – 0,95%
$SIXX(B)$	0.9	0	ABSOLU – 9,6E-6
$SIYY(B)$	0.9	2,7912 E-4	RELATIF – 3,40%
$SIZZ(B)$	0.9	6,6000 E-4	RELATIF – 2,00%

4 Modelization B

4.1 Characteristic of the modelization

the problem are modelled in axisymetry. The contact is treated by the formulation continues.

4.2 Characteristics of the mesh

600 meshes QUAD4
16 0 meshes SEG2

4.3 Quantities tested and results

Identification	Times	Reference	Tolerance (%)
$DX(B)$	0.9	2.14 E-3	1.2%
$SIXX(B)$	0.9.0.0.0.9		
$SIYY(B)$		2.7912 E-4	3.5%
$SIZZ(B)$	0.9	6.66 E-4	2.5%

5 Summary of the results

the results calculated by *Code_Aster* are in agreement with the analytical solutions but very strongly depend on the refinement of the mesh. The two methods of taking into account of the contact (discrete formulation and continues) give the same results.