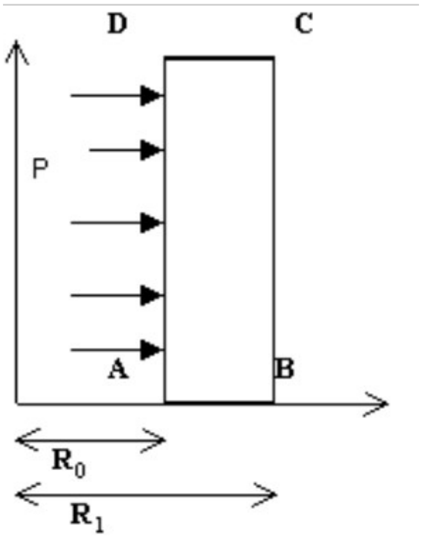
SSNA104 - Hollow roll subjected to a pressure, linear viscoelasticity

Abstract:

This benchmark makes it possible to validate the models of LEMAITRE and LEMA_SEUIL established in *Code_Aster* in the case of viscoelastic behavior linear. The found results are compared with an analytical solution.

1 Problem of reference

1.1 Geometry



Dimensions of the cylinder:

R_0 $1\,m$

R_1 $2\,m$

Appear 1.1-a: Cut hollow roll and loading

1.2 Properties of the materials

Modulus Young: $E=1\,MPa$
Poisson's ratio: $\nu=0.3$

Model of LEMAITRE :

$$g(\sigma,\lambda,T)=\left(\frac{1}{K}\frac{\sigma}{\lambda^{\frac{1}{m}}}\right)^n \text{ with } \frac{1}{K}=1 \quad \frac{1}{m}=0\text{ , } n=1$$

Model LEMA_SEUIL :

$$g(\sigma,\lambda,T)=A\left(\frac{2}{\sqrt{3}}\sigma\right)\Phi \text{ with } A=\frac{\sqrt{3}}{2}\text{ , } \Phi=1 \text{ on all the mesh}$$

$$S=10^{-10}$$

being given the value of different materials parameters, the two models are absolutely identical and can thus be compared with the same analytical solution.

1.3 Boundary conditions and loading

Boundary conditions:
The cylinder is blocked in DY on the sides $[AB]$ and $[CD]$.

Loading:
The cylinder is subjected to an internal pressure on $[DA] P0=1.E-3\,MPa$

2 Reference solutions

2.1 Method of calculating used for the reference solutions

the group of this demonstration can be read with more details in the document [bib1].

In the case of a linear viscoelastic isotropic material, one can describe the behavior in the course of time using two functions $I(t)$ and $K(t)$ so that the strains and the forced can be written:

$$\varepsilon(t)=(I+K)*\frac{d\sigma(t)}{d\tau}-K*\frac{d(Tr(\sigma(t)))}{d\tau}\boldsymbol{I_3}$$

where I_3 indicates the matrix identity of row 3

and $*$ the product of convolution: $(f*g)(t)=\int_0^t f(t-\tau)g(\tau)d\tau$

One finds $I(t)=\frac{1}{E}+kt$, $K(t)=\frac{\nu}{E}+\frac{1}{2}kt$

One imposes the pressure P_0 on time $t=0$, the internal pressure is worth $p(t)=H(t)P_0$ where $H(t)=\begin{cases}0 & si \quad t-\tau<0\\ 1 & si \quad t-\tau\geq 0\end{cases}$ with in this case $\tau=0$

One uses the transform of Laplace Carson $f^+(n)=L(f(t))=n\int_0^\infty f(t)e^{-nt}dt$

From where $p^+=P_0$

the solution of the elastic problem are equivalent is:

$$\sigma^+=\begin{pmatrix}\gamma\left(1-\frac{r_1^2}{r^2}\right) & 0 & 0\\ 0 & \gamma\left(1+\frac{r_1^2}{r^2}\right) & 0\\ 0 & 0 & \sigma_Z^+\end{pmatrix} \text{ where } \gamma=\frac{P_0 r_0^2}{r_1^2-r_0^2}$$

One determines σ_Z^+ by the condition on ε_Z^+ given by the boundary conditions:

$$\varepsilon_Z^+=0=(I^++K^+)\sigma_Z^+-K^+(2\gamma+\sigma_Z^+)=I^+\sigma_Z^+-2K^+\gamma$$

From where $\sigma_Z^+=\gamma\left(1+\frac{(2\nu-1)p}{p+Ek}\right)$.

One finds by the transform of Laplace reverses $\sigma_z(t)=\gamma(1-(1-2\nu)e^{-Eht})$, in the same way by applying the transform of Laplace reverses on σ_r and σ_θ , one finds

$$\sigma^+=\begin{pmatrix} \gamma\left(1-\frac{r_1^2}{r^2}\right) & 0 & 0 \\ 0 & \gamma\left(1+\frac{r_1^2}{r^2}\right) & 0 \\ 0 & 0 & \gamma(1-(1-2\nu)e^{-Eht}) \end{pmatrix}$$

One from of deduced:

$$\dot{\varepsilon_V}=\begin{pmatrix} \frac{3}{2}k\gamma\left(\frac{1-2\nu}{3}e^{-Ekt}-\frac{r_1^2}{r^2}\right) & 0 & 0 \\ 0 & \frac{3}{2}k\gamma\left(\frac{1-2\nu}{3}e^{-Ekt}-\frac{r_1^2}{r^2}\right) & 0 \\ 0 & 0 & -k\gamma((1-2\nu)e^{-Eht}) \end{pmatrix}$$

and while integrating with $\varepsilon_V(0)=0$;

$$\dot{\varepsilon_V}=\begin{pmatrix} \frac{3}{2}\gamma\left(\frac{1-2\nu}{3}e^{-Ekt}-k\frac{r_1^2}{r^2}t\right) & 0 & 0 \\ 0 & \frac{3}{2}\gamma\left(\frac{1-2\nu}{3}e^{-Ekt}-k\frac{r_1^2}{r^2}t\right) & 0 \\ 0 & 0 & -\gamma\frac{(1-2\nu)}{E}(1-e^{-Eht}) \end{pmatrix}.$$

One from of deduced Results

$$w(r,t)=r\gamma\left[\frac{1}{E}\left[(1+\nu)\frac{r_1^2}{r^2}+\frac{1-2\nu}{2}(3-(1-2\nu)e^{-Ekt})\right]+\frac{3}{2}k\frac{r_1^2}{r^2}t\right]$$

2.2 radial displacement from reference

Displacement DX on the node B and the forced $SIXX$, $SIYY$ and $SIZZ$ in B

2.3 Uncertainty on the solution

0% : analytical solution

2.4 bibliographical References

pH. BONNIERES: Two analytical solutions of axisymmetric problems in linear viscoelasticity and with unilateral contact, Note HI-71/8301

3 Modelization A

3.1 Characteristic of the modelization

the problem is modelled in axisymetry.

3.2 Characteristics of the mesh

1000 meshes QUAD4

3.3 Quantities tested and results

Identification	Times	Reference
$DX(B)$	0.9	2.14498 E-3
$SIXX(B)$	0.9.0.0.0.9	
$SIYY(B)$		2.7912 E-4
$SIZZ(B)$	0.9	6.66 E-4

4 Modelization B

4.1 Characteristic of the modelization

the problem are modelled in axisymetry

4.2 Characteristics of the mesh

1000 meshes QUAD4

4.3 Quantities tested and results

Identification	Times	Reference
$DX(B)$	0.9	2.14498 E-3
$SIXX(B)$	0.9.0.0.0.9	
$SIYY(B)$		2.7912 E-4
$SIZZ(B)$	0.9	6.66 E-4

5 Summary of the results

the results calculated by *Code_Aster* are in agreement with the analytical solutions but very strongly depend on the refinement of the mesh.