# SDNV106 – Analyzes with the eigenvalues in DYNA_NON_LINE (stability and oscillatory modes)

**Summarized:**

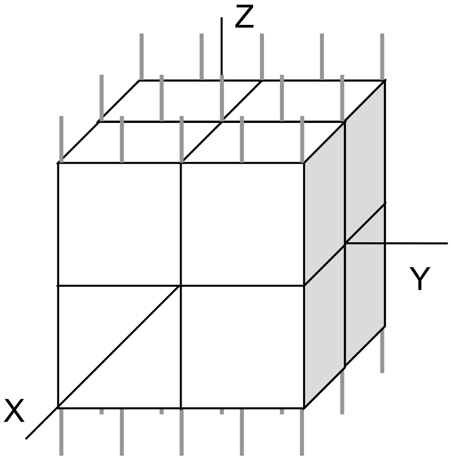
This case test makes it possible to validate the analysis of buckling, as well as the vibratory modal analysis in DYNA_NON_LINE.

Only one modelization is used: Modelization A massive $3D$ made up of meshes HEXA8.

# 1 Problem of reference

## 1.1 Geometry

One considers a cube on side length $2\,m$ subjected to a uniform tension according to the vertical direction $Z$ :




For reasons of symmetry, one will consider only one eighth of the structure, which will be with a grid by only one cubic linear voluminal element.

## 1.2 Properties of the material

the structure is supposed to be homogeneous, composed of an isotropic elastoplastic material, with linear isotropic hardening:

- $E=2.10^4\,MPa$
- $\nu=0.49999$
- $\rho=7900\,kg/m^3$
- $\sigma_y=0,1\,Mpa$ (elastic threshold SY)
- $E_T=200\,Mpa$ (tangent modulus plastic D_SIGM_EPSI)

One thus chooses a material who remains always almost incompressible, that one is in elastic or plastic mode. Moreover, one imposes a relationship 100 between the elastic stiffness and the plastic tangent stiffness.

## 1.3 Boundary conditions

One imposes a uniform loading of standard imposed tension following $Z$ on the upper face of the cube. This force imposed, initially null, grows linearly with time.

The other boundary conditions are of Dirichlet type and translate the conditions of symmetries of the problem (according to the 3 orthogonal planes $(xOy)$ , $(xOz)$ and $(yOz)$ ).

These boundary conditions are sufficient to block all rigid body motions of the system.

## 1.4 Initial conditions

the first computation being quasistatic, one imposes just an initial displacement no one.

# 2 Reference solution

## 2.1 Method of calculating

One wants to check two types of quantities:

- the first critical load of buckling,
- the first eigenfrequency of the system in vibration.

The value of reference of the required critical load is obtained by a quasistatic computation (key word CRIT_STAB of STAT_NON_LINE). One takes this value obtained with the last quasistatic computation step, which corresponds to time $t=1s$ .

The number stored under CHAR_CRIT in data structure result (it is the minimal coefficient multiplying of the loading forced to obtain the buckling load) being proportional to the imposed loading which is monotonous growing linearly with time, one corrects it to have the true value at the first time of computation transient dynamics, that is to say $1{,}001s$ .

One has, by definition of multiplying coefficient CHAR_CRIT :

$F_{critique}=\text{CHAR\_CRIT}(t_i).F_{ext}(t_i)$

The external force is proportional to time: $F_{ext}(t_i)=F_{ext}.t_i$ , therefore $F_{critique}=\text{CHAR\_CRIT}(t_i).F_{ext}.t_i$ .

The assumption is made that on a step, the loading evolves very slowly and thus that one can compare result dynamic computation to a quasistatic evolution during this step. One can then write, for the first dynamic step, which follows quasistatic computation:

$$F_{critique}=\text{CHAR\_CRIT}_{\text{STAT\_NON\_LINE}}(t_i).F_{ext}.t_i\approx F_{critique}=\text{CHAR\_CRIT}_{\text{DYNA\_NON\_LINE}}(t_i).F_{ext}.t_i$$
$$\Rightarrow$$
$$\text{CHAR\_CRIT}_{\text{DYNA\_NON\_LINE}}(t_{i+1})\approx\text{CHAR\_CRIT}_{\text{STAT\_NON\_LINE}}(t_i).\frac{t_i}{t_{i+1}}=\text{CHAR\_CRIT}_{\text{STAT\_NON\_LINE}}(t_i).\frac{1}{1{,}001}$$

For the vibratory analysis, one will make two tests:

- by means of the elastic stiffness matrix,
- by means of the plastic tangent stiffness matrix.

The two values of reference are obtained by two linear modal computations carried out with operator MODE_ITER_SIMULT.
To obtain the first eigenfrequency corresponding to the elastic case, one does a linear elastic design with MODE_ITER_SIMULT and the definite initial material above (of being worth Young modulus $2.10^4 Mpa$ ).

To obtain the first eigenfrequency corresponding to the tangent plastic case, a linear elastic design with MODE_ITER_SIMULT and a fictitious elastic material are done whose Young modulus is worth the definite plastic tangent modulus above: $200\,Mpa$ , that is to say 100 times less than the real elastic modulus. There will be thus an eigenfrequency 10 times weaker than the preceding one.

One knows also the analytical solution of our problem (cubic length 1 constituted of only one linear finite element) which is brought back to a case 1D of tension compression:

$$\omega=\sqrt{2E/\rho}\approx\begin{cases}0{,}358128\,rad/s:\text{matériau élastique}\\0{,}0358128\,rad/s:\text{matériau plastique}\end{cases}$$

## 2.2 Quantities and results of reference

| Quantities | Values | Unit |
|---|---|---|
| multiplying Coefficient of the first critical load of buckling | 2.85714E+01/1.001 | |
| First eigenfrequency elastic | 3.58128E-01 | $Hz$ |
| First plastic eigenfrequency | 3.58128E-02 | $Hz$ |

# 3 Modelization A

## 3.1 Characteristic of the mesh

Number of meshes: 1 HEXA8
Many nodes: 8

## 3.2 Quantities tested and results

| Identification | | Reference | Aster | % plastic |
|---|---|---|---|---|
| difference vibratory Eigenfrequency | $Tps$ = 1.01 | 3.58128E-02 | 3.5812661359567D-02 | -3.87E-04 |
| | $Tps$ = 1.06 | 3.58128E-02 | 3.5812661359997D-02 | -3.87E-04 |
| | $Tps$ = 1.25 | 3.58128E-02 | 3.5812661358541D-02 | -3.87E-04 |
| | $Tps$ = 1.49 | 3.58128E-02 | 3.5812661355801D-02 | -3.87E-04 |
| elastic vibratory Eigenfrequency | $Tps$ = 1.51 | 3.58128E-01 | 3.5812779545194D-01 | -5.71E-05 |
| | $Tps$ = 1.52 | 3.58128E-01 | 3.5812779545194D-01 | -5.71E-05 |
| | $Tps$ = 1.56 | 3.58128E-01 | 3.5812779545194D-01 | -5.71E-05 |
| | $Tps$ = 1.75 | 3.58128E-01 | 3.5812779545194D-01 | -5.71E-05 |
| | $Tps$ = 1.99 | 3.58128E-01 | 3.5812779545194D-01 | -5.71E-05 |
| Coefficient of the first critical load | $Tps$ = 1.001 | 2.854285714E+01 | 2.8570189972986E+01 | 0.096 |

One supplements these tests by two tests on oscillatory mode DEPL_VIBR calculated with MODE_VIBR. More precisely, one will test the value of this field in two nodes:

- GROUP_NO=' A' (node in (0,0,0): who is embedded, one must thus find a displacement identically no one,
- GROUP_NO=' H' (node in (0,1,1) : one makes a test of non regression following the direction $DY$ .

| Identification | | Reference | Aster | % difference |
|---|---|---|---|---|
| DEPL_VIBR in "A" according to $DX$ | $Tps$ = 1.2 | 0. | 0. | 0. |
| DEPL_VIBR in "H" according to $DY$ | $Tps$ = 1.2 | -0.49999288483407 | -0.49999288483077 | 6.61E-10 |

This test makes it possible to validate computations of critical loads of buckling, of vibratory frequencies and eigen modes in DYNA_NON_LINE.

# 4 Summary of the results

This test makes it possible to validate computations of critical loads of buckling, of vibratory frequencies and eigen modes in DYNA_NON_LINE.