SDNL102 - Beam subjected to a velocity field of Summarized

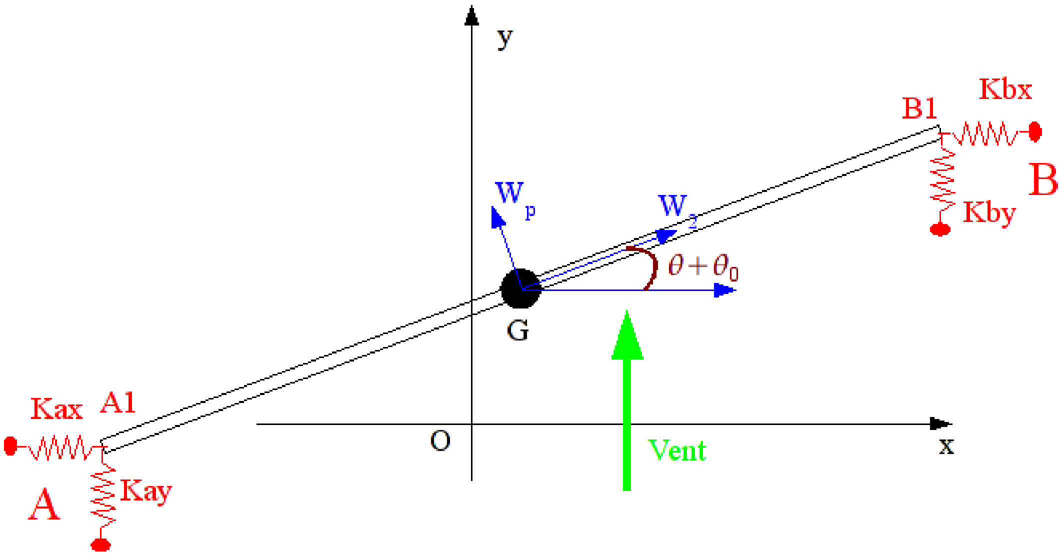
wind:

This test relates to the validation of the application of the loadings of wind on the linear elements. The loading is described by velocity fields of wind.
This problem makes it possible to test:

- linear finite elements [bars, cables, beams (except the curved beams)] with loadings follower of natural "wind",
- loadings using the velocities of wind:
  ◦lecture of the data of the fields of wind,
  ◦projection of the fields of wind attached to the group of dots on the deformed mesh of structure,
  ◦calcul relative velocity,
- the taking into account of the function giving the distributed force according to the relative velocity of structure,
- the reactualization of the geometry to take account of large displacements and large rotations.

1 Problem of reference

1.1 Geometry



Length of beam: 1.5m

Stiffness of the discrete ones: kax kay kbx , kby

1.2 Properties of the material

Material for the linear element: $E=2.0\text{E}+10$, $\rho=1000.0$

Characteristic mechanics of beam: $section='CERCLE'$ $rayon=0.1$, $ep=0.1$

stiffness of springs:

Kxa	Kya	Kxb	Kyb
$10\,N/m$	$20\,N/m$	$25\,N/m$	$22\,N/m$

1.3 Boundary conditions and loadings

At the points A and B : blockings of the degrees of freedom: DX, DY, DZ

At the points $A1$ and $B1$: blockings of the degrees of freedom : DZ, DRX, DRY

Springs are modelled by the discrete ones without dimensions. The nodes A and $A1$, B and $B1$ are geometrically confused.

Characteristics of the velocity field of wind, along the axis y :

$$Vy=20.\sin(\omega.t) \text{, with } \omega=2.\pi.f \text{ and } f=0.2\,Hz$$

1.4 Initial conditions

the beam forms an angle of $30°$ ($\theta_0=30°$) compared to the axis x .

2 Reference solution

2.1 Balance equations

the study is carried out around the initial position of structure in the plane xy. The equations are written at the center of gravity of the beam.
Force of inertia:

$$M.\gamma_g=\begin{Bmatrix} Mx'' \\ My'' \\ \dfrac{M L^2}{12}.\theta'' \end{Bmatrix}$$

Force at the point $A1$

$$Fa=\begin{Bmatrix} -kxa.\delta xa \\ -kya.\delta ya \\ L.(\delta ya.kya.\cos(\theta_0+\theta)-\delta xa.kxa.\sin(\theta_0+\theta))/2 \end{Bmatrix} \begin{matrix} \text{avec les déplacements du point } A1 \\ \delta xa=L.cos(\theta_0)/2-L.cos(\theta_0+\theta)/2+x \\ \delta ya=L.sin(\theta_0)/2-L.sin(\theta_0+\theta)/2+y \end{matrix}$$

Force at the point $B1$

$$Fb=\begin{Bmatrix} -kxb.\delta xb \\ -kyb.\delta yb \\ L.(-\delta yb.kyb.\cos(\theta_0+\theta)+\delta xb.kxb.\sin(\theta_0+\theta))/2 \end{Bmatrix} \begin{matrix} \text{avec les déplacements du point } B1 \\ \delta xb=-L.cos(\theta_0)/2+L.cos(\theta_0+\theta)/2+x \\ \delta yb=-L.sin(\theta_0)/2+L.sin(\theta_0+\theta)/2+y \end{matrix}$$

Force due to the wind

- Relative velocity of a point M

$$V_r=\begin{Bmatrix} Vvx+s.\sin(\theta_0+\theta).\theta'-x' \\ Vvy-s.\cos(\theta_0+\theta).\theta'-y' \\ 0 \end{Bmatrix}$$

with s : the curvilinear abscisse of the point M on the beam $s\in[-L/2, L/2]$
Vvx Vvy : velocity of the wind following axis X and centers it y.

- Relative velocity perpendicular to the bar at the point M:

$$V_p=\begin{Bmatrix} \sin(\theta_0+\theta).(-Vvy.\cos(\theta_0+\theta)+Vvx.\sin(\theta_0+\theta)+s.\theta'-\sin(\theta_0+\theta).x'+\cos(\theta_0+\theta).y') \\ \cos(\theta_0+\theta).(Vvy.\cos(\theta_0+\theta)-Vvx.\sin(\theta_0+\theta)-s.\theta'+\sin(\theta_0+\theta).x'-\cos(\theta_0+\theta).y') \\ 0 \end{Bmatrix}$$

Force due to the wind in a point M

$Fvent_{(M)}=Fcx_{(M)}.\dfrac{V_p}{\|V_p\|}$ in our case one chooses $Fcx_{(M)}=\|V_p\|$

one thus obtains $Fvent_{(M)}=V_p$

- Resultant of the force due to the wind on the bar

$$Fvent=\begin{Bmatrix} L.\sin(\theta_0+\theta).((-Vvy+y').\cos(\theta_0+\theta)+(Vvx-x').\sin(\theta_0+\theta)) \\ L.\cos(\theta_0+\theta).((Vvy-y').\cos(\theta_0+\theta)+(-Vvx+x').\sin(\theta_0+\theta)) \\ -L^3.\theta'/12 \end{Bmatrix}$$

final Equation of the dynamics

$$M.\gamma_g=Fa+Fb+Fvent$$

2.2 Quantities and results of reference

Displacements and rotation of the point G with times: $2.0\sec$ $3.0\sec$ $4.0\sec$, $5.0\sec$ and $6.0\sec$.

2.3 Uncertainties on the solution

None. The resolution of the balance equation is done by an integration method of Runge Kutta of order 4.

3 Modelization A

3.1 Characteristic of the modelization and the mesh

the linear element: "beam" cut out into 12 meshes.
The discrete ones: "DIS_T"

3.2 Quantities tested and results

Time 2.0sec	Analytical	Absolute error	relative Error
$x(m)$	– 0.27571	0.00070	0.00255
$y(m)$	0.46478	0.00120	0.00259
$Rz(rd)$	– 0.04851	0.00001	0.00027

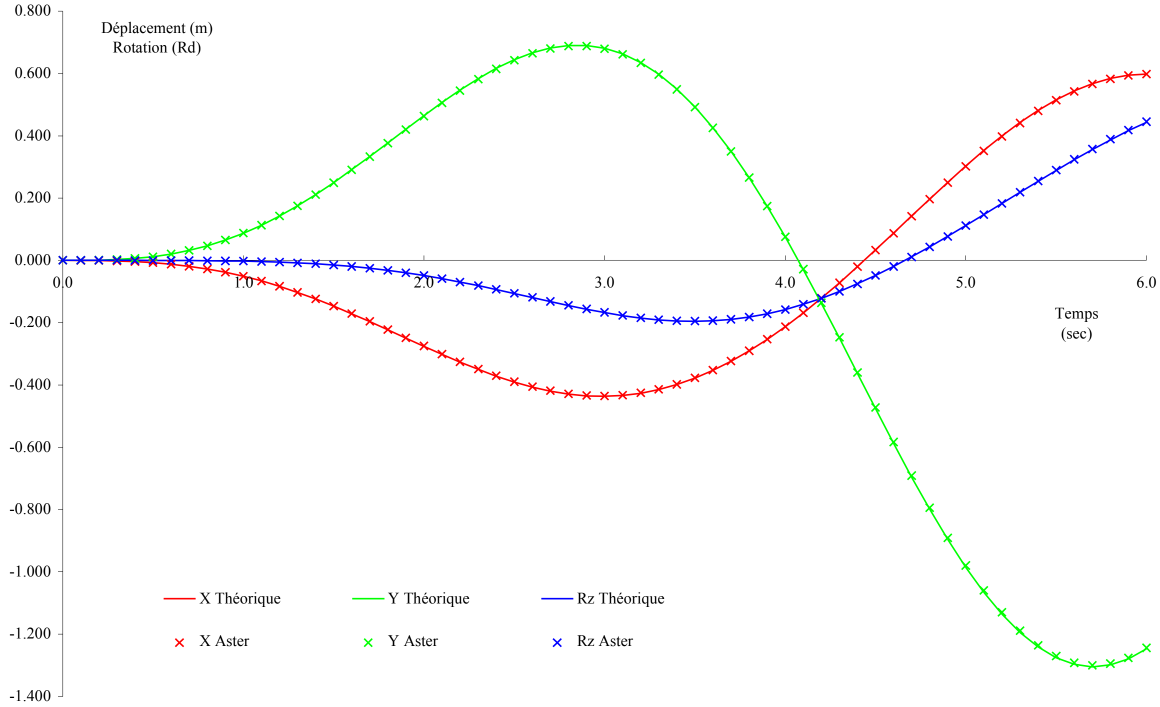
Time 3.0sec	Analytical	Absolute error	relative Error
$x(m)$	– 0.43640	0.00118	0.00271
$y(m)$	0.68149	0.00190	0.00279
$Rz(rd)$	– 0.16767	0.00079	0.00472

Time 4.0sec	Analytical	Absolute error	relative Error
$x(m)$	– 0.21266	0.00043	0.00201
$y(m)$	0.07494	0.00111	0.01476
$Rz(rd)$	– 0.15769	0.00026	0.00163

Time 5.0sec	Analytical	Absolute error	relative Error
$x(m)$	0.30290	0.00108	0.00357
$y(m)$	– 0.98487	0.00536	0.00544
$Rz(rd)$	0.11188	0.00027	0.00241

Analytical	Temps6.0sec	Absolute error	relative Error
$x(m)$	0.59847	0.00032	0.00054
$y(m)$	– 1.24735	0.00322	0.00258
$Rz(rd)$	0.44284	0.00251	0.00566

4 Summary of the Comparison



results enters the theoretical results and those of *Code_Aster.*