SDND121 – Spring-mass system with shocks under forced excitation

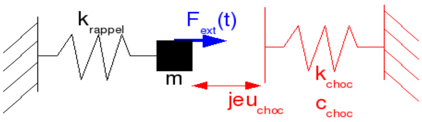
Summarized:

This problem corresponds to a transient analysis by modal recombination of a nonlinear discrete system to a degree of freedom. Non-linearity consists of a contact with shock on a rigid level. The mass is subjected to a forced excitation. The problem is stiff, i.e. the stiffness is very different between the phases from coasting flight and the phases of contact. This problem makes it possible to test the energy assessment for the various diagrams of temporal integration, as well as the kinematics.

1 Problem of reference

1.1 Geometry

the structure is a rigid shoe modelled by only one node. The shoe is provided with a come out from recall. It is subjected to a harmonic external force $F_{ext}(t)=a\sin(2\pi f t)$. It carries a condition of normal shock against a rigid plane, treated by penalization.



Appear 1.1-a: Geometry

1.2 Properties of the materials

Properties of structure:

$m=156\,kg$

$k_{rappel}=2\cdot 10^{6}\,N/m$

Properties of the thrust of shock:

$k_{choc}=10^{10}\,N/m$

$c_{choc}=0\,Ns/m$

$jeu_{choc}=1\,mm$

1.3 Initial conditions, in extreme cases and loading

the shoe leaves with null initial conditions: $x_0=x_{t=0}=0$ and $\dot{x}_0=\dot{x}_{t=0}=0$.
It moves in only one direction.
The external force is sinusoidal: $F_{ext}(t)=Fa\cdot\sin(2\pi f t)$, with $Fa=3\cdot 10^{3}N$ and $f=5\,Hz$.

2 Reference solution

2.1 Method of calculating used for the reference solution

During a phase of coasting flight, the equation of motion is written $m\ddot{x}+k_{rappel}x=F_{ext}(t)$. Taking into account the sinusoidal form of $F_{ext}(t)$, this equation admits an analytical solution. One can calculate numerically, with an accuracy as large as one wants, the time t_{in} of entry in the contact, checking $x(t_{\mathrm{in}})=jeu_{choc}$.
One is then in a phase of contact, whose equation is $m\ddot{x}+c_{choc}\dot{x}+(k_{rappel}+k_{choc})x=F_{ext}(t)$. There exists an analytical solution there too. With the boundary conditions resulting from the time of contact, one can calculate numerically the time t_{out} of output of the contact.
While proceeding thus repeatedly, one obtains the total solution of the problem.

Note: the analytical formulas here are not given. The files containing the formulas and making it possible to calculate the total solution are joined with the command file.

2.2 Results of reference

One tests the energy assessment, the adequacy between the forces of contact and the kinematics, as well as the kinematics.

For the energy assessment, one calculates the energies kinetic $E_i^{cin}=\frac{1}{2}m\dot{x}_i^2$, potential $E_i^{pot}=\frac{1}{2}k_{rappel}x_i^2$, of shock $E_i^{choc}=\frac{1}{2}k_{choc}p_i^2$ (p is the penetration; this statement is valid only if there is no damping of shock), not injected by the external force $E_i^{inj}=\sum_{j=1}^{i}f_j^{ext}\dot{x}_{j+\delta\frac{1}{2}}\Delta t$ (with $\delta=1$ for the diagram of Eulerian, $\delta=0$ for the diagram of the central differences). Total energy is obtained $E_i^{tot}=E_i^{cin}+E_i^{pot}+E_i^{choc}$. One calculates finally the total error on the energy assessment by $erreur_{globale}^{énergie}=\sqrt{\frac{\sum_i\left(E_i^{tot}-E_i^{inj}\right)^2}{\sum_i\left(E_i^{inj}\right)^2}}$, which is worth 0 ideally.

For the adequacy between the forces of contact and the kinematics, one calculates the quantity $erreur_{globale}^{force}=\sqrt{\frac{\sum_i\left(F_i^{choc}-k_{choc}p_i\right)^2}{\sum_i\left(k_{choc}p_i\right)^2}}$, which is worth 0 ideally.
For the kinematics, one compares calculated times of entry and output of contact, at analytical times.

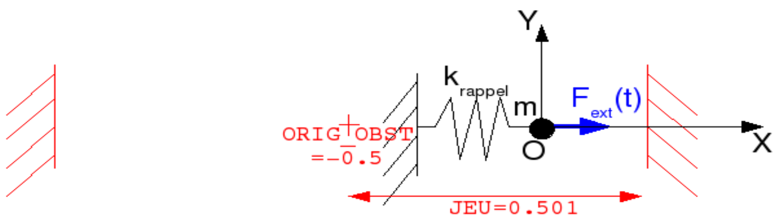
2.3 Uncertainty on the solution

the solution is analytical per pieces. Times of entry and output of contact are numerically given with $10^{-9}s$ near.

3 Modelization

3.1 Characteristics of the modelization

the shoe is modelled by a mesh with a node (of type POI1), located at rest in $O=(0,0,0)$. The obstacle is of type "PLAN_Z". Not to have a shock with the symmetric plane, one shifts sufficiently the center of the two planes: ORIG_OBST= (- 0.5, 0. , 0. ,). As real clearance between the shoe and the plane of right must be of $1\,mm$, an artificial clearance JEU= (0.5+jeu_choc) is used.



Appear 3.1-a: Modelled geometry

temporal integration is carried out over a period $T=4\,s$ either with the diagram of Eulerian, or with the diagram of the central differences ("ADAPT_ORDRE2" that one forces with time step constant thanks to parameters COEF_MULT_PAS=1.0, COEF_DIVI_PAS=1.0). The problem is stiff (the ratio of the stiffness between the phases of coasting flight and contact is about 5000), one is thus brought to use one time step very weak being worth $\Delta t=4\cdot 10^{-6}\,s$.

One carries out also a computation with "ADAPT_ORDRE2" with indeed variable step, of which the goal is not the accuracy, but simply the checking of the adequacy between the forces of contact and the kinematics.

3.2 Characteristics of the mesh

The mesh consists of a single node and a single mesh of the type POI1.

3.3 Quantities tested and Energy assessment

3.3.1 results and kinematics

In the following tables, one gives the error values de l' on energy and a few times of input/output of contact.

- Diagram of Eulerian:

Values	Reference	Aster	Tolerance
$erreur_{globale}^{énergie}$	0 J	0.092 J	0.1 J
1st entry of contact (S)	2.4867876 E-02 S	2.4868 E-02 S	1.2E-5 S
1st output of contact (S)	2.5260518 E-02 S	2.5264 E-02 S	1.2E-5 S
last entry of contact (S)	3.886525493 E+00 S	3.886528 E+00 S	1.2E-5 S
last output of contact (S)	3.886916559 E+00 S	3.886916 E+00 S	1.2E-5 S

Table 4.1-1: Results for the diagram of Eulerian

- Diagram of the central differences:

Values	Reference	Aster	Tolerance
$erreur_{globale}^{énergie}$	0 J	0.063 J	0.1 J
1st entry of contact (S)	2.4867876 E-02 S	2.4868 E-02 S	1.2E-5 S
1st output of contact (S)	2.5260518 E-02 S	2.5264 E-02 S	1.2E-5 S
last entry of contact (S)	3.886525493 E+00 S	3.886528 E+00 S	1.2E-5 S
last output of contact (S)	3.886916559 E+00 S	3.886916 E+00 S	1.2E-5 S

Table 4.1-2: Results for the diagram of the central differences

3.3.2 Adequacy between force of contact and kinematical

Value	Reference	Aster	Tolerance
$erreur_{globale}^{force}$	0 N	2.22 E-10 N	1.E-8 N

Table 4.2-1: Results for the adequacy between force of contact and kinematical

4 Summary of the results

One observes as the results are very close to the analytical solution, as well for the diagram of Eulerian as for the diagram of the central differences. Times of change of stiffness obtained by these two diagrams are identical.

The energy assessments present an error lower than $0,1\,J$ in both cases. This value is to be relativized taking into account time step the very weak one which tends to reduce the variations, but time step also weak is necessary to deal with this stiff problem.

Lastly, there is a perfect adequacy (with the numerical accuracy near) between the forces of contact and the kinematics, which means that the processing of the contact by penalization is correctly carried out.