SDND105 – Shock of a material point against a wall with the plastic behavior Summarized

buckling:

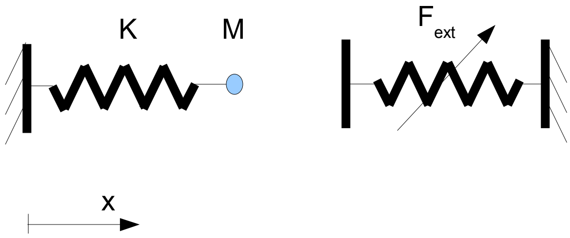
This test validates the behavior of an obstacle of type shock with possible buckling. The resolution was carried out with operator DYNA_VIBRA by means of key word FLAMBAGE.

One calculates the time of beginning of buckling (this time corresponds to the first time when the shock force exceeded the threshold of buckling), the cumulated plastic strain and the time of sharpening to the initial position.

The got results are in agreement with the analytical results.

1 Problem of reference

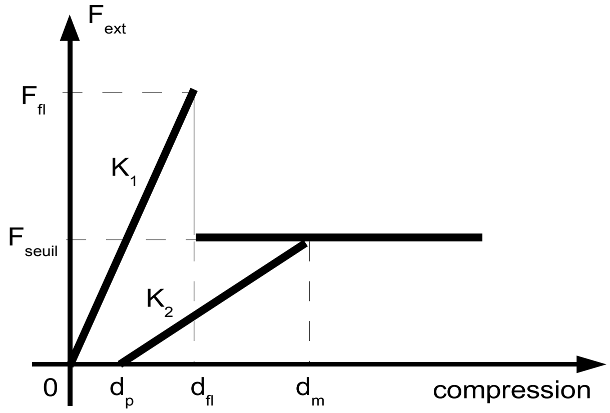
1.1 Geometry



1.2 Properties of the material

$K = 10^{-7} N/m$
$M = 1\, kg$

the reaction force F_{ext} depends on the compression of the wall which follows the model below:



With:

$K_1 = 1\, N/m$
$F_{fl} = 1\, N$
$F_{seuil} = 0.5\, N$
$K_2 = 0.5\, N/m$

1.3 Boundary conditions and loadings

One imposes the displacement of the mobile mass M along the axis Des. x

1.4 Initial conditions

On initial time, the mobile mass is right in contact against the wall (initial clearance no one) with an initial velocity equalizes with $2\, m/s$.
The wall did not undergo yet plastic strain (wall not flambe).

2 Reference solution

2.1 Method of calculating

the problem consists in analyzing the response of a mobile mass subjected at an initial velocity non-zero and coming to shock against a wall which follows a constitutive law of type buckling.

The balance equation of the system is written:

$$M\frac{\partial^2 x}{\partial t^2}+Kx=F_{ext} \quad (1)$$

the resolution of (1) proceeds in several phases: before buckling of the wall, loading after buckling, unloading after buckling and coasting flight.

2.1.1 First phase

Before buckling, the reaction force is worth: $F_{ext}=-K_1 x$

The solution of the equation (1) is put in the following form: $x(t)=A_1\sin\omega_1 t+B_1\cos\omega_1 t$

With: $\omega_1^2=\frac{K+K_1}{M}$

By taking account as of initial conditions: $x(0)=0$ and $\frac{\partial x}{\partial t}(0)=v_0$

One identifies: $A_1=\frac{v_0}{\omega_1}$ and: $B_1=0$

That is to say: $x(t)=\frac{v_0}{\omega_1}\sin\omega_1 t$

One reaches the limiting threshold of buckling F_{fl} at time t_{fl}, such as: $x(t_{fl})=d_{fl}=\frac{F_{fl}}{K_1}$.

From where: $t_{fl}=\frac{1}{\omega_1}Arc\sin(\frac{F_{fl}\omega_1}{K_1 v_0})$

and $\frac{\partial x}{\partial t}(t_{fl})=v_{fl}$

2.1.2 Second phase

the following phase is the phase of loading after buckling; the reaction force is constant as long as the velocity remains positive. This reaction force is worth: $F_{ext}=-F_{seuil}$ and the solution of the equation (1) is written: $x(t)=A_2\sin\omega_0 t+B_2\cos\omega_0 t-\frac{F_{seuil}}{M\omega_0^2}$

With: $\omega_0^2=\frac{K}{M}$

Like $\omega_0^2\ll 1$, one can carry out a restricted development of the goniometrical functions.

That is to say: $x(t)=A_{21}t^2+B_{21}t+C_{21}$

By taking account as of initial conditions $x(t_{fl})=d_{fl}$ and $\frac{\partial x}{\partial t}(t_{fl})=v_{fl}$, one obtains:

$$A_{21}=\left(\frac{v_{fl}t_{fl}}{2}-\frac{d_{fl}}{2}-\frac{F_{seuil}^2}{4M}\right)\omega_0^2-\frac{F_{seuil}}{2M}$$

$$B_{21}=v_{fl}+\frac{F_{seuil}t_{fl}}{M}+\left(d_{fl}t_{fl}-\frac{v_{fl}t_{fl}^2}{2}\right)\omega_0^2$$

$$C_{21}=d_{fl}-\frac{F_{seuil}t_{fl}^2}{2M}-v_{fl}t_{fl}-\frac{d_{fl}t_{fl}^2}{2}\omega_0^2$$

The time to which the velocity is cancelled is: $t_m=-\frac{B_{21}}{2A_{21}}$

By neglecting the terms in ω_0^2 , one deduces: $t_m=t_{fl}+\frac{M\ v_{fl}}{F_{seuil}}$,

and maximum displacement is worth: $x(t_m)=d_m$

That makes it possible to obtain the cumulated plastic strain: $d_p=d_m-\frac{F_{seuil}}{K_2}$

2.1.3 Third phase

This phase corresponds to unloading, the reaction force is worth: $F_{ext}=-K_2(x-d_p)$

The solution of the equation (1) is written: $x(t)=A_3\sin\omega_2t+B_3\cos\omega_2t+\frac{K_2d_p}{K+K_2}$

With: $\omega_2^2=\frac{K+K_2}{M}$

By taking of account the initial conditions $x(t_m)=d_m$ and $\frac{\partial x}{\partial t}(t_m)=0$,

one obtains: $A_3=\left(d_m-\frac{K_2d_p}{K+K_2}\right)\sin\omega_2t_m$ and $B_3=\left(d_m-\frac{K_2d_p}{K+K_2}\right)\cos\omega_2t_m$

the reaction force is cancelled when the displacement of the material point reaches the value of the cumulated plastic strain d_p .

Like $K\ll K_2$, one makes the following approximation: $\frac{K_2d_p}{K+K_2}\approx d_p$

Thus, the time t_d which corresponds to the cancellation of the reaction force is such that:
$x(t_d)=d_p=A_3\sin\omega_2t_d+B_3\cos\omega_2t_d+d_p$

That is to say: $\sin\omega_2t_m\sin\omega_2t_d+\cos\omega_2t_m\cos\omega_2t_d=\cos\omega_2(t_d-t_m)=0$

From where: $t_d = t_m + \frac{\pi}{2\omega_2}$

2.1.4 Fourth phase

the following phase corresponds to the phase of coasting flight $F_{ext} = 0$.

The solution of the equation (1) is written: $x(t) = A_4 \sin\omega_0 t + B_4 \cos\omega_0 t$

The initial conditions are:

$x(t_d) = d_p$ and: $\frac{\partial x}{\partial t}(t_d) = v_d = \omega_2 (x_m - \frac{K_2 d_p}{K + K_2}) \sin\omega_2 (t_m - t_d)$

What gives:

$$A_4 = d_p \sin\omega_0 t_d + \frac{v_d}{\omega_0} \cos\omega_0 t_d$$

$$B_4 = d_p \cos\omega_0 t_d - \frac{v_d}{\omega_0} \sin\omega_0 t_d$$

Like $\omega_0^2 \ll 1$, by carrying out a restricted development of the goniometrical functions until order 2, the solution is put in the following form:

$$x(t) = A_{41} t^2 + B_{41} t + C_{41}$$

With:

$$A_{41} = \frac{\omega_0^2}{2} [v_d t_d - d_p (1 - \frac{\omega_0^2 t_d^2}{2})]$$

$$B_{41} = d_p t_d \omega_0^2 + v_d (1 - \frac{\omega_0^2 t_d^2}{2})$$

$$C_{41} = d_p (1 - \frac{\omega_0^2 t_d^2}{2}) - v_d t_d$$

By neglecting the terms in ω_0^2 , one obtains:

$$x(t) = v_d t + d_p - v_d t_d$$

And one deduces time t_0 from transition to the initial position ($x = 0$).

That is to say: $t_0 = t_d - \frac{d_p}{v_d}$

2.2 Quantities and results of reference

One proposes to test the following quantities:
t_{fl} : time of beginning of buckling
d_p : cumulated plastic strain
t_0 : time of sharpening to the initial position (after buckling and discharge)

Taking into account the numerical values of the data input, one obtains:
$t_{fl}=\frac{\pi}{6}$ (expressed in seconds)
$d_p=3$ (expressed in meters)
$t_0=\frac{\pi}{6}+2\sqrt{3}+\frac{\pi+6}{\sqrt{2}}$ (expressed in seconds)

2.3 Uncertainties on the solution

the reference solution is analytical (with the second order near).

3 Modelization A

3.1 Characteristic of the modelization

One models the system with a material point and an obstacle of the type PLAN_Y.

One by means of evaluates the quantities obtained following buckling due to the shock key word FLAMBAGE of operator DYNA_VIBRA.

One also checks the various methods of resolution (EULER, ADAPT_ORDRE2 and DEVOGE). With adaptive time scheme ADAPT_ORDRE2 one defines (in seconds):

- the time step initial one: NOT = 0.0002,
- the maximum value of time step: PAS_MAXI = 0.001,
- the minimal value of time step: PAS_MINI = 2.E-8.

3.2 Characteristics of the mesh

Many nodes: 2
Number of mesh: 1 SEG2

3.3 Quantities tested and results

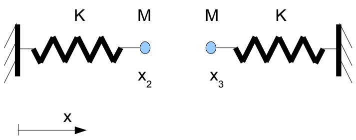
One tests the values of the quantities related to the buckling of the wall.

Identification	Reference	*Aster*	variation
t_{fl}	$\frac{\pi}{6}s$	0.52411 S	0.097%
d_p	3 m	2.998584 m	0.047%
$x(t_0)=x(\frac{\pi}{6}+2\sqrt{3}+\frac{\pi+6}{\sqrt{2}})$	0 m	-2.154 E-3 m	2.2 E-3 m

4 Modelization B

4.1 Characteristic of the modelization

For this modelization, one considers two mobile material points according to the following diagram:



The system is perfectly symmetric. One obtains the same formulation as the modelization A if one Indeed chooses normal stiffness of shock equal to half of the stiffness chosen for modelization

A., if one notes: $x_2=-x_3=x$
The reaction force F_{ext} is put in the following form:

During the first phase: $F_{ext}=-K_1(x_2-x_3)=-2K_1 x$
During the second phase: $F_{ext}=-F_{seuil}$
During the third phase: $F_{ext}=-K_2(x_2-x_3-2d_p)=-2K_2(x-d_p)$
During the fourth phase: $F_{ext}=0$

One models the problem with an obstacle of the type `BI_PLAN_Y`.

At initial time, the two material points are in contact with an initial velocity equal to $2m/s$.

One by means of evaluates the quantities obtained following buckling due to the shock key word `FLAMBAGE` of operator `DYNA_VIBRA`, with time schemes `EULER` and `ADAPT_ORDRE2`.
With adaptive time scheme `ADAPT_ORDRE2` one defines (in seconds):

- the time step initial one: `NOT = 0.001`,
- the maximum value of time step: `PAS_MAXI = 0.005`.

4.2 Characteristics of the mesh

Many nodes: 4
Number of meshes: 2 `SEG2`

4.3 Quantities tested and results

One tests the values of the quantities related to the behavior of buckling during the shock.

Identification	Reference	*Aster*	variation
t_{fl}	$\frac{\pi}{6}s$	0.524 S	0.077%
d_p	3 m	2.9984 m	0.052%
$x(t_0)=x(\frac{\pi}{6}+2\sqrt{3}+\frac{\pi+6}{\sqrt{2}})$	0 m	-1.930 E-3 m	1.93 E-3 m

5 Summary of the results

the differences between the solutions obtained with Aster and the analytical solutions are very weak.