

---

## SDND102 - seismic Response of a multimedia nonlinear spring-mass system

---

### Summarized

the problem consists in analyzing the response of a mechanical structure, modelled by two systems masses - spring undamped, subjected to a seismic loading of harmonic type, with possibility of shock.

One tests the discrete element in traction and compression, the computation of the eigen modes and the static modes, the computation of the transient response by nonlinear modal recombination of a structure subjected to an accelerogram (modelization A) as well as computation of the direct transitory seismic response of a nonlinear structure (modelization B).

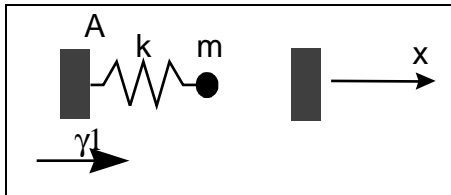
This case test is also used to validate a computation with explicit resolution on accelerations and shock (modelizations C and D) by comparing the results respectively resulting from `DYNA_NON_LINE` with a diagram of implicit time, then clarifies nondissipative central differences and, finally, clarifies dissipative `TCHAMWA`.

The got results are in very good agreement with the results of reference.

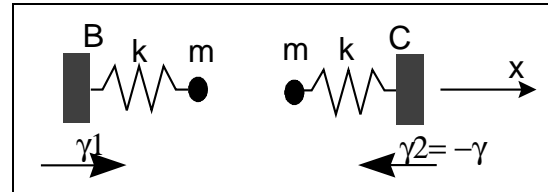
## 1 Problem of reference

### 1.1 Geometry

One compares the seismic response of a spring-mass system with a degree of freedom which can impact a fixed wall (problem 1) with that of two systems mass-spring identical being able to clink and subjected to the same seismic request (problem 2).



Problem 1



Problem 2

### 1.2 Material properties

Stiffness of springs:  $k = 98696 \text{ N/m}$ .  
Point mass:  $m = 25 \text{ kg}$ .

For problem 1 (impact on a rigid wall), the normal stiffness of shock is worth  $K_{choc} = 5,76 \cdot 10^7 \text{ N/m}$ .  
As for problem 2 (shock of two deformable structures), it is worth  $K_{choc} = 2,88 \cdot 10^7 \text{ N/m}$ .  
In both cases, the damping of shock is null.

### 1.3 Boundary conditions and loadings

#### Boundary conditions

only authorized displacements are the translations according to the axis  $x$ .  
The points  $A$ ,  $B$  and  $C$  are clamped:  $dx = dy = dz = 0$ .

#### Loading

the points of anchorage  $A$  and  $B$  are subjected to an acceleration according to the direction  $x$ :  $\gamma_1(t) = \sin \omega t$  with  $\omega = 20 \cdot \pi \text{ s}^{-1}$  and the point  $C$  with an acceleration  $\gamma_2(t) = -\sin \omega t$ .

### 1.4 Initial conditions

In both cases, the systems mass-spring are initially at rest:  
at  $t=0$   $dx(0)=0$ ,  $dx/dt(0)=0$  in any point.

For problem 1, the mass is separated from the fixed wall of clearance  $j = 5 \cdot 10^{-4} \text{ m}$ . As for problem 2, the masses are separated from clearance  $J = 2 \cdot j = 10^{-3} \text{ m}$ .

## 2 Reference solution

---

### 2.1 Method of calculating used for the reference solution

It acts to compare the response of a symmetric system consisted two systems mass-spring identical to the response of a spring-mass system. The two problems, exposed in detail in the reference [bib2], are requested by the same accelerogram.

One initially calculates the eigenfrequencies  $f_i$ , the eigenvectors associated standardized compared to the modal mass  $\Phi_{Ni}$  and the static modes  $\Psi$  with the system (analytical values). One calculates then the generalized response of the system multimedia by solving analytically the integral of Duhamel [bib1]. Lastly, one restores on physical base the relative displacement of the nodes of shock what allows us, after having calculated the field of displacements of training, to calculate the field of absolute displacements.

One calculates the function *diff* defined as being the difference between absolute displacement of the node shocking on a mobile obstacle and that of the node shocking on a fixed obstacle. It is checked that it is well null for various times.

### 2.2 Results of reference

Displacements relative and absolute with the nodes of shock.

### 2.3 Uncertainty on the solution

Comparison between two equivalent modelizations.

### 2.4 Bibliographical references

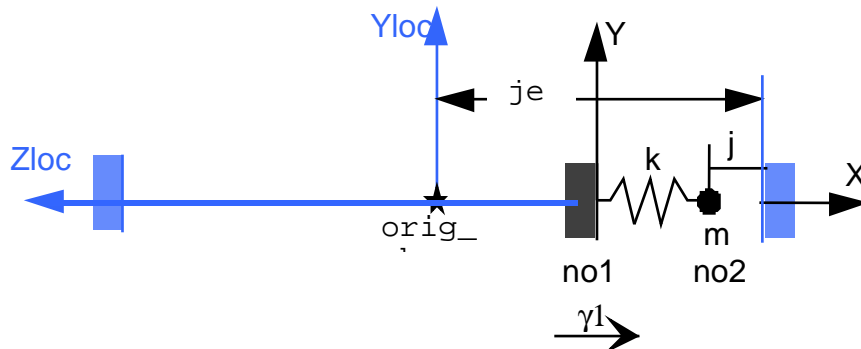
- 1) J.S. PRZEMIENIECKI: Structural Theory of matrix analysis New York, Mac Graw - Hill, 1968, p. 351-357.
- 2) Fe. WAECKEL: Use and validation of the developments carried out to compute: the seismic multimedia structure response - HP52/96.002.

## 3 Modelization A

### 3.1 Characteristic of the modelization

the systems mass-spring are modelled by discrete elements with 3 degrees of freedom `DIS_T`.

#### Modelization of problem 1:



Appear 3.1-a: Modelization of a spring-mass system impacting a rigid wall

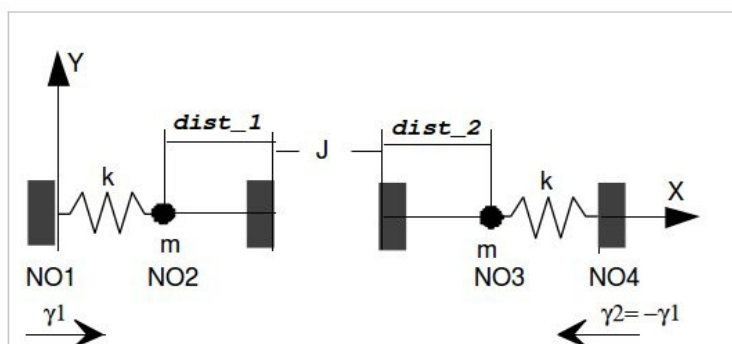
the node `no1` is subjected to an imposed acceleration  $\gamma_1(t)$ . One calculates the relative displacement of the node `no2`, his displacement of training and his absolute displacement.

An obstacle of the type `PLAN_Z` (two parallel planes) is retained to simulate the impact of the spring-mass system on a rigid wall. The norm with the plane of shock is the axis `Z`, `NORM_OBST`: (0. 0. 1.). Not to be constrained by the rebound of the oscillator on the symmetric level, one pushes back very this one far (cf [Figure 3.1-a]).

From where:

- [1] the origin of obstacle `ORIG_OBST`: (- 1. 0. 0.);
- [2] and clearance corresponding `clearance`: 1.1005

#### Modelization of problem 2:



Appear 3.1-b: Modelization of two systems mass-spring which clink

the node `NO1` is subjected to an imposed acceleration  $\gamma_1(t)$ , the node `NO4` with  $\gamma_2(t) = -\gamma_1(t)$ . One calculates the relative displacement of the nodes `NO2` and `NO3`, their displacement of training and their absolute displacement.

The conditions of shock between the two systems mass-spring are simulated by an obstacle of the type `BI_PLAN_Z` (plane obstacle between two mobile structures). The norm with the plane of shock is selected according to the axis `Z`, that is to say `NORM_OBST = (0. 0. 1.)`.

The thickness of matter surrounding the nodes of shock in the direction considered is specified by operands `DIST_1` and `DIST_2`. In the treated case, one chooses `DIST_1 = DIST_2 = 0.4495` so that at initial time, the two nodes of shock are separated from clearance  $J=2 j=10^{-3} mm$  (cf [Figure 3.1-b]).

Temporal integration is carried out with the algorithm of Eulerian and time step of  $2,5 \cdot 10^{-4s}$ . Computations are filed all the 8 time step.

One considers a reduced damping  $\xi$  of 7% for all the calculated modes.

## 3.2 Characteristics of the mesh

One calls `model` the mesh associated with the problem made up of a spring-mass system butting against a fixed wall and `bichoc` that which is associated with problem 2.

Mesh associated with the model `model`:

many nodes: 2;  
number of meshes and types: 1 `DIS_T`.

Mesh associated with the model `bichoc`:

many nodes: 4;  
number of meshes and types: 2 `DIS_T`.

## 4 Results of the modelization A

### 4.1 Values tested of the modelization A

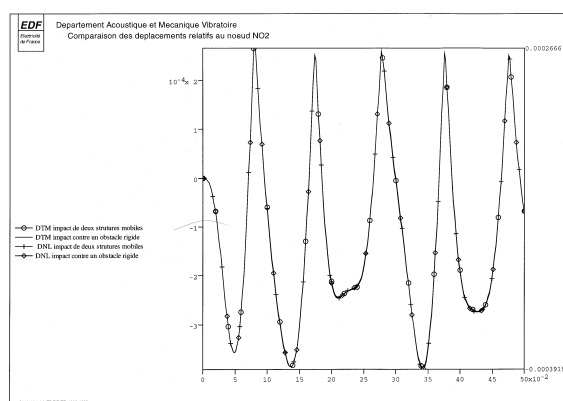
One calculates the function *diff* defined as being the difference between absolute displacement of the node `NO2` and that of the node `no2`. And it is checked that it is well null for various times.

Time (S)	Reference
0,1,0,0,3	
	0,0,0,5,0,0
0,7,0,0	
1,0,0	

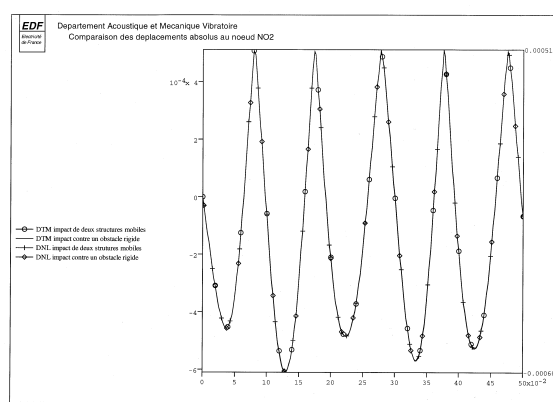
One also tests the value of the absolute displacement of the node *NO2* for various times.

Time (S)	Reference (problem 2)
0,05	- 3,58082E-04
0,156	- 1,22321E-04
0,25	- 1,8876E-04
0,4	- 1,89772E-04
0,5	- 6,84454E-05
0,8	- 1,11982E-04
0,9	- 1,20103E-04
1	- 1,07178E-04

One represents Ci below the pace of displacements relative and absolute with the node *NO2* :



**Absolute displacements**



**relative Displacements**

## 5 Modelization B

### 5.1 Characteristic of the modelization

the systems mass-spring are modelled, as in the modelization A, by a discrete element with 3 degrees of freedom `DIS_T`.

#### Modelization of problem 1:

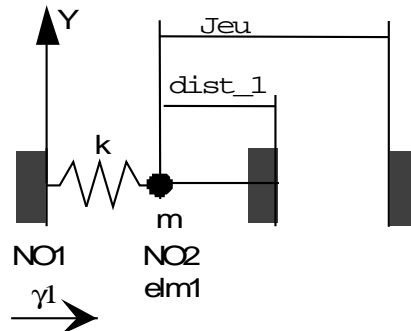


Figure 5.1-a : Modelization of a spring-mass system impacting a rigid wall

the node `NO1` is subjected to an imposed acceleration  $\gamma_1(t)$ . One calculates the relative displacement of the node `NO2`, his displacement of training and his absolute displacement.

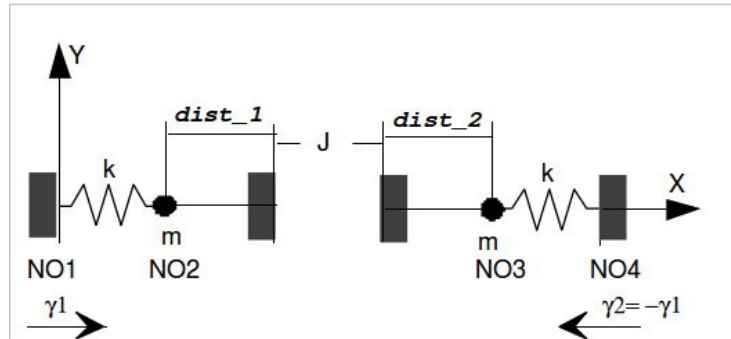
Element of type `a DIST_T` on a mesh `POI1` is retained to simulate the impact of the beam on a rigid wall: the possible shocks between the beam and the obstacle are taken into account as being internal forces with this element. One of the command affects to him a nonlinear behavior of standard shock (stiffness) via constitutive law `DIS_CONTACT DEFI_MATERIAU`.

The thickness of matter surrounding the node of shock in the direction considered is specified by operand `DIST_1` of the command `DEFI_MATERIAU`. In the treated case, one chooses `DIST_1 = 0.4495` and `JEU = 0.45` so that at initial time, the node of shock and the obstacle are separated from clearance  $j = 5 \cdot 10^{-4} \text{ mm}$  (cf [Figure 5.1-a]).

The seismic loading, due to imposed displacements of the node `NO1`, is calculated by the operator `CALC_CHAR_SEISME`. One creates then a concept `charges` from operand `VECT_ASSE` of the command `AFFE_CHAR_MECA`.

One uses the implicit diagram of integration of `NEWMARK` of `DYNA_NON_LINE` with key word `SCHEMA_TEMP` (`FORMULATION=' DEPLACEMENT'`) with time step of  $10^{-3} \text{ s}$  and the default settings.

## Modelization of problem 2:



Appear 5.1-b: Modelization of two systems mass-spring which clink

the node  $NO1$  is subjected to an imposed acceleration  $\gamma_1(t)$ , the node  $NO4$  with  $\gamma_2(t) = -\gamma_1(t)$ . One calculates displacements relative and absolute of the nodes  $NO2$  and  $NO3$ , their displacement of training and their absolute displacement.

The possible shocks between the two beams are taken into account as being internal forces with an element with two nodes. One of the command assigns to this element a nonlinear behavior of standard shock (stiffness) via key word `RIGI_NOR` of constitutive law `DIS_CONTACT` `DEFI_MATERIAU`. The normal direction of contact is the local axis  $x$  of the discrete element with two nodes.

The thickness of matter surrounding the nodes of shock in the direction considered is specified by operands `DIST_1` and `DIST_2` of the command `DEFI_MATERIAU`. In the treated case, one chooses `DIST_1 = DIST_2 = 0.4495` so that at initial time, the two nodes of shock are separated from clearance  $J = 2 \cdot j = 10^{-3} m$  (cf [Figure 5.1-a]).

The seismic loading, due to imposed displacements of the anchorages (node  $NO1$  and  $NO4$ , is calculated by the operator `CALC_CHAR_SEISME`. One creates a concept `charges` from operand `VECT_ASSE` of the command `AFFE_CHAR_MECA`.

Temporal integration is carried out with the algorithm of Newmark and time step of  $10^{-3} s$ . Computations are filed all the 8 time step. One considers a reduced damping  $\xi$  of 7% for all the calculated modes (key word `AMOR_MODAL` of operator `DYNA_NON_LINE`).

## 5.2 Characteristics of the mesh

The mesh associated with the model `bichoc` consists of 4 nodes and 3 meshes of type `DIS_T`.



## 6 Results of the modelization B

### 6.1 Values tested of the modelization B

One calculates the function *diff* defined as being the difference between absolute displacement of the node *NO2* and that of the node *no2*. And it is checked that it is well null for various times.

Time (S)	Reference
0,1.0,0.0,2	
	0,0.0,3.0,0
0,4.0,0.0,5	
	0,0

One also tests the maximum value of the force of impact to the node *NO2*.

Type of impact	Reference
against a rigid wall	6,29287E+02
between two mobile structures	6,29287E+02

One also tests the values of the absolute fields to the node *NO2* and at time  $t=0.01$ .

Standard	field	Reference	Tolerance
DEPL_ABSOLU	-1.488877E-004	NON REGRESSION	1,00E-010
VITE_ABSOLU	-1.287591E-002	NON REGRESSION	1,00E-010
ACCE_ABSOLU	5.877853E-001	NON REGRESSION	1,00E-010

One also tests functionality *OBSERVATION*. *L* be absolute fields with the node *NO2* and at time  $t=0.01$  must be identical to the preceding fields:

Standard	field	Reference	Tolerance
DEPL_ABSOLU	-1.488877E-004	NON REGRESSION	1,00E-010
VITE_ABSOLU	-1.287591E-002	NON REGRESSION	1,00E-010
ACCE_ABSOLU	5.877853E-001	NON REGRESSION	1,00E-010

One tests finally option *SUIVI\_DDL* by visually comparing the values obtained with those extracted the array of *OBSERVATION* generated. These checks relate to displacement and the velocity (fields *DEPL* and *QUICKLY*) with the node *NO2* at time  $t=0.1$ . One tests also the option *MIN* of *SUIVI\_DDL* on mesh group *RESSORT1* at the same time. This time was selected so that the minimal value of the fields of displacement and velocity on this group of mesh is obtained with the node *NO2*.

For displacement one thus finds the value:  $-3.99791E-05 m$  and for the velocity the value:  $-1.51040E-02 m/s$ .

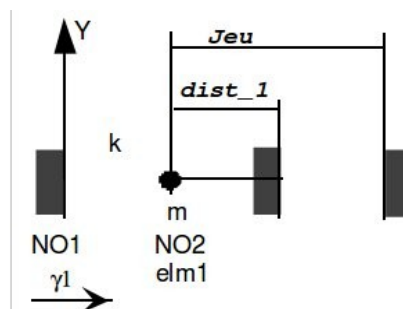
## 7 Modelization C

### 7.1 Characteristic of the modelization

The modelization C is before a whole test of `DYNA_NON_LINE` with key word `SCHEMA_TEMPS` (`FORMULATION='ACCELERATION'`), whose results are compared with `DYNA_NON_LINE` with key word `SCHEMA_TEMPS` (`FORMULATION='DEPLACEMENT'`).

The systems mass-spring are modelled, as in the modelization A, by a discrete element with 3 degrees of freedom `DIS_T`. Only the modelization with a degree of freedom is tested.

**Modelization of problem:**



**Appear 7.1-a: Modelization of a spring-mass system impacting a rigid wall**

the node `NO1` is subjected to an imposed acceleration  $\gamma_1(t)$ . One calculates the relative displacement of the node `NO2`, his displacement of training and his absolute displacement.

Element of type `a DIST_T` on a mesh `POI1` is retained to simulate the impact of the beam on a rigid wall: the possible shocks between the beam and the obstacle are taken into account as being internal forces with this element. One of the command affects to him a nonlinear behavior of standard shock (stiffness) via constitutive law `DIS_CONTACT DEFI_MATERIAU`.

The thickness of matter surrounding the node of shock in the direction considered is specified by operand `DIST_1` of the command `DEFI_MATERIAU`. In the treated case, one chooses `DIST_1 = 0.4495` and `JEU = 0.45` so that at initial time, the node of shock and the obstacle are separated from clearance  $j = 5 \cdot 10^{-4} \text{ mm}$  (cf [Figure 5.1-a]).

The seismic loading, due to imposed displacements of the node `NO1`, is calculated by the operator `CALC_CHAR_SEISME`. One creates then a concept `charges` from operand `VECT_ASSE` of the command `AFFE_CHAR_MECA`.

One uses the diagram of explicit integration of `NEWMARK` of Central differences type with time step of  $10^{-3} \text{ s}$ . The computation by `DYNA_NON_LINE` with key word `SCHEMA_TEMPS` (`FORMULATION='ACCELERATION'`) is carried out in modal space, non-linearity being due to the shock and thus resident local.

### 7.2 Characteristics of the mesh

The mesh associated with the model consists of 2 nodes, a mesh `SEG2` of the type `DIS_T` and of a specific mesh `POI1` of the type `DIS_T`.

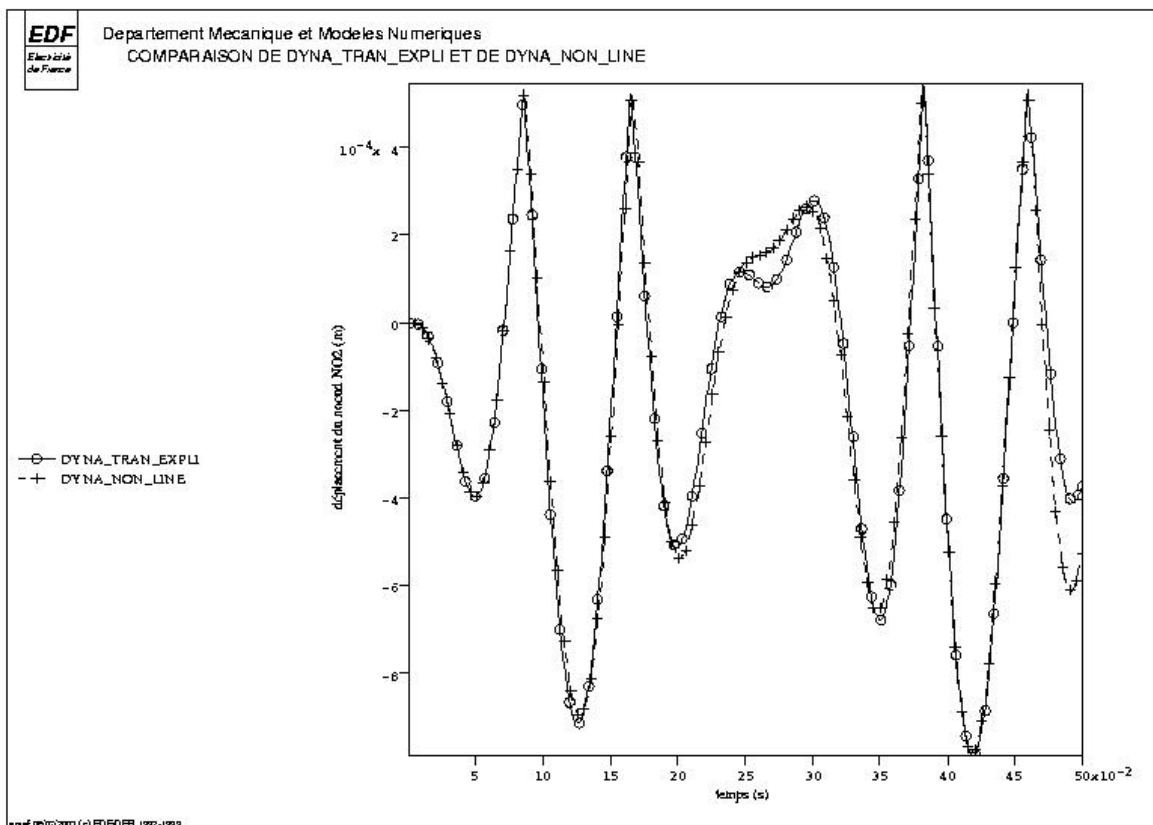
## 8 Results of the modelization C

### 8.1 Values tested of the modelization C

The computation is nonlinear because of shock and one does not have analytical solution. One thus tests computation on values of NON-regression on displacement according to  $x$  node *NO2*.

Time (S)	Reference
0,1	- 15,6520E-3
0,2	- 51,4832E-3
0,3	28,1291E-3
0,4	- 44,9343E-3
0,5	- 37,7508E-3

One compares absolute displacements resulting from `DYNA_NON_LINE` with key word `SCHEMA_TEMPS` (`FORMULATION='ACCELERATION'`) with those given by `DYNA_NON_LINE` with key word `SCHEMA_TEMPS` (`FORMULATION='DEPLACEMENT'`).



## 9 Modelization D

---

### 9.1 Characteristic of the modelization

The modelization D is an alternative of the modelization C where one will replace the nondissipative time scheme of the central differences by the dissipative explicit diagram of TCHAMWA (with the parameter  $\text{PHI} = 1.05$ ). Unlike the modelization C, one solves here on physical base and not on modal base (key word `PROJ_MODAL` of `DYNA_NON_LINE` is not thus more present in the modelization D).

## 10 Results of the modelization D

---

### 10.1 Values tested of the modelization D

The computation is nonlinear because of shock and one does not have analytical solution. Compared with the modelization C the values obtained are different because the time scheme introduced a numerical dissipation. One tests computation on values of NON-regression on displacement according to  $x$  of the node *NO2*.

Time (S)	Reference	Tolerance
0,1	- 15,6911E-3	0.10%
0,2	- 49,4505E-3	0.10%
0,3	30,2638E-3	0.10%
0,4	- 38,0509E-3	0.10%
0,5	- 39,2295E-3	0.10%

the fact of changing time scheme and of introducing an additional dissipation, modifies some of the instantaneous values tested of about a 10%.

## 11 Summary of the results

---

the results got with *Code\_Aster* are in conformity with those expected (error lower than the thousandths).

On this example, direct nonlinear computation is much more expensive in computing times, of a factor 20, that on modal base.

The modelization C watch which one obtains many similar results with an explicit temporal integration method with key word `SCHEMA_TEMPS (FORMULATION=' ACCELERATION')` and implicit (`DYNA_NON_LINE` with key word `SCHEMA_TEMPS (FORMULATION=' ACCELERATION')`).

The modelization D proves that one gets also close results with a dissipative explicit temporal integration method of `TCHAMWA` (the variation being due to this added numerical damping).