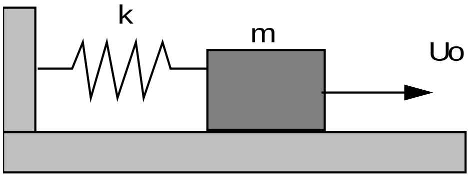
SDND100 - To release of a shoe rubbing with friction of Summarized the Coulomb

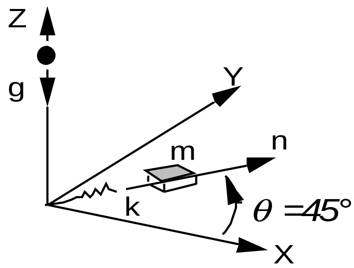
type

One considers the one-way system with a degree of freedom made up of a mass in contact rubbing of Coulomb type on a rigid level, and of a spring attaching it to a fixed point. The mass is released in an initial position except equilibrium. It oscillates until the complete stop at the end of a finished time.
The first two modelizations correspond to the transient response by modal recombination of the rubbing shoe, the third corresponds to its direct transient response. Three computations are compared with the analytical solution.

1 Problem of reference

1.1 Geometry





Direction of displacement: $\theta = 45°$ in the plane XY

1.2 Material properties

Stiffness of spring:	$k = 10\,000\,N/m$
Point mass:	$m = 1\,kg$
Gravity:	$g = 10\,m/s^2$
Coefficient of Coulomb:	$\mu = 0{,}1$

1.3 Boundary conditions and loadings

the system rests on the plane $Z = 0$ on which it can slip with a coefficient of kinetic friction of Coulomb of $\mu = 0{,}1$.

1.4 Initial conditions

initial Displacement of the mass: $r_0 = 0{,}85\,mm$ according to the direction θ .

Initial velocity null.

2 Reference solution

2.1 Method of calculating used for the reference solution

For a system without damping, the differential equation to solve is written:

$$\begin{cases} m\ddot{r}+k\,r=\mu|F_n| \\ r(t=0)=r_0\geq 0 \\ \dot{r}(t=0)=0 \end{cases} \qquad \text{with } F_n=-mg\,sign(\dot{r})$$

One shows [bib1] that the solution of the differential equation is written:

$$r(t)=\frac{\mu|F_n|}{k}+(r_0-\frac{\mu|F_n|}{k})\cos\omega_0 t$$

The amplitude of the extrema, which all come them $t_{n+1}=\frac{n\pi}{\omega_0}$, obeys the model of following recurrence:

$$r(t_{n+1})=(-1)^n\left[r_0-\frac{\mu|F_n|}{k}\right]\cos\omega_0 t$$

with $n=1,2,\ldots,N$ such as $\left|\frac{r(t_{n+1})}{r_0}\right|<\frac{\mu|F_n|}{k\,r_0}$

motion stops when $\left|\frac{r(t_{n+1})}{r_0}\right|<\frac{\mu|F_n|}{k\,r_0}$ with the position $r(t_{n+1})$.

2.2 Results of reference

Values of displacements in the direction θ for times of change of sign velocity ($r(t_1),r(t_2),\ldots,r(t_5)$ established above).

2.3 Uncertainty on the analytical

solution Solution.

2.4 Bibliographical references

F. AXISA - Méthodes d'analyze in nonlinear dynamics of structures: non-linearities of contact - Courses IPSI from May 28th to May 30th, 1991

3 Modelization A

3.1 Characteristic of the modelization

element of type a `DIS_T` on a mesh `POI1` are used to model the system.
Conditions of relations between degrees of freedom are employed to impose with motion being one-way in the direction θ :

```
LIAISON_DDL = _F (NOEUD = ("NO1", "NO1"),
                  DDL = ("DX" "DY"),
                  COEF_MULT = (0.707, -0.707),
                  COEF_IMPO = 0.)
```

An obstacle of the type `PLAN_Z` (two parallel planes separated by a clearance) is used to simulate the slip surface. One chooses to take for generator of this plane the axis Oy , that is to say `NORM_OBST = (0. , 1. , 0.)`. The origin of the obstacle is `ORIG_OBST = (0. , 0. , 1.)`. It remains to define its clearance which gives the half - spacing between the planes.

So that there exists a reaction force of the plane on the system it is necessary that this last is slightly inserted in the plane obstacle of a distance δn such as: $F_n = K_n \cdot \delta n$.
Like $F_n = mg$, one has then $\delta n = mg / K_n$.
One has considered stiffness of shock normal of $20\,N/m$ (fictitious stiffness which has meaning only to generate a reaction force of the plane on the system), one thus has $\delta n = 0{,}5$. Obstacle `PLAN_Z` having for origin $Z=1$ and the solid being in $Z=0$; a clearance of $0{,}5\,m$ will create a depression $\delta n = 0{,}5\,m$ from where `JEU: 0.5`

tangential Stiffness of shock: $K_T = 400\,000\,N/m$: it is large in front of the stiffness of the oscillator so that the phase of stop is modelled correctly.

Time step used for temporal integration: 5.10^{-4s} .

3.2 Characteristics of the mesh

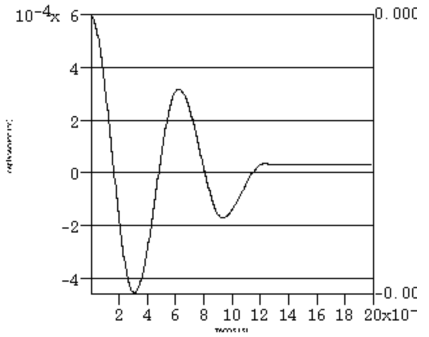
Many nodes: 1
Number of meshes and types: 1 `POI1`

3.3 Quantities tested and Values

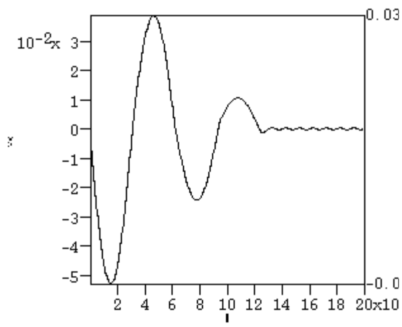
results of displacements (in meters) in the direction θ for times of change of sign the velocity over the period of time $(0;0.3s)$.

Identification	time (S)	Reference
$DY=r2\,cos45$	$\pi\times10^{-2}$	– 4.596E-4
$DY=r3\,cos45$	$2\pi\times10^{-2}$	3.182E-4
$DY=r4\,cos45$	$3\pi\times10^{-2}$	– 1.768E-4
$DY=r5\,cos45$	$4\pi\times10^{-2}$	3.536E-5

One presents Ci below the evolution of displacement and the velocity at the point $NO1$



Displacement of the point $NO1$



Velocity of the point $NO1$

4 Modelization B

4.1 Characteristic of the modelization

In the modelization B, one regards the shoe and the plane as two mobile structures. Each structure is then modelled by a node and element of type a POI1. The node $NO2$ is supposed to be blocked, it materializes the plane of friction. One imposes conditions of relations between degrees of freedom on the node $NO1$ (which models the shoe) so that motion is one-way in the direction θ .

An obstacle of the type BI_PLAN_Z (two mobile parallel planes separated by a clearance) is used to simulate the slip surface. One chooses to take for generator of this plane axis OY, that is to say NORM_OBST = (0. , 1. , 0.) . By default, the origin of the obstacle is located at semi distance from the nodes $NO1$ and $NO2$. It remains to define parameters DIST_1 and DIST_2 which represent the thickness of matter around the nodes of shock.

So that there exists a reaction force of the plane on the system it is necessary that this last is slightly inserted in the plane obstacle of a distance δn such as: $F_n = K_n \cdot \delta n$.
Like $F_n = mg$, one has then $\delta n = mg / K_n$.
One has considered stiffness of shock normal of $20 \, N/m$ (fictitious stiffness which has meaning only to generate a reaction force of the plane on the system), one thus has $\delta n = 0{,}5 \, m$. Knowing that the two nodes $NO1$ and $NO2$ are geometrically confused, one chooses for example DIST_1 = DIST_2 = $\delta n / 2$.

Tangential stiffness of shock: $K_T = 400\,000 \, N/m$: it is large in front of the stiffness of the oscillator so that the phase of stop is modelled correctly.

Time step used for temporal integration: $5.10^{-4} s$.

4.2 Characteristics of the mesh

Many nodes: 2
Number of meshes and types: 2 POI1

4.3 Quantities tested and Values

results of displacements (in meters) in the direction of the oscillator for times of change of sign the velocity over the period of time $(0 ; 0.3 s)$.

Identification	time (S)	Reference
$DY = r2 \cos 45$	$\pi \times 10^{-2}$	– 4.596E-4
$DY = r3 \cos 45$	$2\pi \times 10^{-2}$	3.182E-4
$DY = r4 \cos 45$	$3\pi \times 10^{-2}$	– 1.768E-4
$DY = r5 \cos 45$	$4\pi \times 10^{-2}$	3.536E-5

5 Modelization C

5.1 Characteristic of the modelization

This modelization corresponds to the direct transient response of the rubbing shoe.

The normal direction of contact is the local axis X which corresponds in the case test to the total axis Z. The slip surface is the local plan (Y,Z) is the plane (X,Y) in the total reference. One thus directs the element of shock to a node, with key word ORIENTATION of operator AFFE_CARA_ELEM in the following way:

```
ORIENTATION= _F (MAILLE=' EL1',  CARA: "VECT_X_Y",
                 VALE = (0. , 0. , -1. , 0. , 1. , 0. ))
```

to be able to obtain a reaction force of the plane on the system it is necessary that this last is slightly inserted in the plane obstacle of a distance δn such as: $F_n = K_n \cdot \delta n$.
The reaction balances the weight of the shoe, one thus has: $F_n = mg$ i.e $\delta n = mg / K_n$.
One has considered stiffness of shock normal of $20\,N/m$ (fictitious stiffness which has meaning only to generate a reaction force of the plane on the system), one thus has $\delta n = 0{,}5$ from where DIST_1 = 0.5.

The tangential stiffness of shock considered is $K_T = 400\,000\,N/m$, the coefficient of Coulomb is worth 0,1.

The constitutive law of shock is thus in the following way defined in DEFI_MATERIAU :

```
DIS_CONTACT =_F (RIGI_NOR= 20. ,
                  DIST_1 = 0.5,
                  RIGI_TAN = 400000. ,
                  COULOMB = 0.1)
```

One time step uses one $5.10^{-4} s$ for temporal integration.

5.2 Characteristics of the mesh

Many nodes: 1
Number of meshes and types: 1 POI1

5.3 Quantities tested and Values

results of displacements in the direction of the oscillator for approximate times of change of sign the velocity over the period of time $(0 ; 0.2 s)$.

Identification	times (S)	Reference
$DY = r2\ cos45$	$\pi \times 10^{-2}$	– 4,585E-04
$DY = r3\ cos45$	$2\pi \times 10^{-2}$	3,173E-04
$DY = r4\ cos45$	$3\pi \times 10^{-2}$	– 1,754E-04
$DY = r5\ cos45$	$4\pi \times 10^{-2}$	3,550E-05

6 Summary of the results

the analytical solution of the problem with friction is reproduced with a very good accuracy ($<0.5\%$). That compared to the asks for nevertheless the use of a parameter of tangent stiffness rather high stiffness of the system as well as time step of integration relatively reduced.
On this example, direct nonlinear computation is much more expensive in computing times than that on modal base.